Kindergarten Activities for Early Mathematics

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Abstract

This paper focuses on activities and games for Kindergarten that follows the Singapore approach. Major themes of Kindergarten Mathematics are explored.

Introduction

Some people, including mathematicians, think that Kindergarten Mathematics is light and not very important; "serious" Mathematics only starts in the first grade. This idea is far from the truth.

Exploring Mathematics can help the young children satisfy and deepen their natural curiosity about the numbers, sizes, shapes, and relationships they encounter in everyday life. Mathematics in kindergarten classrooms begins with play. As the children interact with one another, mathematical situations develop naturally. Although young children can and do learn from play and from day-to-day life experiences, educators cannot assume that mathematical learning will automatically unfold. It must be planned. Kindergarten lays a vital foundation for children's mathematical learning and reasoning, which can be decisive for their future relation with Mathematics.

So, Kindergarten Mathematics can be considered light in the sense that the younger ones naturally learn Mathematics from play-based activities and day-to-day life experiences. However, it is really very important and it must be carefully planed. Initial Mathematics can be, at same time, easy and sophisticated. Important seminal concepts should be treated with young children (3-5 years old).

Some educational systems integrate important steps related to Kindergarten Mathematics. A paradigmatic case is the well known *Singapore Math*, which refers to the instruction and curriculum for teaching Mathematics in the small island country. This approach has become popular due to its top rankings on the international assessment of student math achievement TIMSS (*Trends in International Mathematics and Science Study*). Singapore has been among the top ranking countries in that assessment since 1995.

In Figure 1, we outline the four basic principles of the Singapore approach in Kindergarten.

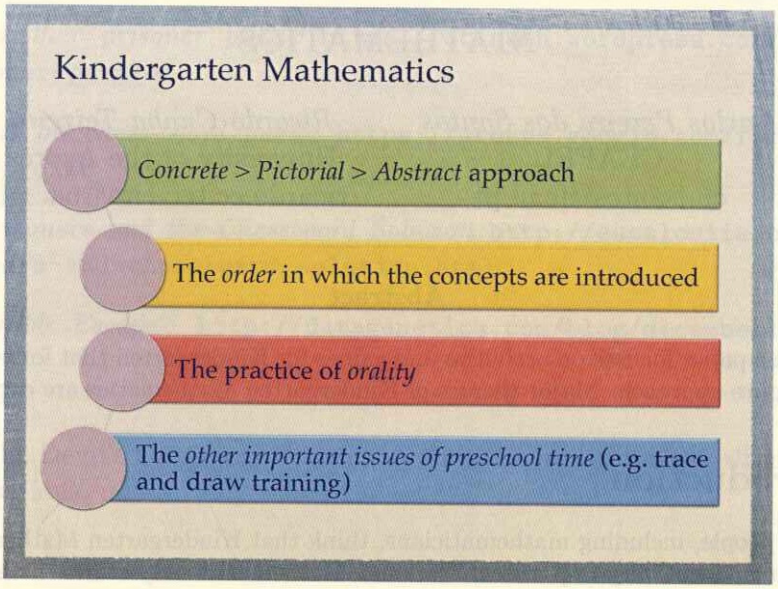

Figure 1: The 4 principles of Kindergarten Mathematics (Singapore approach).

Generally speaking, the order in which the issues are treated is crucial. No "jumps" are allowed; if B needs A, it is fundamental to work A first. Also, the practice of orality is encouraged; we think with words! Children must be able to build complete sentences, explaining their ideas. The concrete-pictorial-abstract approach is very appropriate when we think about Mathematics. Care with the transition from concrete to abstract is needed since it is part of the nature of Mathematics, whatever its level of application. Finally, the other important issues of preschool time should be integrated in the activities (e.g. trace and draw training).

In Kindergarten, special attention is paid to 8 major subjects (see Figure 2). Activities should be well planned, having a playful component suited to 3-5 years old kids. This is a professional and skilled work since activities should be developed and tested over the years.

In the following sections, some examples designed by students of the Master Course in Preschool Education and Elementary Teaching (University of the Azores, Portugal) are shown. The order of the sections follows the list of the subjects mentioned in Figure 2, inspired by the Singapore Math. In the final section, we explore some games for kindergarten children.

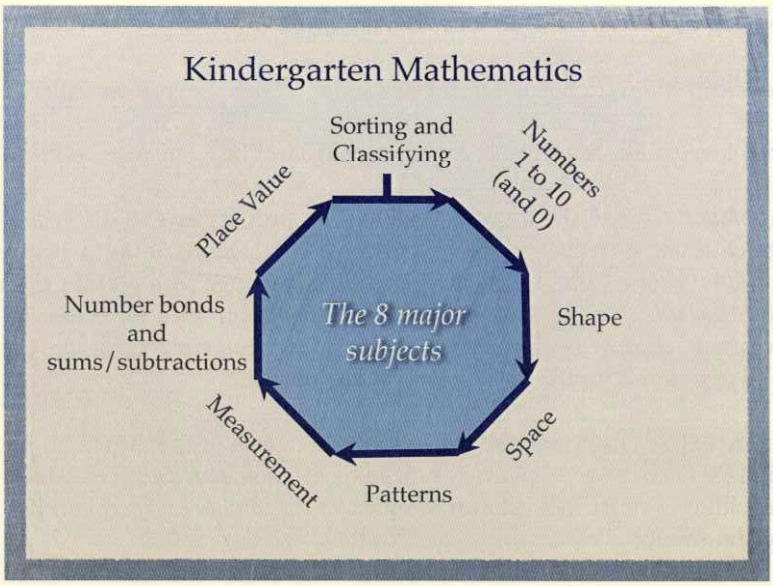

Figure 2: The 8 subjects of Kindergarten Mathematics (Singapore approach).

Sorting and Classifying

Children at this stage (age 3):

- ◇ recognize objects and their properties;
- ◇ sort objects according to several criteria;
- ◇ recognize criteria;
- ◇ establish criteria.

All occasions are good to improve vocabulary (kid's home, school, street, vehicles, circus, animals, farm, human body, hygiene, five senses, ...). Imagination is mandatory! We must collect a good number of different environments, diversifying the activities.

Everything must be done by giving the utmost importance to orality, tracing skills (matching tasks) and drawing/painting skills. When facing a common matching task, we should say: "First with your finger, only then with your pencil". With this easy but important procedure, we can understand how the child is thinking (providing a kind of training to the child). Encouraging imagination throughout the establishment of sorting criteria is very important.

When planning the activities, we must think about the dialogue with the children. The educator should make questions that can promote a nice and rich conversation. It is important to think in advance in children's responses... Each image should be chosen with a clear and defined purpose. For example, in matching tasks, the image should present many interesting possibilities. This allows

you to keep alive the dialogue with the children.

The educator should choose logical examples. Logic is very useful! For example, a red cat to teach colors is something very weird. Why not a strawberry? Children love to apply logical-mathematical reasoning, and they do it well.

An important part of the activities related to properties and criteria is to encourage children to verbalize their ideas presenting more detailed descriptions. For example, if one child states that an object belongs to a certain group, it is important to ask: "What this thing have in common with all these objects?". On the other hand, it is also crucial to focus children's attention on the differences: "Why this object does not belong to this set?".

