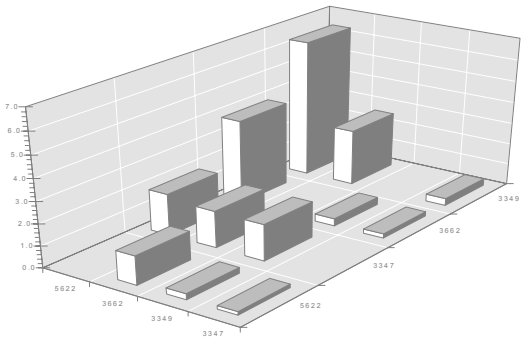
EURO XVI



MARKET SHARE MODELS FOR CONSUMER GOODS

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Brussels, July 2002

⇨MARKET SHARE MODELS FOR CONSUMER GOODS

ABSTRACT

- Market share models
  - “classical“;
  - attraction;
- Relative marketing variables;
- Expressions for direct and cross elasticities;
- Are direct elasticities robust?
- Case study:
  - Compare alternative market share models;
  - Compare alternative relative marketing variables;
  - Compare elasticities obtained by alternative market share models and relative marketing variables.

⇨ MARKET SHARE ELASTICITIES

Direct:

$$e_{ik} = \frac{\partial m_i}{\partial X_{ik}} \cdot \frac{X_{ik}}{m_i}$$

Cross:

$$e_{ijk} = \frac{\partial m_i}{\partial X_{jk}} \times \frac{X_{jk}}{m_i}$$

where:

e_{ik} = direct elasticity of brand i with respect to marketing variable X_{ik};
e_{ijk} = cross elasticity of brand i with respect to marketing variable X_{jk};
m_i = market share of brand i;
X_{ik} = marketing variable k for brand i;

⇨ MARKET SHARE:
"CLASSICAL MODELS"

Linear Model:

$$m_i = \alpha_i + \sum_k \beta_{ik} \cdot X_{ik}$$

Multiplicative Model:

$$m_i = \alpha_i \cdot \prod_k X_{ik}^{\beta_{ik}}$$

Exponential Model:

$$m_i = \exp\left(\alpha_i + \sum_k \beta_{ik} \cdot X_{ik}\right)$$

$$\sum m_i = ? \ (\approx 1)$$

⇨ MARKET SHARE: ATTRACTION MODELS

Multiplicative Competitive Interaction Model (MCI):

$$m_i = \frac{\alpha_i \cdot \prod_k X_{ik}^{\beta_{ik}}}{\sum_{j=1}^{n} \left(\alpha_j \cdot \prod_k X_{jk}^{\beta_{jk}} \right)}$$

MultiNomial Logit Model (MNL):

$$m_i = \frac{\exp(\alpha_i + \sum_k \beta_{ik} \cdot X_{ik})}{\sum_{j=1}^{n} \left(\exp(\alpha_j + \sum_k \beta_{jk} \cdot X_{jk}) \right)}$$

$$\sum m_i = 1$$

⇨RELATIVE MARKETING VARIABLES

$$X_{ik}^{*} = \frac{X_{ik}}{\frac{1}{n}\sum_{l=1}^{n} X_{lk}}$$

(1) Non-weighted average

$$X_{ik}^{*} = \frac{X_{ik}}{\sum_{l=1}^{n} m_l \cdot X_{lk}}$$

(2) Weighted average

$$X_{ik}^{*} = \frac{X_{ik}}{\sqrt[n]{\prod_{l} X_{lk}}}$$

(3) Geometric mean

$$X_{ik}^{*} = X_{ik} - \frac{1}{n}\sum_{l} X_{lk}$$

(4) Difference to mean

⇨ DIRECT ELASTICITIES
COMPLETE REFERENCE

Market Share Models				
Linear	**Multiplica.**	**Exponent.**	**MCI**	**MNL**
Comom fraction				
$\beta_{ik} \cdot X_{ik}^* / / m_i \cdot ①$	$\beta_{ik} \cdot ①$	$\beta_{ik} \cdot X_{ik}^* \cdot ①$	$\beta_{ik} \cdot (1-m_i) \cdot ① + + \sum_{l \neq i} \beta_{lk} \cdot m_l \cdot ②$	$\beta_{ik} \cdot X_{ik}^* \cdot (1-m_i) \cdot ① + \sum_{l \neq i} \beta_{lk} \cdot m_l \cdot X_{lk}^* \cdot ②$

Non-relative variable

$①=1$ $②=0$

(1) Non-weighted average

$①=1-X_{ik}^*/n$ $②=X_{ik}^*/n$

(2) Weighted average

$①=1$ $②=X_{lk}^* \cdot X_{ik}/X_{lk} \cdot m_i'/(1-m_l')$

(3) Geometric mean

$①=(n-1)/n$ $②=1/n$

(4) Difference to mean

$①=(n-1)/n \cdot X_{ik}/X_{ik}^*$ $②=1/n \cdot X_{ik}/X_{lk}^*$

Note: for non-relative variables X_{ik} substitutes X_{ik}^*.