It is important to encourage the students to recognize properties in objects. The first kind of activity to explore is normally a "look and talk" one (see Figure 3). The children must look at two animals in the image and explain if they are exactly the same.

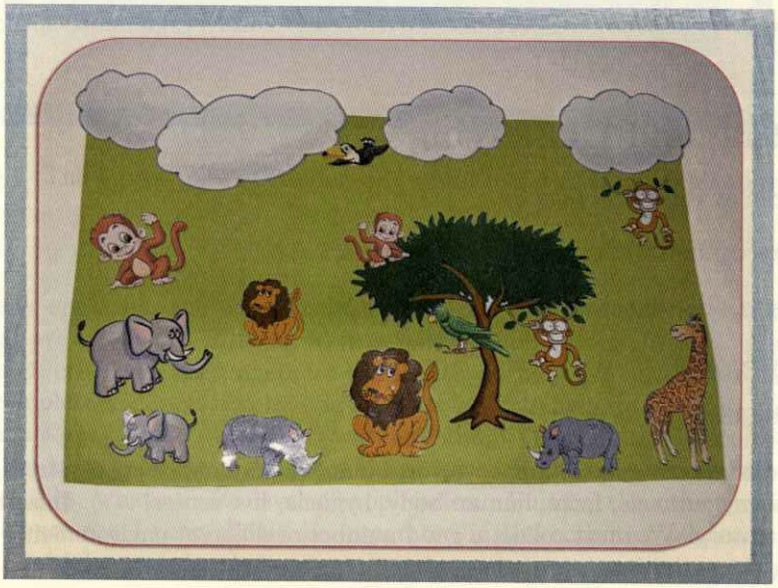

Figure 3: Look and talk.

In the next activities, children must sort objects according to some criteria (see Figure 4). Ask the children to use their finger to show where each object belongs. These activities require children to organize items into groups based on a common characteristic such as size, color, shape, texture, or flavor and also explain why they grouped the items as they did.

Other important activities involve pairwising the objects (see Figure 5). Ask the students to use their index finger to match the objects.

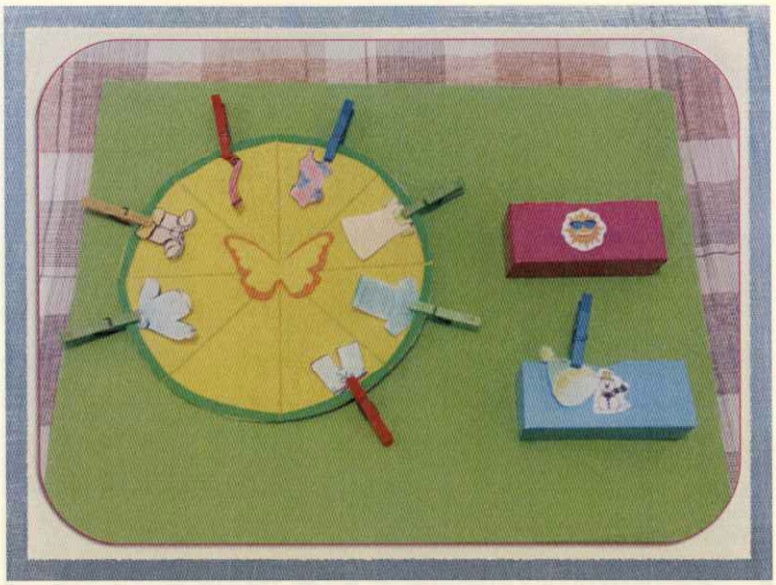

Figure 4: Sort the clothes (Winter/Summer).

Figure 5: Match the objects accordingly to their patterns.

Some important explorations are based on the challenge: "One of these things is not like the others. Can you find it?". Start by introducing a group of four items in which all items but one are exactly the same. The children must recognize and establish criteria to identify the object that is different from the others, with increasing difficulty levels: "One object, one criterion"; "One object, more than one criterion"; and, finally, "More than one object" (see Figure 6).

Does the imagination have a limit for this kind of activities? Actually, imagination can fly to the horizon! Another example of a "Not like the others" activity: a soup, a tea, a spaghetti and an ice cream.

Numbers 1 to 10 (and 0)

Children at this stage (ages 3-4):

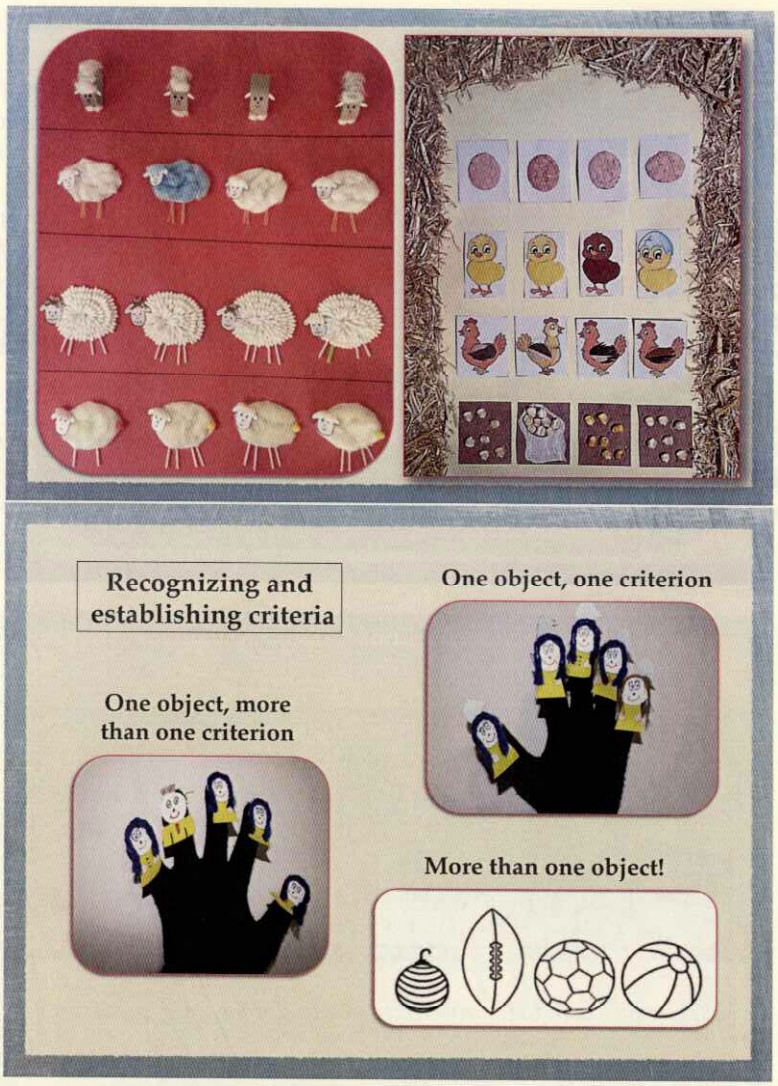

Figure 6: "Not like the others" activities: different levels of difficulty.

- ◇ count objects (cardinality);
- ◇ recognize and write numerals (1–10 and 0);
- ◇ sort objects, using the ordinals (ordinality).

There are some important aspects related to counting objects [1]: Stable order; One-to-one correspondence; Cardinality; Order irrelevance principle; and Abstraction. Initially, the counting words are simply "a song to sing". The educator must say "Count out loud" and "Point at objects with your index finger". Also, the child should understand the basic abstraction related to the numbers, that is, everything can be counted. "The counting" does not belong to specific

objects, such as pencils or strawberries.

In the first activity, the children must count the objects and mark the corresponding numeral (see Figure 7). Ask the children to count out loud as they point to each object.

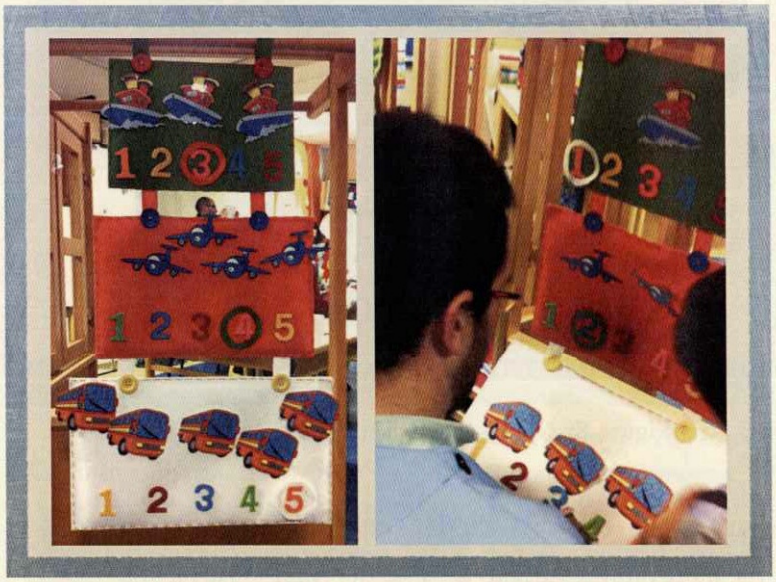

Figure 7: Count the objects and mark the corresponding numeral.

In Figure 8, there's a similar activity in which the children have to count the objects and color the boxes.

It is important for the children to develop the recognition of small quantities without counting (*subitizing*). If we attend to the arrangement of the items, the question may be visual and not a counting task. Several studies showed that we can determine the *exact* number of items in a set, for small amounts, up to 3 or 4 objects, only with the eyes, without counting [2]. The educator should vary gradually the arrangements of the objects in order to allow different explorations of the same amount. For young children, objects in a line are easiest, then rectangular arrangements (pairs of objects in rows) and dice or domino arrangements, then scrambled arrangements.

It is also important to vary the objects to count, using not only images on paper, but also 3D objects and sounds. It is easier for a child to count 3D objects, since he can move the objects already counted. Occasionally, the educator should mix various types of objects, for example, oranges, bananas and pears, and ask the child to count only the pears.

In Figure 9, some activities exploring the concrete-pictorial-abstract approach are presented.

Figure 8: Count the objects and color the boxes.

About the numeral recognition/writing, the child must be able

- ◇ to identify the starting point of the writing;
- ◇ to write the numerals on dotted worksheets.

It is important to use dotted worksheets with large symbols, in order to illustrate the *motor plan* to be performed in each numeral (see Figure 10).

Again, we should prefer logical examples. For instance, there is *one* (1) sun; There are *two* (2) eyes; A tricycle has *three* (3) wheels; A car has *four* (4) wheels; We have *five* (5) fingers on each hand; *Six* (6) eggs in a box; *Seven* (7) musical notes; An octopus has *eight* (8) legs; *Nine* (9) months of pregnancy. Note that there are two numerals that can cause confusion (6 and 9), so the educator should pay particular attention to this particular situation.

It is also fundamental to explore "inverse situations". A possible example: show the numeral 3 and invite the child to draw 3 marbles. We highlight the following suggestions with respect to the writing of the numerals:

- ◇ First, encourage the child to "make the numerals" in the air;
- ◇ Then encourage the child to simulate the gestures with the index finger over the table;
- ◇ Finally, encourage the child to write several things on dotted worksheets (circles and lines), before the numerals.

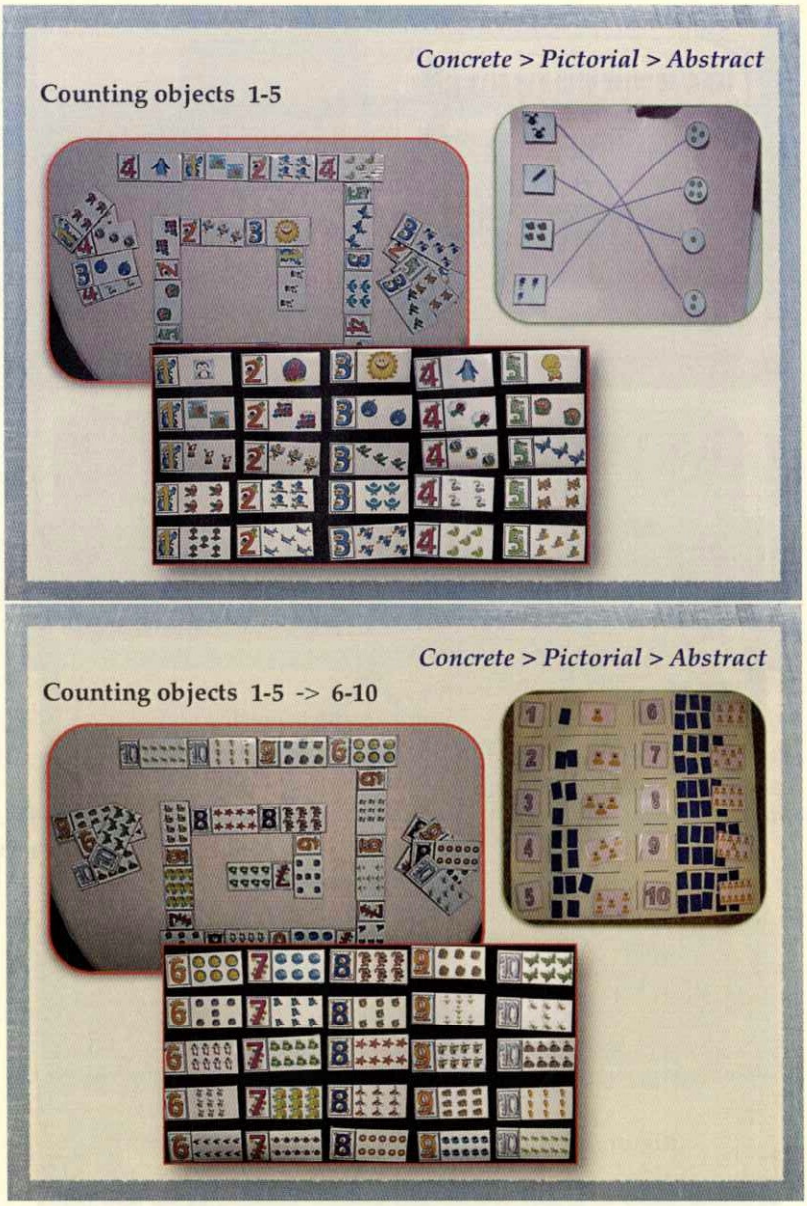

Figure 9: Concrete > Pictorial > Abstract approach.