⇨ DIRECT ELASTICITIES

Non-relative variable *Relative variable (2)*

Linear Model:

$$e_{ik} = \beta_{ik} \cdot X_{ik} / m_i$$

* varies directly with marketing variables;

$$e_{ik} = \beta_{ik} \cdot X_{ik}^{*} \cdot (1 - m'_i \cdot X_{ik}^{*}) / m_i$$

* mixed variation with marketing variables;

Multiplicative Model:

$$e_{ik} = \beta_{ik}$$

* time invariable;
* market share invariable;

$$e_{ik} = \beta_{ik} \cdot \left(1 - m'_i \cdot X_{ik}^{*}\right)$$

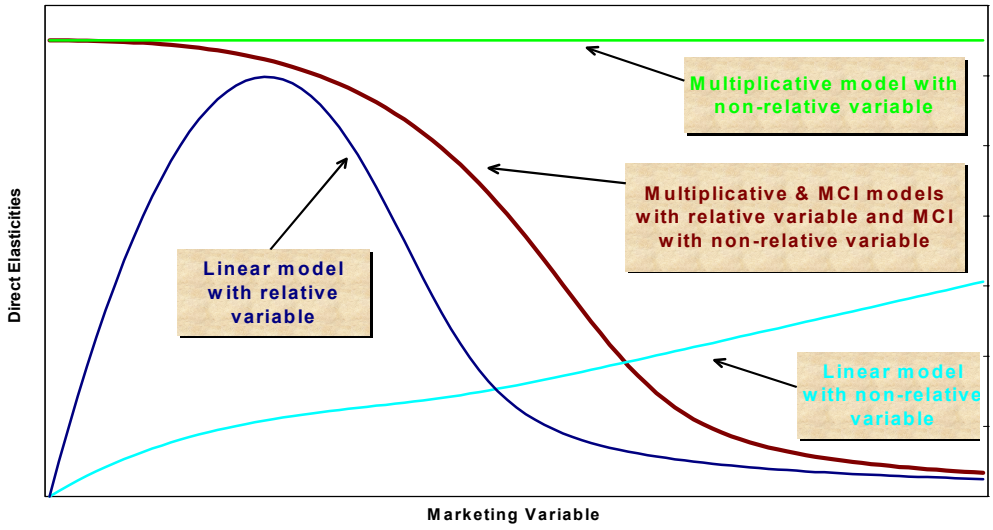
* decreases with marketing variables;

MCI Model:

$$e_{ik} = \beta_{ik} \cdot (1 - m_i)$$

$$e_{ik} = \beta_{ik} \cdot (1 - m_i) \cdot (1 - m'_i \cdot X_{ik}^{*}) + \sum_{l \neq i} \beta_{lk} \cdot m_l \cdot m'_i \cdot X_{ik}^{*}$$

* decreases with marketing variables of brand i;



⇨ CROSS ELASTICITIES
COMPLETE REFERENCE

Market Share Models				
Linear	**Multiplica.**	**Exponent.**	**MCI**	**MNL**
Comom fraction				
$-\beta_{ik}\cdot X_{ik}^{*}/$ $/m_i\cdot①$	$-\beta_{ik}\cdot①$	$-\beta_{ik}\cdot X_{ik}^{*}\cdot$ $\cdot①$	$-\beta_{jk}\cdot m_j\cdot①+$ $+\sum_{l\neq j}\beta_{lk}\cdot(m_l-\delta_{il})\cdot②$	$-\beta_{jk}\cdot X_{jk}^{*}\cdot m_j\cdot①+$ $\sum_{l\neq j}\beta_{lk}\cdot(m_l-\delta_{il})\cdot X_{lk}^{*}\cdot②$
Non-relative variable				
$①=1$			$①=1$ $②=0$	
(1) Non-weighted average				
$①=X_{jk}^{*}/n$			$①=1-X_{jk}^{*}/n$ $②=X_{jk}^{*}/n$	
(2) Weighted average				
$①=X_{jk}^{*}\cdot m_j'$			$①=1-m_j'\cdot X_{jk}^{*}$ $②=X_{jk}^{*}\cdot m_j'$	
(3) Geometric mean				
$①=1/n$			$①=(n-1)/n$ $②=1/n$	
(4) Difference to mean				
$①=1/n\cdot X_{jk}/X_{ik}^{*}$			$①=(n-1)/n\cdot X_{jk}/X_{jk}^{*}$ $②=1/n\cdot X_{jk}/X_{lk}^{*}$	

Note: for non-relative variables X_{ik} substitutes X_{ik}^{*}.

⇨ CROSS ELASTICITIES

Non-relative variable	*Relative variable (2)*

Linear Model:

$$e_{ijk} = -\beta_{ik} \cdot X_{ik} / m_i$$

* invariable with brand j;
* increases with share and brand i marketing variables;

$$e_{ijk} = -\beta_{ik} \cdot X_{ik}^{*} \cdot X_{jk}^{*} \cdot m'_j / m_i$$

* decreases with brand j variables;
* increases with share and decreases with brand i marketing variables;

Multiplicative Model:

$$e_{ijk} = -\beta_{ik}$$

* time invariable;
* invariable with brand j;

$$e_{ijk} = -\beta_{ik} \cdot X_{jk}^{*} \cdot m'_j$$

* decreases with brand j variables;

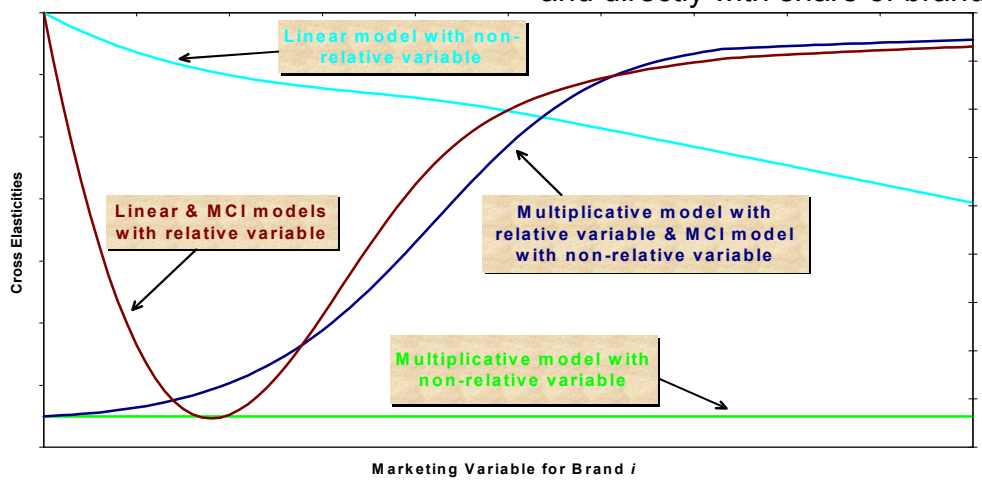
MCI Model:

$$e_{ijk} = -\beta_{ik} \cdot m_j$$

* decreases with brand j share;

$$e_{ijk} = -\beta_{jk} \cdot m_j \cdot (1 - m'_j \cdot X_{jk}^{*}) + \\ + \beta_{ik} \cdot (m_i - 1) \cdot m'_j \cdot X_{ik}^{*} \cdot X_{jk}^{*} + \\ + \sum_{l \neq i,j} \beta_{lk} \cdot m_l \cdot m'_j \cdot X_{lk}^{*} \cdot X_{jk}^{*}$$

* varies directly with marketing variables of brand j and brand i;
* varies inversely with share of brand j and directly with share of brand i;

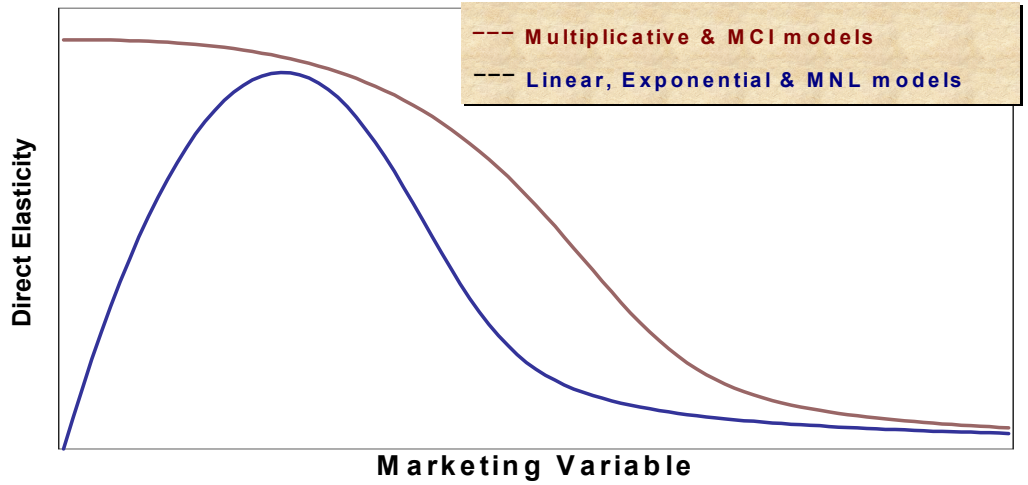


⇨ ARE DIRECT ELASTICITIES ROBUST?

Legend: **a)** $\lim_{m_i \to 1} e_{ik}$ **b)** $\lim_{X_{ik} \to \pm\infty} e_{ik}$

	Market Share Models				
	Linear	**Multiplica.**	**Exponent.**	**MCI**	**MNL**
Non-relative	**a)**=$\beta_{ik} \cdot X_{ik}$ **b)**=*1*	**a)**= β_{ik} **b)**= β_{ik}	**a)**= $\beta_{ik} \cdot X_{ik}$ **b)**= $\pm\infty$	**a)**=*0* **b)**=*0*	**a)**=*0* **b)**=*0*
equation (1)	**a)**=$\beta_{ik} \cdot X_{ik}^* \cdot (1-X_{ik}^*/n)$ **b)**=*0*	**a)**=$\beta_{ik} \cdot (1-X_{ik}^*/n)$ **b)**=*0*	**a)**=$\beta_{ik} \cdot X_{ik}^* \cdot (1-X_{ik}^*/n)$ **b)**=*0*	**a)**=*0* **b)**=*0*	**a)**=*0* **b)**=*0*
equation (2)	**a)**=*0* **b)**=*0*	**a)**=*0* **b)**=*0*	**a)**=*0* **b)**=*0*	**a)**=*0* **b)**=*0*	**a)**=*0* **b)**=*0*
equation (3)	**a)**=$\beta_{ik} \cdot X_{ik}^* \cdot (n-1)/n$ **b)**=$(n-1)/n$	**a)**=β_{ik} $(n-1)/n$ **b)**=$\beta_{ik}(n-1)/n$	**a)**=$\beta_{ik} \cdot X_{ik}^* \cdot (n-1)/n$ **b)**=$\pm\infty$	**a)**=*0* **b)**=*0*	**a)**=*0* **b)**=*0*
equation (4)	**a)**=$\beta_{ik} \cdot X_{ik} \cdot (n-1)/n$ **b)**=*1*	**a)**=$\beta_{ik} \cdot X_{ik}/X_{ik}^* \cdot (n-1)/n$ **b)**=β_{ik}	**a)**=$\beta_{ik} \cdot X_{ik} \cdot (n-1)/n$ **b)**=$\pm\infty$	**a)**=*0* **b)**=*0*	**a)**=*0* **b)**=*0*