The number zero should be introduced separately. If not, children can start counting from zero and this is not good... The zero should appear after the numbers $1-10$: for example, showing images and asking for the number of carrots, with no carrots. Further interesting activities are illustrated in Figure 11.

By 4 years of age, the child learns about ordinal relations ("first", "second", ..., "last", "before", "after", "previous", "next", ...).

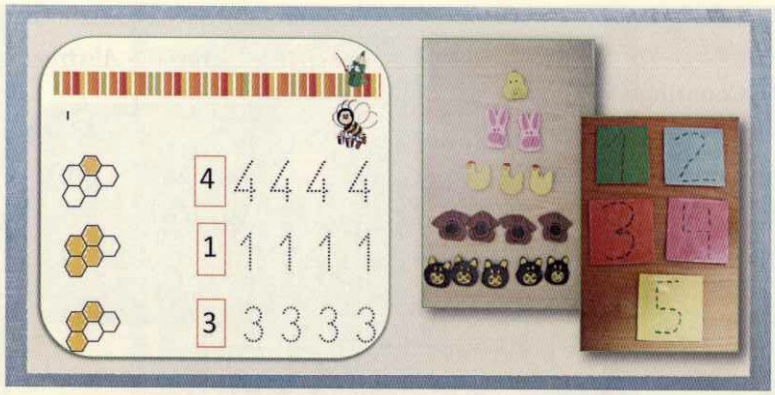

Figure 10: Count, match and trace the numerals.

Figure 11: Counting objects: introducing zero.

Cardinality is related to ordinality and the educator should focus on that fact. For example, "Starting here (starting point of the order), I'll count 1, 2, 3 (pointing with the index finger); Three; this is the third!". The educator should encourage the child to know and use terms of order. The child must be able

- ◇ to label objects (with a not clear correspondence);
- ◇ to count ordinally;
- ◇ to count in descending order, alternately, ...

In Figure 12, some activities are presented.

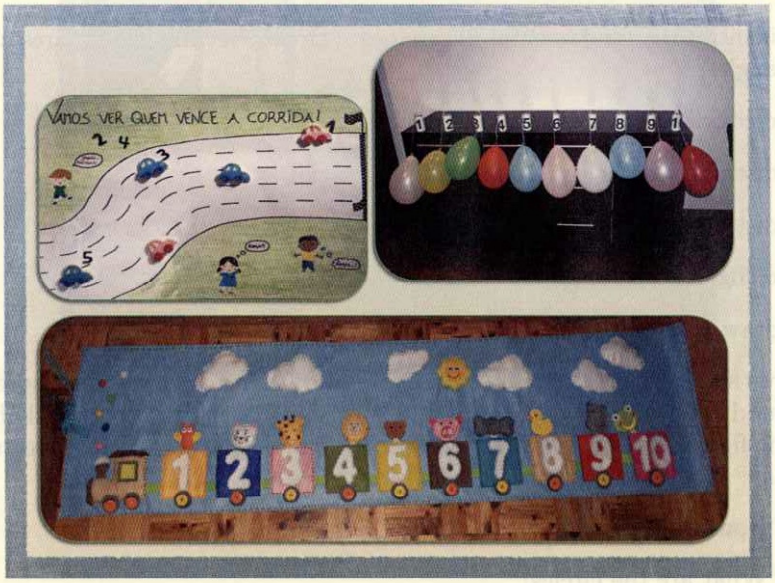

Figure 12: Ordering numbers.

Also, the child must be able to sort sequences of events. It is important to choose examples with daily life situations (see Figure 13).

Figure 13: Ordering events.

All the ideas covered in this section provide an introduction to the well known concept of *number sense*. In fact, *number sense* is an integrated understanding of number that we upgrade lifelong. The kindergarten child should recognize numerals and number words (cardinals and ordinals), learn to count objects and to draw the numerals, relate zero to "nothing", learn to order objects in a collection, and so on.

Shape

Children at this stage (ages 3-4):

- ◇ identify the fundamental 2D shapes – circles, rectangles, squares and triangles;
- ◇ recognize 2D shapes in everyday life objects;
- ◇ distinguish between square and rectangle;
- ◇ distinguish between 2D and 3D shapes;
- ◇ recognize 2D shapes under sensory stimulation;
- ◇ draw 2D shapes.

Children should learn typical "shape vocabulary". Considering Singapore Math as a good inspiration, the images used in the activities are very well thought out; the figures have different 2D shapes, aiming to promote nice conversations, such as:

- ◇ "How are the spots of the giraffe?"
- ◇ "What are the circles?"

Also, "Look at the forms and tell what they are." is not enough (as in so many books for children...). The identification of the 2D shapes in everyday life objects, describing them with complete sentences is a much better option ("That door is a rectangle!"; "That lamp is a circle!").

Children can assign colors to shapes in a logical way. For example, yellow for the Sun (circle). The shapes should also be recognized under sensory stimulation. For example, a child (or the educator) with the index finger "draws" a shape on the back of another child; the goal is to have a guess about the drawn shape. Another interesting example: give a closed rope to two children and ask for a square, a rectangle and a triangle. This kind of tasks paves way for a more abstract reasoning (three sides and three corners to identify a triangle, for example). The motor skills related to the drawing of fundamental shapes are also very important. In Figure 14, some possible explorations are presented.

The distinction "square *versus* rectangle" should be simplified. The inclusion concept is difficult for children, as evidenced by several studies. A famous experiment consists in showing 3 lemons and 5 apples to a young child and ask: "Which has more, apples or fruits?". Many will give a wrong answer... Therefore, the straightforward statement, "All squares are rectangles, but not all rectangles are squares", usually is a bad idea. We should adapt the *sentence-type* to the age of the children. A good example is: "That rectangle is not a square, it has a much bigger side than the other".

"Match and mark" activities are also very common in Singapore Math (an example is given in Figure 15). For each row, the child must find the object with the same shape as the 2-dimensional form in the left column. After this task, he has to match the objects left over in the different rows.

Figure 14: Recognizing 2D shapes.

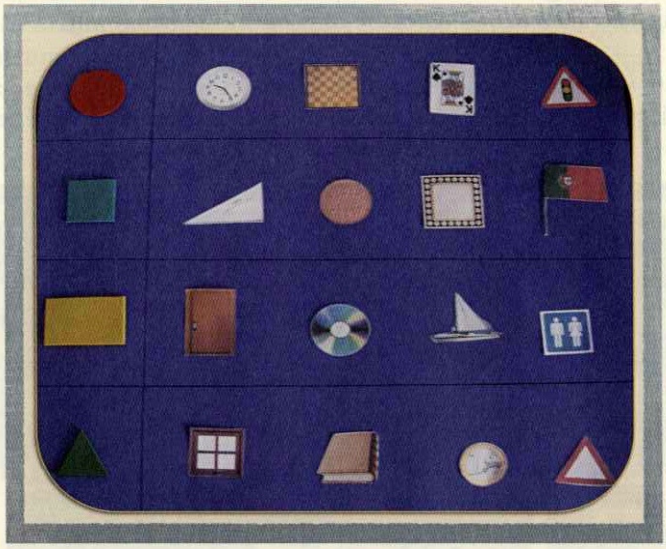

Figure 15: Match and mark.

Space

Children at this stage (ages 4-5):

- ◇ identify the fundamental 3D shapes: cubes, cuboids, cones, cylinders and spheres;
- ◇ recognize 3D shapes in everyday life objects;

- ◇ know and use spatial location vocabulary: "inside/outside", "on top", "below/above", "in front/in back", "up/down", "next to", "near/far" and "left/right";
- ◇ know and use motion words: "to roll", "to jump", "to slide", "to swing", "to pull", "to twist", "to throw" and "to balance";
- ◇ learn to orientate-build-transform objects;
- ◇ distinguish between 2D and 3D shapes;
- ◇ distinguish between circle and sphere;
- ◇ recognize 3D shapes under sensory stimulation.

Children should learn spatial vocabulary and relate it to everyday life situations, for instance: "That toy is a cube!". Some sensory recognition is very important (kids close their eyes and try to guess the nature of solids only with their hands).

Spatial vocabulary is mandatory. Observe that lateralization is a suitable subject for 4-5 years old. It should be in educator's mind that to be on the left or on the right is a relative issue. Always raises the question: "In relation to whom?". The referential is important; one should try to use a child's own reference. For example, the hand holding the fork, the kicking foot, a birthmark, the heartbeat,... A great idea is to explore stories in which the character has his back turned. This allows all to be aligned in the same way.

In Figure 16, some explorations are presented. In this context, the use of everyday life objects is very important as it calls for children own experiences.

Figure 16: Recognizing 3D shapes.

A final remark for the distinction "3D *versus* 2D" that must be worked (see Figure 17).

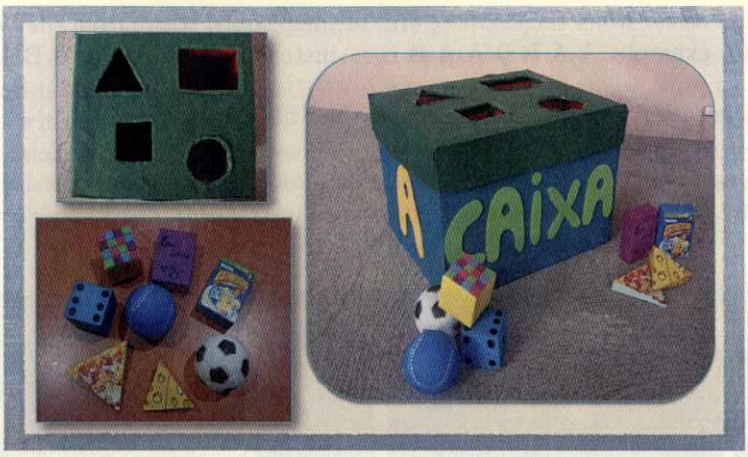

Figure 17: 2D shapes *vs* 3D shapes.

Patterns

Children at this stage (ages 4-5):

- ◇ identify and understand sequential repeated patterns, progressive patterns and symmetric patterns;
- ◇ learn to focus attention on details and to detect mixtures;
- ◇ explain logical rules;
- ◇ build and explain patterns.

One should start with the common *sequential repeated patterns*:

A B A B A B ...

A A B A A B A A B ...

A A B B A A B B A A B B ...

A B C A B C A B C ...

...

These patterns should be explored in several ways and contexts. It is important to make sure that the children really understand how the patterns are designed. For that they have to detect the smallest unit of the pattern, i.e. the "core" of the pattern (for example, A B; A A B; A A B B; ...). Let's focus on some important aspects:

- ◇ It is mandatory to show clearly the smallest unit of the pattern and, at least, one more of its copies. There are books which have, for example, a red circle, followed by a blue circle, asking to continue the pattern. This is a completely inadequate task because we have to understand the pattern; we don't have to guess the pattern;

- ◇ It is easier when the smallest unit of the pattern starts from the beginning (for example, A A B B A A B B ... instead of A B B A A B B ...);
- ◇ It is easier to continue a sequence than fill it in the middle.

Regarding the latter two topics, the degree of difficulty should be introduced gradually. In Figure 18, some activities are illustrated.

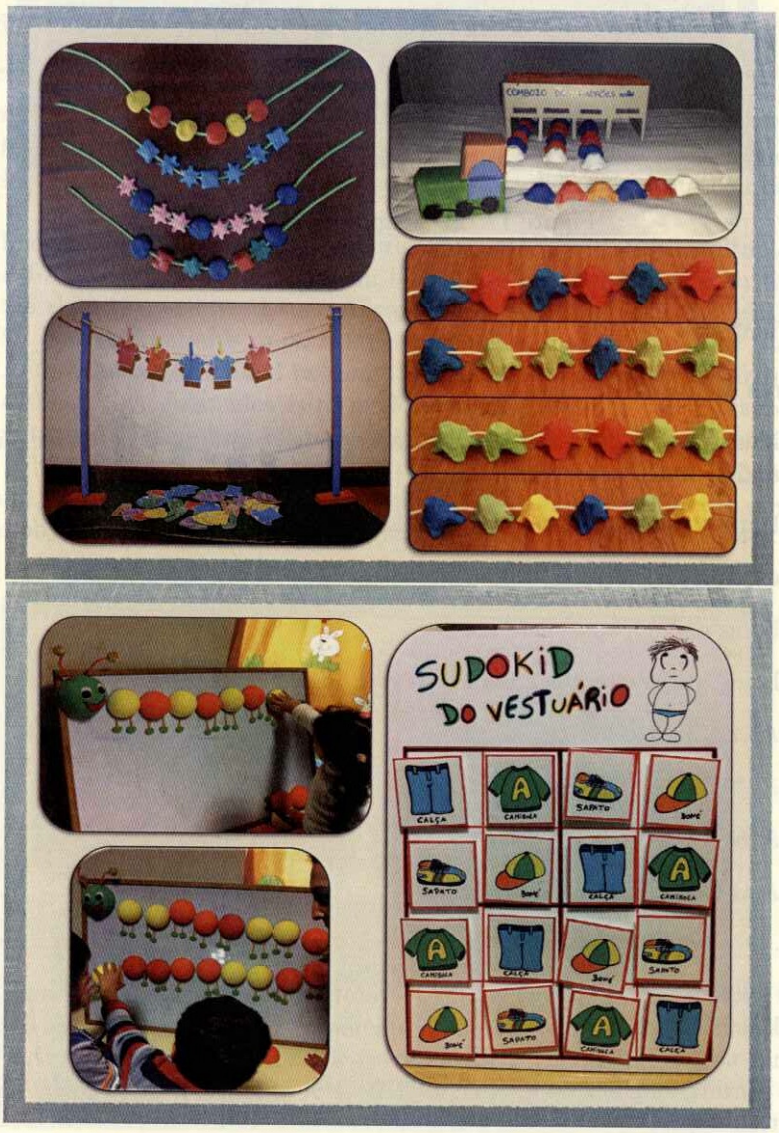

Figure 18: Recognizing different patterns.