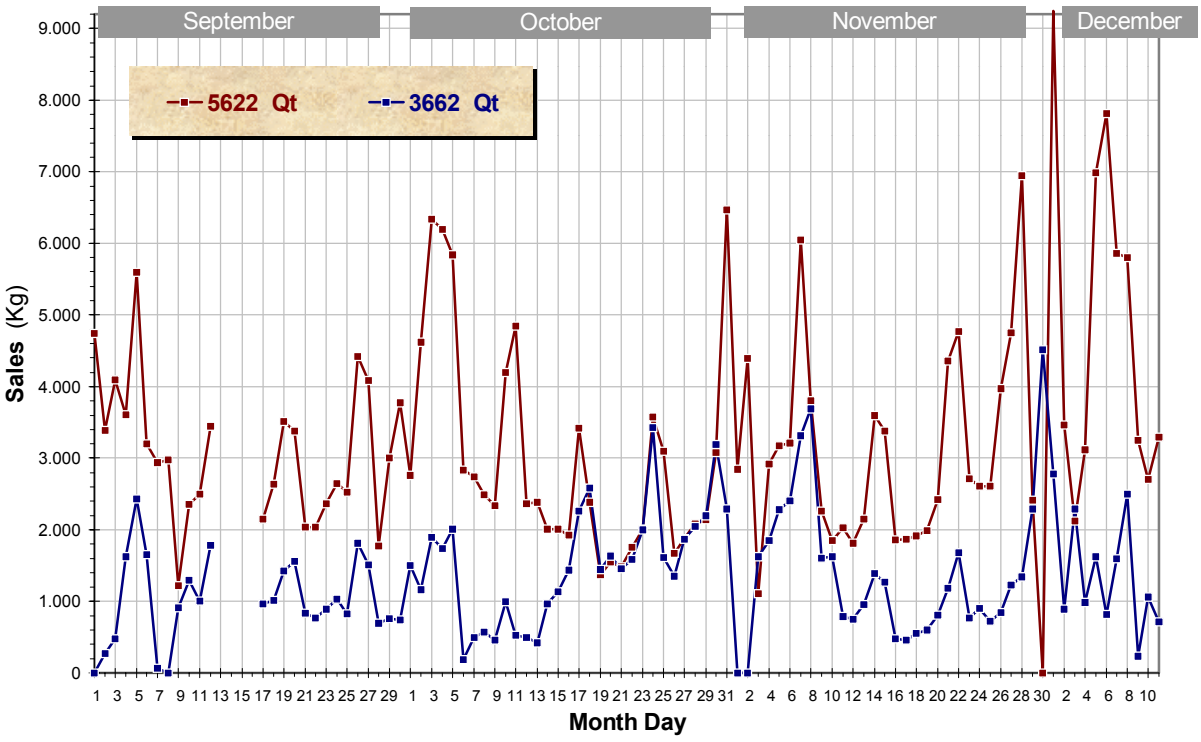
⇨ BEHAVIOUR OF ROBUST MODELS



⇨ THE RICE FAMILY:

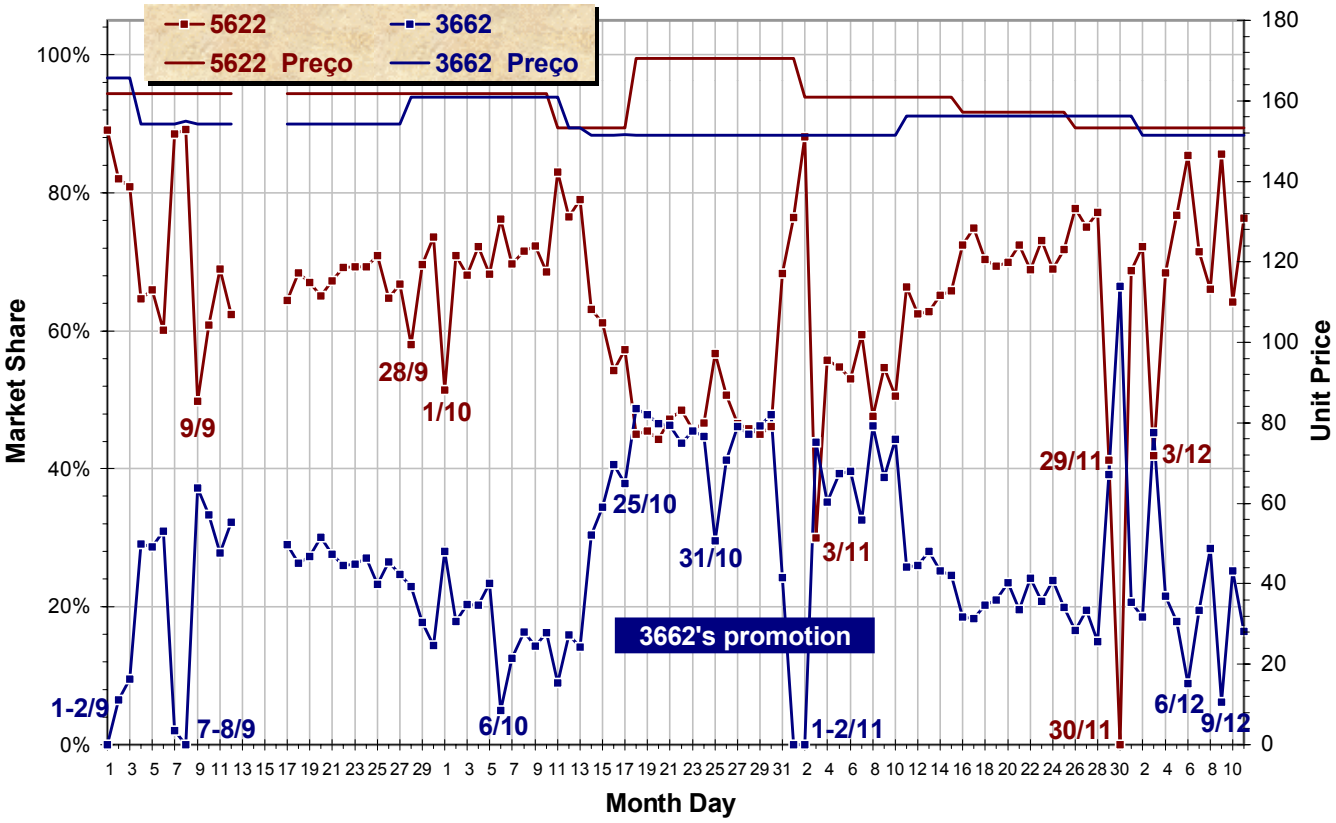
Code	Type	Brand	Quantity	Mean share
5622	Rice Extra Long	Saludães	1 Kg	63%
3662	Rice Extra Long	Malandrinho	1 Kg	28%
3349	Rice Carolino	Grão de Ouro	1 Kg	5%
3347	Rice Carolino	D. Ana	1 Kg	3%
5626	Rice Extra Long	Oriente	1 Kg	1%