The work with manipulatives and the discussions of regularities help children to recognize patterns. Also, chanting the patterns with children is a very good

strategy (e.g. "square, triangle, square, triangle, ...").

There are thousands of good activities: symmetrical drawings, sudoku figures, patterns with *lego*, puzzles where you have to choose the right piece to complete a given image (the kid should explain his choice).... There are patterns with irrelevant elements (for example, color may be not important, only the position and the objects themselves).

Measurement

Children at this stage (age 5):

- ◇ learn to quantify things ("How many times it is?");
- ◇ realize the concept of unity;
- ◇ "feel" the quantities and make comparisons;
- ◇ measure objects with non-standard units and build objects with a certain measure;
- ◇ know and use words related to measurement: "large", "small", "more", "less", "heavy", "light", "tall", "short" and "long".

A typical activity is to show a drawing of a doll, side-by-side with a column of small squares; the column may be positioned horizontally or vertically; the measurement may be done from different reference points. First, the educator should trace with his index finger, aligning the top of the doll with the right position of the column, and the same with a pencil. Then, slowly, he leaves the child to do the job (first with the index finger and only then with a pencil). An interesting idea is to adjust the square sizes to the well known *Cuisenaire* rods (each small square should match the face of the unit cube). Children can also use 3D objects (e.g. *lego* pieces, ice cream sticks, pens, ...) as the unit to measure other objects. It is important to ensure that each object has a whole number of units as its measure.

Educators should chose "easy" quantities to be sensory worked as *length*, *weight* and *capacity*; the child already has an intuition about these quantities. In Figure 19 some examples are presented.

It is possible to use balance scales for measurement activities. We should choose objects weighing a whole number of units. To explore the volume (capacity), educators may use cups of various shapes. It is important to establish relationships between different containers (for example, the volume of two small cups may match the volume of a large cup).

The same object can be used for various measurements. For example, a *lego* piece may be useful to measure lengths and to measure weights as well. This is important: the child will realize that the same object may have many measurable attributes.

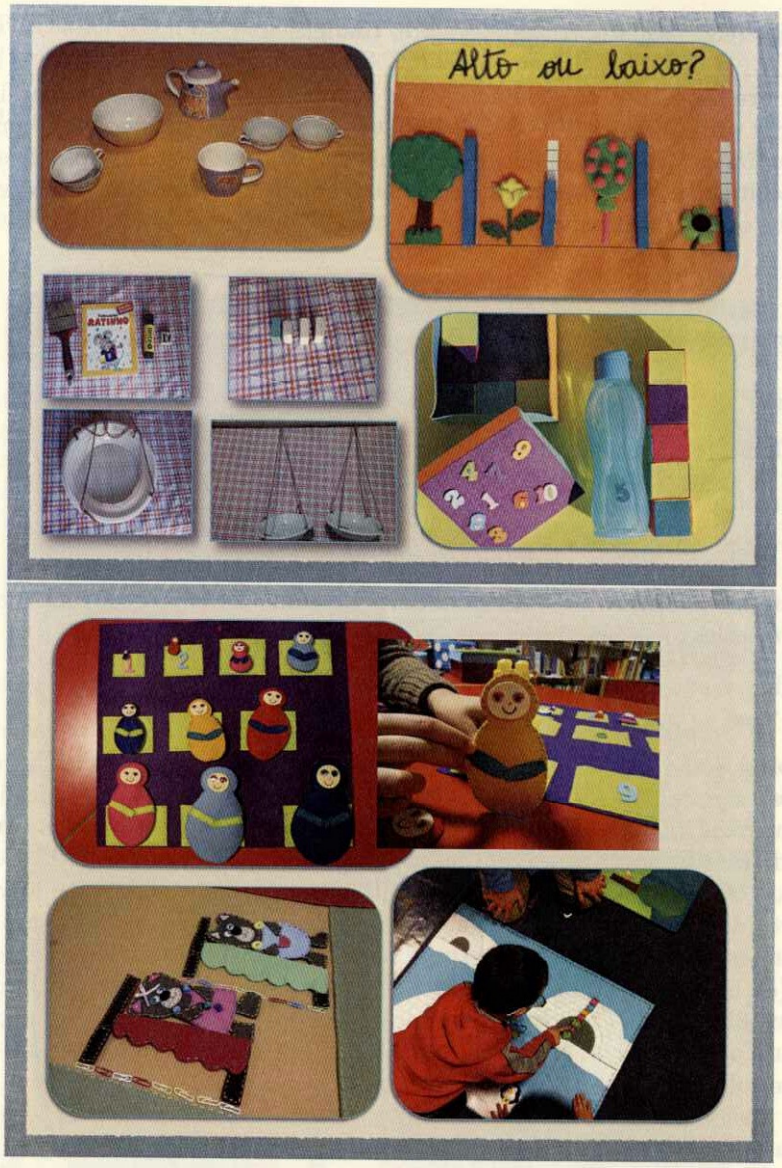

Figure 19: How long? How tall? How heavy? How many liquid? Making comparisons.

If the child developed the skills expected for this subject, the educator can try the usual inverse activity: starting with a column of small squares, he may ask for a drawing with 4 squares in height. In this inverse activity, the drawing is not there to be measured; a drawing with a particular height is the target.

Measurement is an important real-world area of Mathematics. We apply length, weight and capacity in our everyday lives. Also, by its very nature it connects the two most important domains of early mathematics: geometry and number.

Number bonds and sums/subtractions

Children at this stage (age 5):

- ◇ decompose small numbers – less or equal than 10 (tell a "story" from a given decomposition; tell a story after presenting a decomposition);
- ◇ add (join);
- ◇ subtract (remove);
- ◇ know the symbols "+", "−" and "=".

We highlight the following rule: use only the whole numbers up to 10 (including 10 is very important because our number system is base ten). Numbers greater than 10 relate to more sophisticated mental processes (to be worked at the end of Kindergarten and in 1st grade).

The understanding of the *part-whole* model is crucial. When we add, we start from the parts in order to obtain the whole; on the other hand, when we decompose, we start from the whole in order to obtain parts (usually there is more than one way to do the task). Decomposition is the opposite procedure of addition.

Again, the orality is a key issue, when exploring number bonds. When a image is presented (a family of rabbits, for instance) and the child observes 5 decomposed into 2 and 3, it is not a counting task. It is a "detective work" in which the child has to realize what 5, 2 and 3 have to do with the image. This process requires more maturity, so it is not advisable to work such activities with children under 5 years old.

There are different levels of difficulty:

- ◇ "Unique story decomposition" (for example, from $7 \mapsto 2+5$, there is only one way to build the story);
- ◇ "One decomposition, several stories" (for example, from the single decomposition $7 \mapsto 2+5$, there are several possible stories);
- ◇ "Several decompositions" (for example, $7 \mapsto \ldots + \ldots$ is presented and the child chooses the number bonds and tells the stories accordingly).

In Figure 20, some examples are presented with the famous Singapore *number bonds*.