⇨ SALES FOR THE TWO MAJOR BRANDS

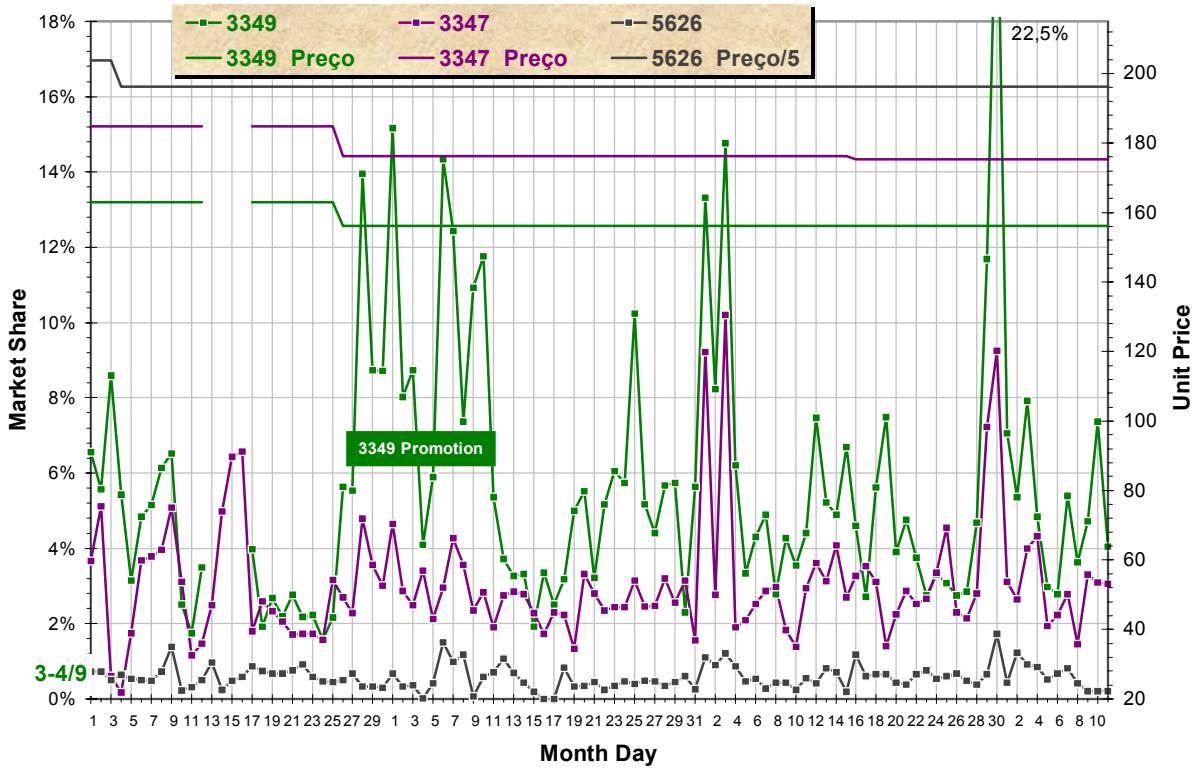


⇨ MARKET SHARE & PRICE SERIES

MAJOR BRANDS



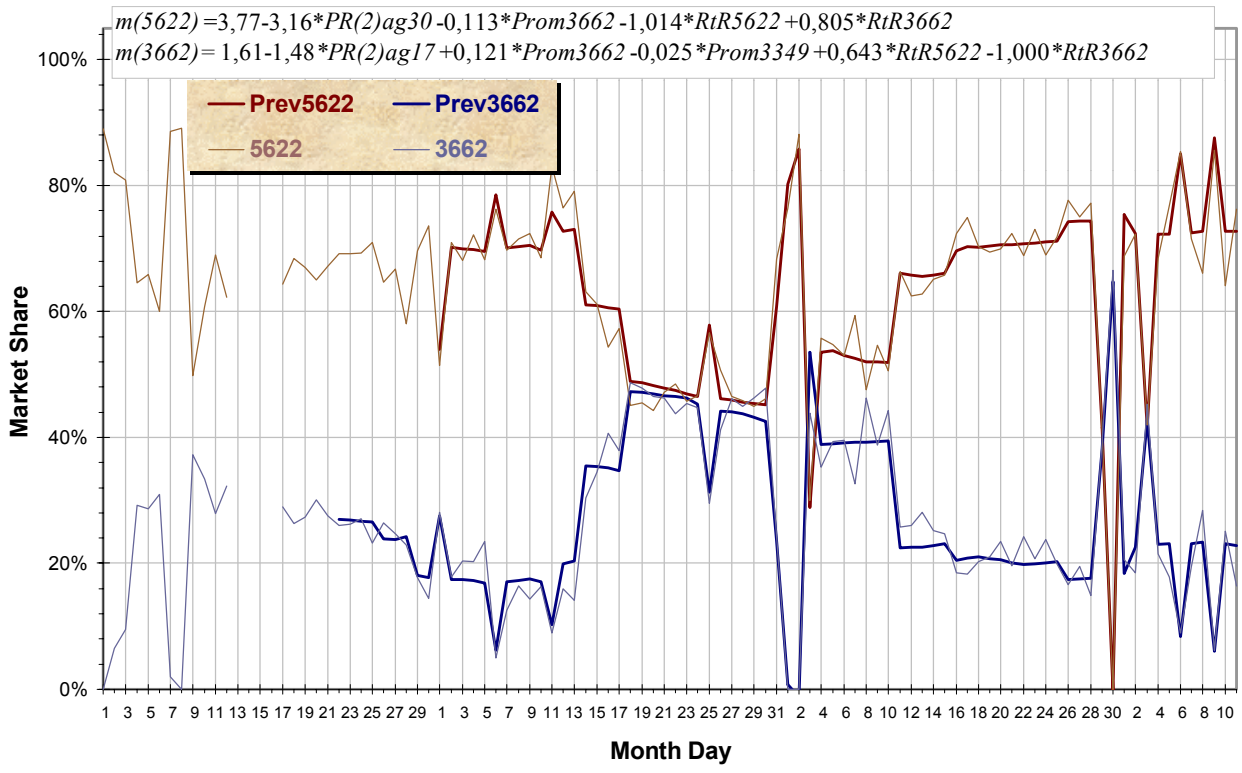
MINOR BRANDS



⇨ QUALITY OF FIT FOR CLASSICAL MODELS

ADJUSTED R^2 FOR MAJOR BRANDS

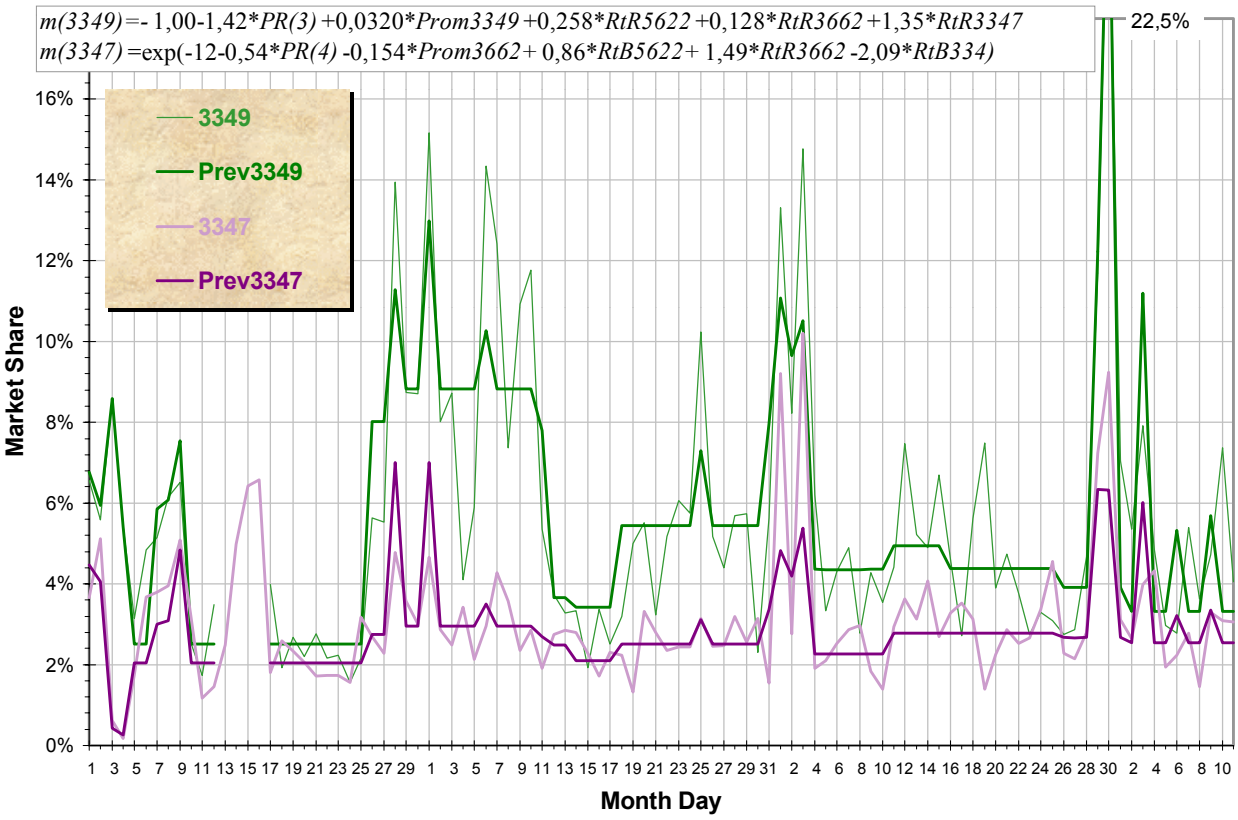
Explanatory Variable	Market Share Models	
	Linear	Multiplicative
Brand 5622 - Rice Extra-Long Saludães		
Non-relative	92,90%	91,70%
Equation (2)	**95,12%**	93,80%
Brand 3662 - Rice Extra-Long Malandrinho		
Non-relative	86,64%	79,78%
Equation (2)	**93,65%** √	87,03% √