Regarding Singapore Math, $2+3$ is to memorize, but $8+5$ is not. The difference is that, in the second case, the whole exceeds 10. After the work with number bonds and the place value concept (five and six years old), the child is prepared to think about $8+5$ using a three-step approach: first decompose $5 \mapsto 2+3$; second, join 8 and 2 in order to compose the ten; third, 10 and 3 is 13. This

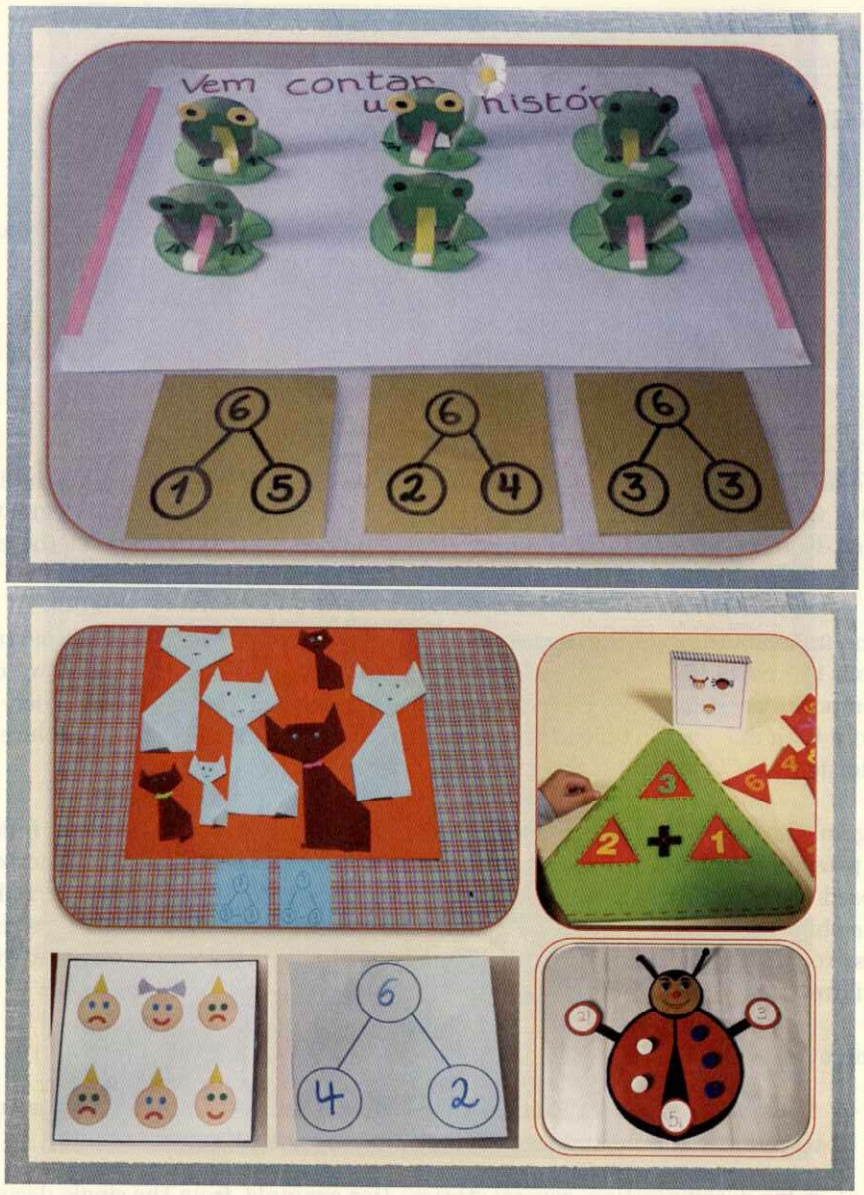

Figure 20: Number Bonds.

mental calculation is quite complex, but it is one of the first major goals of the method. Sometimes a kind of "barrier" is presented; two units of the 5 jump the barrier to form the ten.

About "short" additions and "short" subtractions ("short" in the sense that the whole does not exceed ten), in kindergarten times we only use these operations to join and to remove. As usual, we must contextualize the activities. That is, nice cartoons asking for interpretations and stories. The mathematical signs +,

– and = should appear. In Figure 21, some examples are shown.

Figure 21: Sums and subtractions.

A last note about finger counting (or reciting). Adding and subtracting by counting is only encouraged when one of the parts is very small (up to 3). Usually, when we want to find how much is $41 + 2$, we "walk" two units forward from 41. This is a kind of finger counting, but it is a very good one because, often, it is more or less instantaneous. When the parts are not small, there are better strategies. That is what we want for children.

If one carefully analyzes the Singapore manuals, there are many animals, fruits and other concrete things. But, occasionally, things get more abstract with dots, bars and dashes. This is not done at random. It is a subtle route to abstraction.

Place Value

Children at this stage (age 5):

- ◇ count up to 20;
- ◇ understand the concept of place value (particulary, the ten's place).

Regarding this subject, the main goal is to make children realize that a number such as "12" follows the positional main rule of the decimal number system. The symbol "1" is one ten and the symbol "2" represents two units. The symbol's value depends on its position in the numeral. Understanding the positional number system is very difficult for kindergarten kids, since it is related to the importance of the position and the mixed nature of our numerical writing. Telling a five years old child that the value of "1" is ten is more or less science fiction...

Having this goal in mind, typical activities are the following:

a) "Keep apart ten and say the number";

b) "Paint ten and say the number";

c) Use devices with moving symbols (usual in Singapore books).

In Figure 22, a concrete example is presented. A child is invited to look at a tree with 15 apples. Next, the educator asks the child to count 10 apples (loudly and pointing at the same time). Then the child take these 10 apples and put them in a box, previously prepared. Following, the child records the numeral 10 in the device (1 in the tens and 0 in the units).

Figure 22: A tree of apples.

The educator must repeat the information "The box has 10 apples.", pointing with his index finger and, then, ask how many apples still remained on the tree. The child should use the device to put the numeral "5" in a box over the symbol "0". In the end, the child counts out loud all the apples (10 in the box

and 5 still in the tree). The educator should say "We have 15 apples, 10 in the box and 5 in the tree.". And, after, "That is what fifteen is, ten and five.". When the educator says "ten and five", he should use the device and make a movement with the "5" to superimpose it on the "0". The children should have the clear understanding that the "1" from "15" is the "1" from "10". When the educator teaches what "15" is, he should show "10" and cover the "0" with the "5". Thus, the "1" from "10" is no longer a "strange mystery". The child previously saw the number 10...

This is a non trivial subject. Note that Singapore Math prepares children in relation to this concept for a year and a half. In Figure 23, some explorations are illustrated.

Figure 23: Place value.

In the first grade, composing ten units into a ten and decomposing a ten into ten units are the most important procedures to be taught. These procedures are fundamental for the understanding of all the usual and unusual algorithms.

Hence, of course, much effort and many strategies are dedicated to this subject.

In Singapore Math, having related actions for the composition/decomposition procedures is very usual. For example,

- ◇ 10 beads form a necklace – a complete necklace more two beads add up 12 beads;
- ◇ 10 eggs form a full box – a full box more two eggs add up 12 eggs.

Other nice manipulatives: Play money (10 euros bills and 1 euro coins); Base ten blocks; *Cuisenaire* rods.