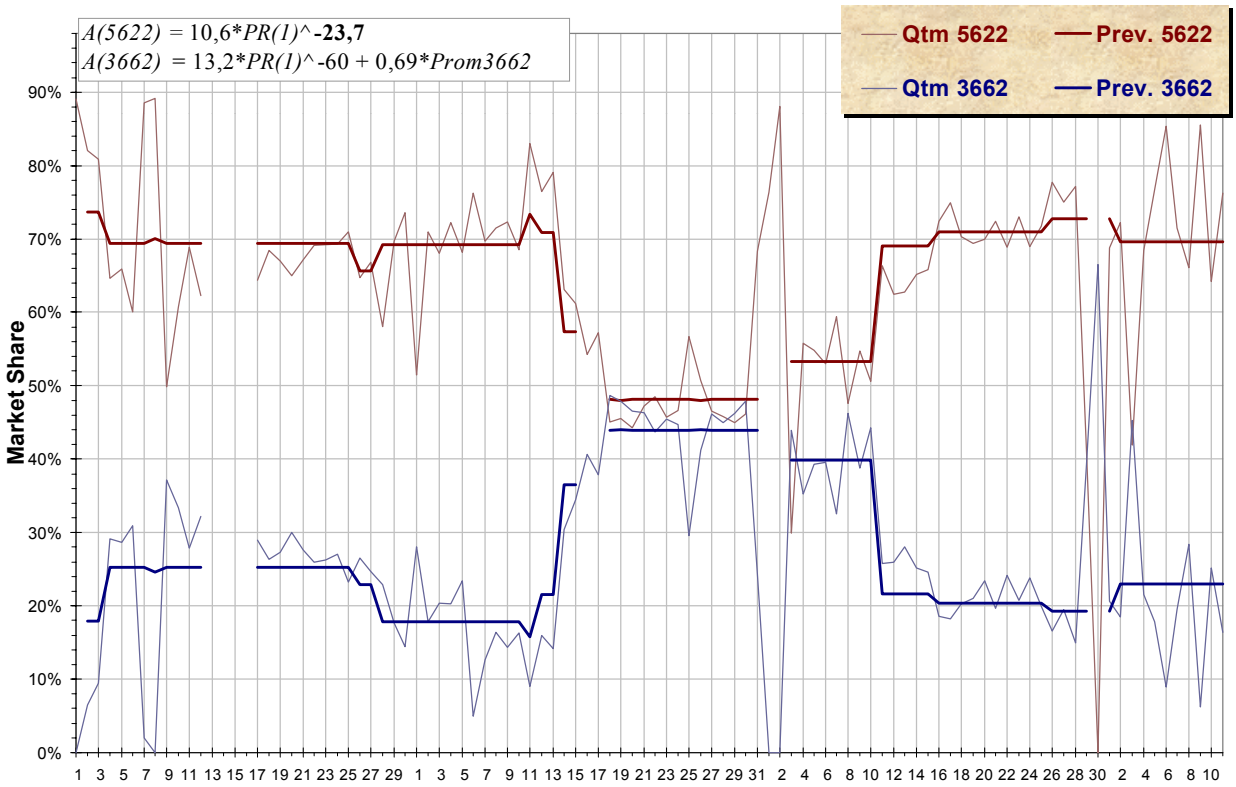
⇨ QUALITY OF FIT FOR CLASSICAL MODELS

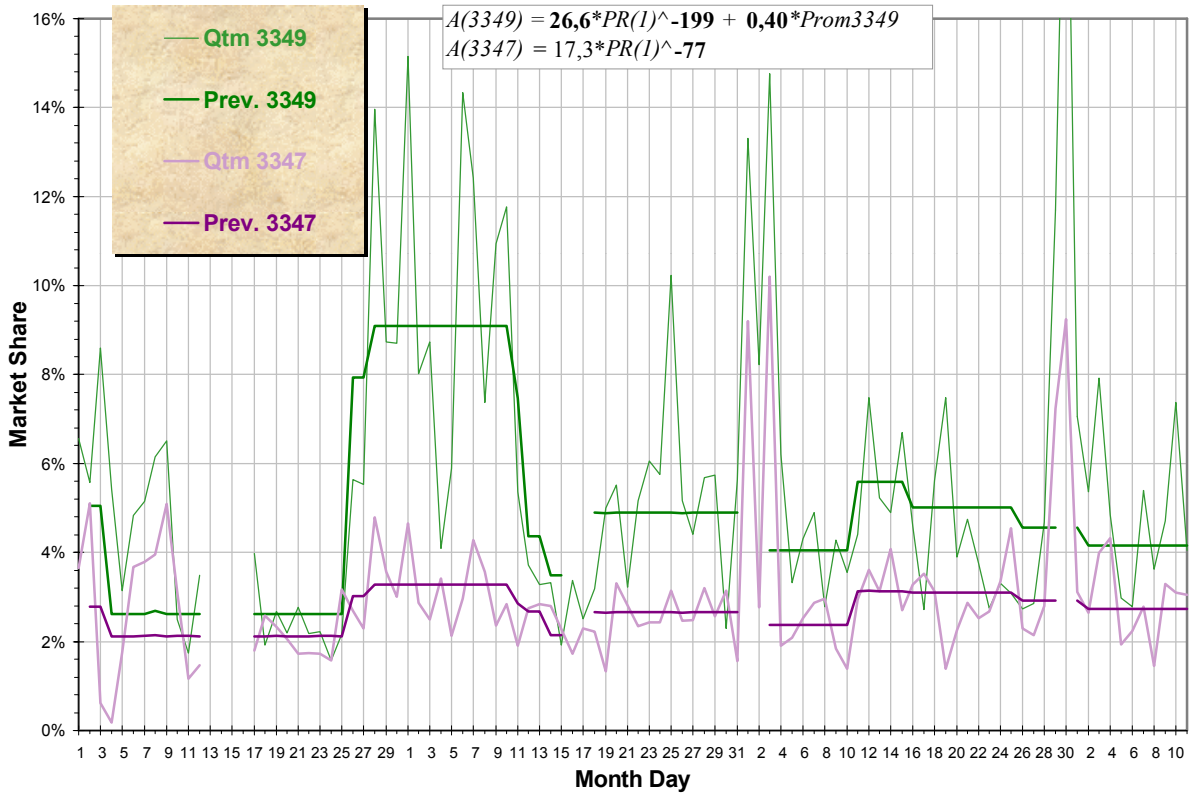
ADJUSTED R^2 FOR MINOR BRANDS

Explanatory Variable	Market Share Models	
	Linear	Multiplicative
Brand 3349 - Rice Carolino Grão de Ouro		
Non-relative	71,38%	59,76%
Equation (2)	**74,51%**	63,02%
Brand 3347 - Rice Carolino D. Ana		
Non-relative	~~*53,48%*~~	~~*59,66%*~~
Equation (2)	~~*53,98%*~~	***60,13%***