In *Knowing and Teaching Elementary Mathematics*, Liping Ma [3] proposes the idea of "concept knot" – a single concept that ties several important ones together. The composing/decomposing procedure and the place value concept are good examples of concept knots, very useful for several things; in particular to compute $8 + 5$.

Games

The most common kindergarten games are the following:

- ◇ Concentration & Memory games;
- ◇ Overlapping games;
- ◇ Make it fit games;
- ◇ Orientation games;
- ◇ Silhouette games;
- ◇ Path finding games;
- ◇ Dominoes, card decks and cubes.

In this section, although not exploiting all the cases, we present some examples. First we have some Dominoes, Puzzles and Memory Games for the Concrete > Pictorial > Abstract approach (see Figure 24).

Another interesting idea is to use a game similar to the traditional Goose Game, in which the player rolls the dice to move forward on the board, and there are challenges along the way (see figure 25).

Figure 26 illustrates an applet available on the Web page: `www.sarahorta.pt.vu`. The player has to match each object with the correspondent solid.

In Figure 27, we can see a game board with 3 different dices: the first with 6 characters, the second with 6 houses, and the third refers to the character's position in relation to the house: front-back; left-right; near-far. Each player receives one point for each correct position. The winner is the player who scores

Figure 24: Puzzles, Dominoes and Memory Games.

Figure 25: A "Goose Game".

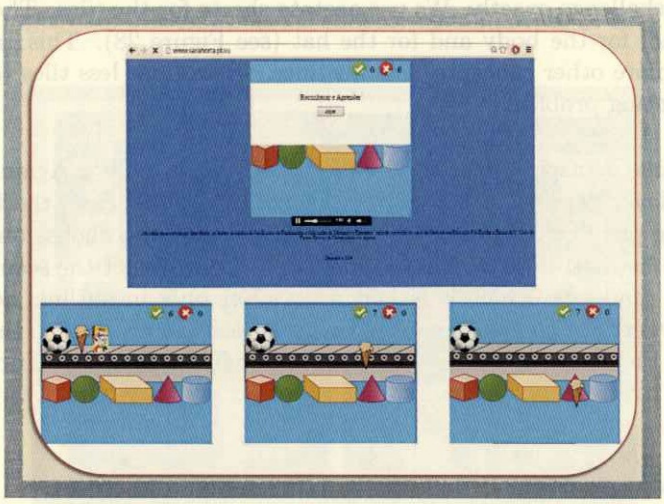

Figure 26: Recognizing 3D shapes (applet).

Figure 27: Spatial orientation game.

more in less time.

The next game is inspired in *Colour Code* of *SmartGames*[1]. This could be a one player game or a two players game. In the last case, the faster wins. A player starts to choose one challenge. Then he must pick the tiles needed and stack them up, one by one, in the tile display holder. The composition must match the challenge exactly. We use acetate sheets for the tiles. There are tiles for the head, for the body and for the hat (see Figure 28). This game can be used to explore other concepts. For instance, we can use less tiles to introduce simple counting problems.

The last game is inspired in *Day and Night* of *SmartGames*[2]. Again, this could be a one player game or a two players game. In the last case, the faster wins. The playing pieces are the basic forms. A player starts to choose one challenge card. Then he must arrange the playing pieces to construct the scene shown on the chosen challenge, which is either in detail or only in outline, according to the skill level of the challenge card. The composition must match the challenge exactly. In Figure 29, we can see some of the challenges, in detail or in outline.

Final remarks

Although informal mathematics occurs regularly in day-to-day routines and in children's play, learning advances when teachers are prepared with interesting

[1]See `http://www.smartgames.eu/en/smartgames/colour-code`.
[2]See `http://www.smartgames.eu/en/smartgames/day-and-night`.

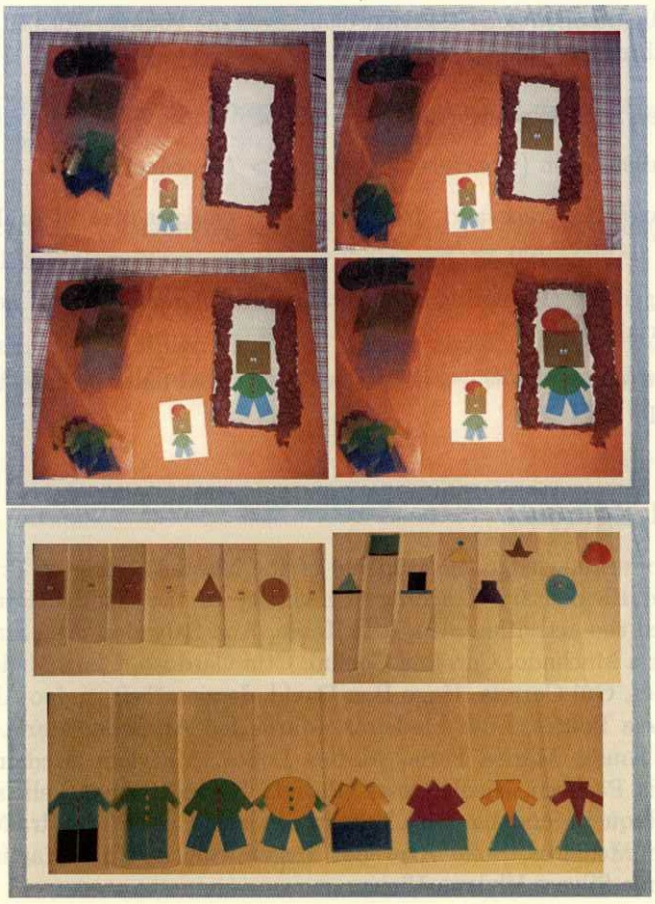

Figure 28: Recognizing 2D shapes: combinations of tiles.

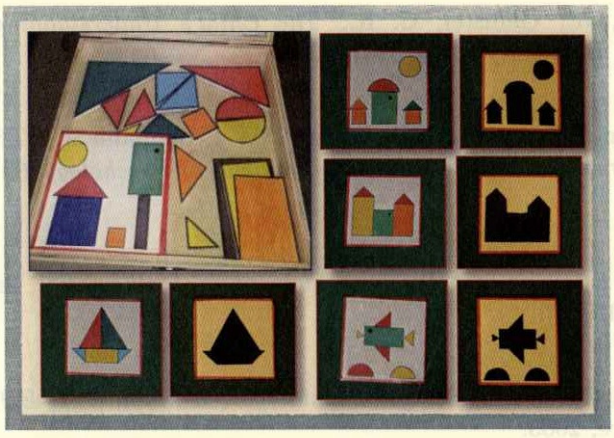

Figure 29: Recognizing 2D shapes: in detail or only in outline.

and carefully planned activities that challenges the younger ones to focus on the 8 major subjects of Singapore Math.

The children must explore multiple means of:

- ◇ Representation (Concrete > Pictorial > Abstract);
- ◇ Expression (Trace and draw training; The practice of orality: "Math Talk");
- ◇ Engagement (Exploring a long range of different types of activities and materials).

Also the order in which the concepts are introduced must be carefully planned, step by step.

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