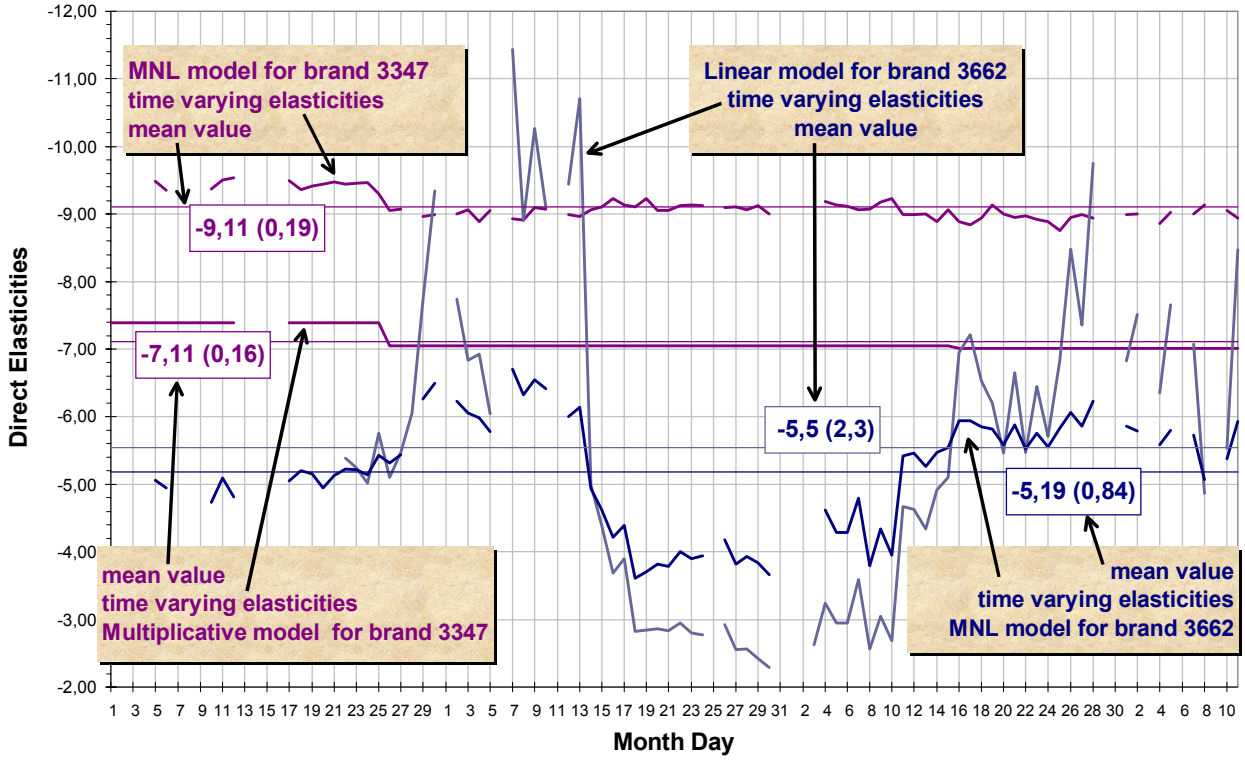


⇨ QUALITY OF FIT FOR ATTRACTION MODELS





⇨ TIME VARIATION OF DIRECT ELASTICITIES



⇨COEFFICIENT OF VARIATION FOR DIRECT ELASTICITIES

$\hat{e}_{ik}$ | $\sigma_{ik}/\hat{e}_{ik}$	Linear		Multiplicative		MNL		MCI	
	elasti.	c. v.	elasti.	c. v.	elasti.	c. v.	elasti.	c. v.
brand 5622 - Rice Extra-Long Saludães								
Non-relative	-1,8	0,12	-1,8	0,12	-1,4	0,26	----	
equation (2)	**-1,8**	**0,10**	-1,9	0,10	-2,7	0,25	-2,70	0,25
brand 3662 - Rice Extra-Long Malandrinho								
Non-relative	-4,7	0,19	-7,3	0,18	-5,7	0,26	----	
equation (2)	**-4, 7**	**0,12**	-5,3	0,17	-5,9	0,23	-5,9	0,23
equation (3)	-4,6	0,12	-5,6	0,15	-4,7	0,27	-4,7	0,27
brand 3349 - Rice Carolino Grão de Ouro								
Non-relative	-6,4	0,36	-9,3	0,26	-14	0,15	----	
equation (2)	**-10**	**0,23**	-12	0,21	-14	0,17	-14	0,17
equation (3)	-14	0,20	-16	0,16	-20	0,16	-20	0,16
brand 3347 - Rice Carolino D. Ana								
Non-relative	-2,3	0.77	-3,2	0,52	-6,9	0,26	----	
equation (2)	-1,1	1,89	-2,3	0,77	-6,3	0,32	-6,3	0,32
equation (4)	-5,3	0,36	**-7,7**	**0,23**	-9,9	0,25	----	

⇨DIRECT & CROSS ELASTICITIES

e_{ij}	ΔP_j			
Δm_i	*5622*	*3662*	*3349*	*3347*
Classical Models				
5622	-1,8	1,32	0,28	0,16
3662	3,92	-4,7	0,35	0,20
3349	6,39	2,61	-14	0,33
3347	1,92	1,70	1,67	-7,7
Attraction Models : MNL with relative price				
5622	-1,3	1,72	0,76	*-0,20*
3662	1,91	-5,6	1,24	0,24
3349	4,08	3,99	-20	2,16
3347	2,21	2,40	1,54	-8,8
MNL with non-relative price				
any brand	2,86	2,27	0,70	0,17

⇨ COMPETITIVE MAP: