



# A ordem da Natureza

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Departamento de Matemática da Universidade dos Açores





# Regras do CAOS

## Sensível às condições iniciais

Por menor que seja a alteração no início do evento, a sua evolução produz sempre um evento diferente.

## Interdependência entre os eventos

A sequência de eventos está tão interligada que é impossível identificar qual foi o primeiro a ocorrer.

# Em busca da regularidade

49

*p* *pp* *ppp*

This system contains measures 49 through 53. The right-hand part features a complex melodic line with many accidentals and slurs. The left-hand part provides a steady accompaniment. Dynamic markings *p*, *pp*, and *ppp* are indicated across the measures.

54 fuga *mf*

*s* *tr*

This system contains measures 54 through 57. Measure 54 is marked *fuga* and *mf*. The right-hand part has a rhythmic pattern with slurs and trills (*tr*). The left-hand part has a similar rhythmic pattern. A *s* (sforzando) marking is present in measure 56.

58

*tr* *s*

This system contains measures 58 through 60. The right-hand part continues with slurs and trills (*tr*). The left-hand part has a steady accompaniment. A *s* marking is present in measure 60.

61

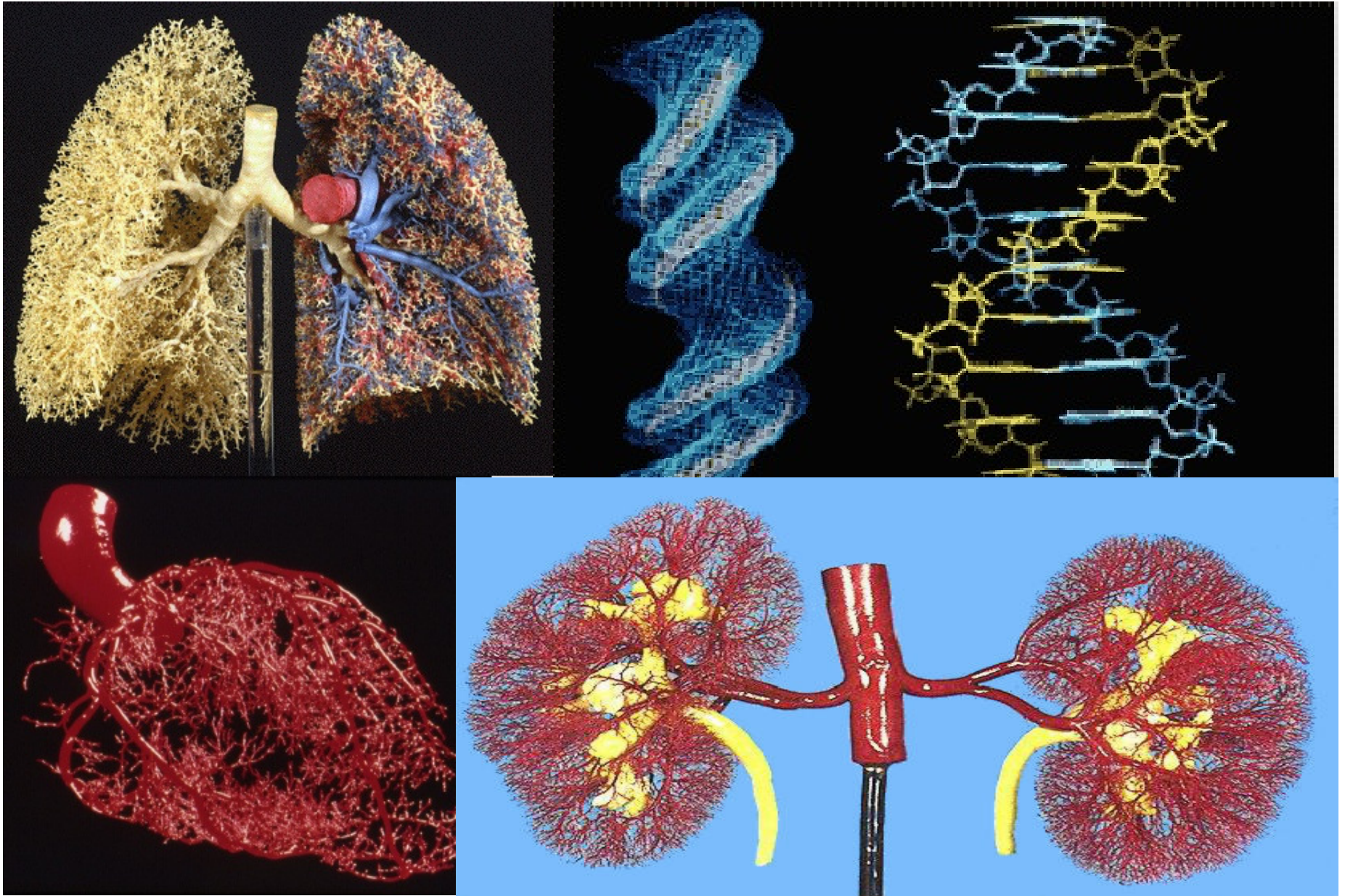
*tr* *s* *6* *6* *6*

This system contains measures 61 through 64. The right-hand part features slurs, trills (*tr*), and sixteenth-note runs. The left-hand part has a steady accompaniment. A *s* marking is present in measure 61, and *6* markings are present in measures 62, 63, and 64.

# Em busca da regularidade



# Em busca da regularidade



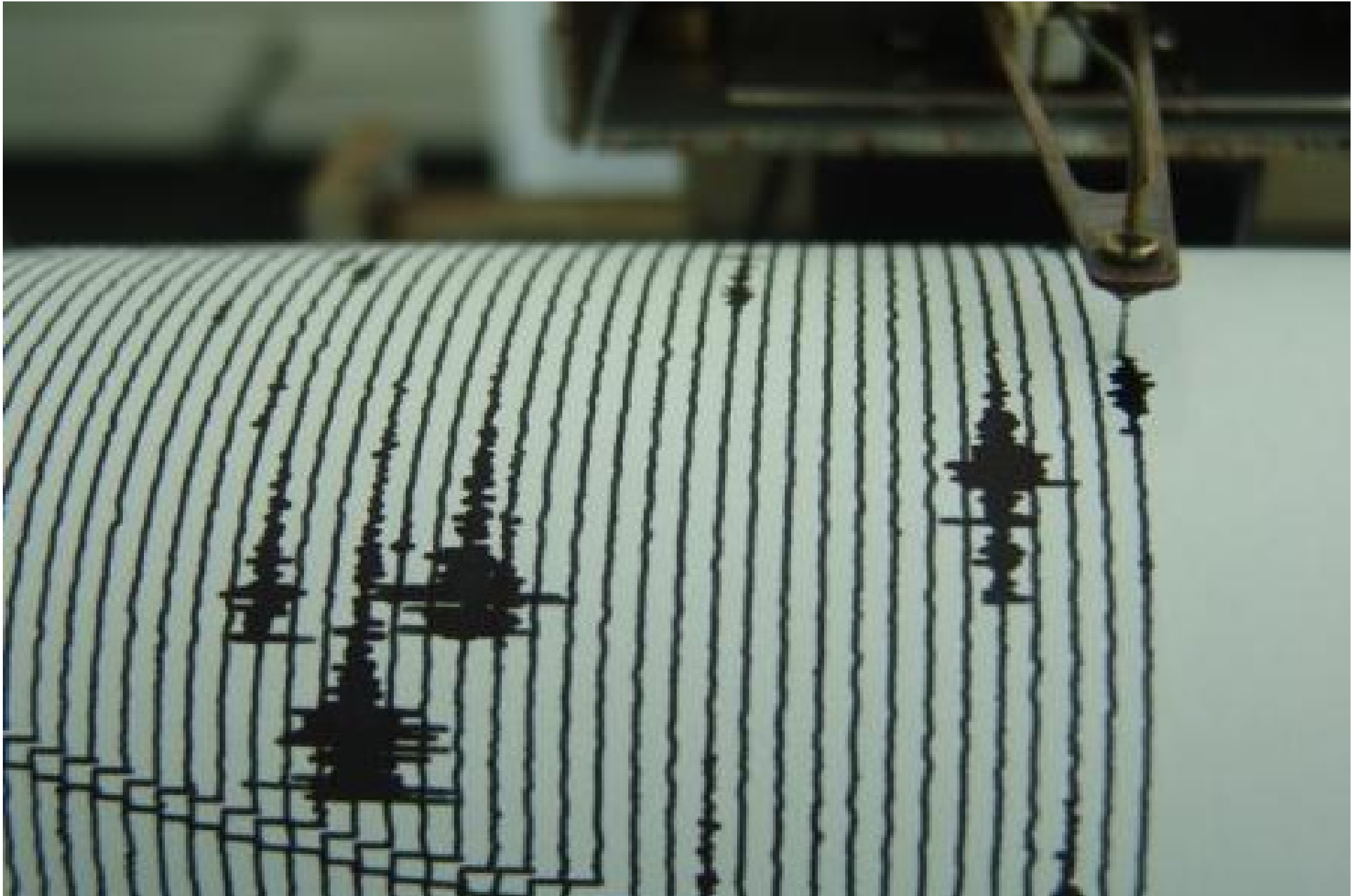
## Em busca da regularidade



**Para que serve estudar a regularidade?**



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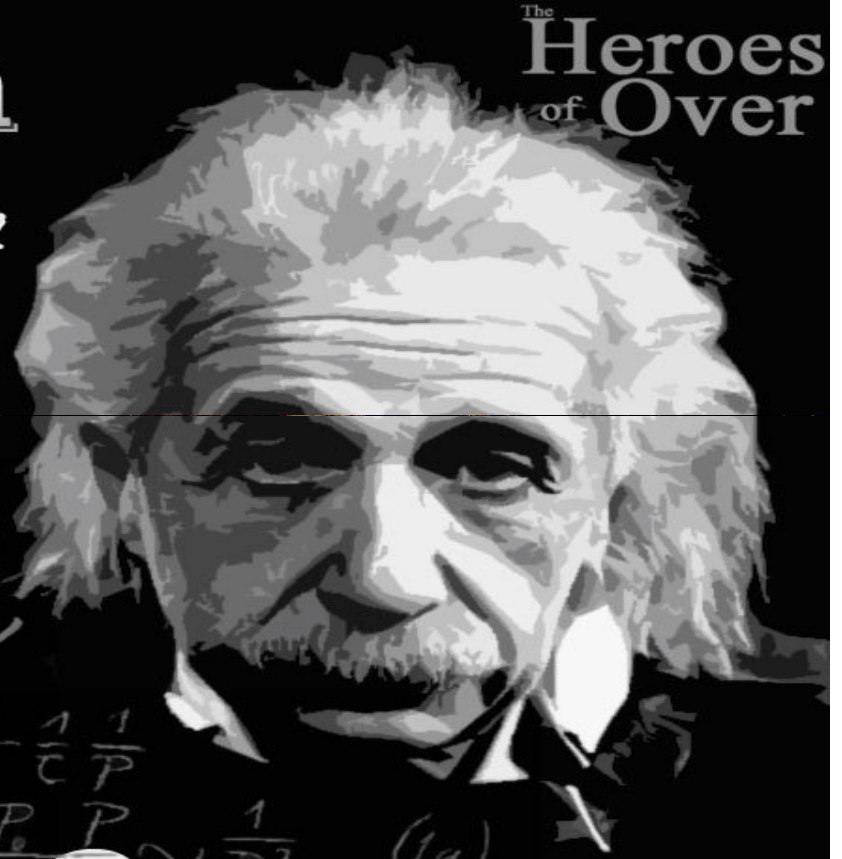


“Existe uma estranha ordem por detrás de toda a imprevisibilidade”

Einstein andava sempre à procura de uma simples fórmula que governasse tudo!

# Einstein

*“A mente que se abre a  
uma nova idéia jamais  
voltará ao seu tamanho  
original.”*



The  
Heroes  
of Over

$$E = mc^2$$

$$R_{\mu\nu} = 0$$
$$D \frac{1}{c} \frac{1}{c} \frac{dt}{dt} - \frac{1}{c} \frac{1}{c} \frac{1}{P}$$
$$D^2 = \frac{1}{p^2} \frac{P_0}{P} \sim \frac{1}{p^2} \quad (1a)$$
$$D = \frac{1}{c} \frac{P_0}{P} \sim \frac{1}{c} \quad (2a)$$
$$D^2 \sim 10^{53}$$

# Evolução de um evento natural

Dinâmica do evento

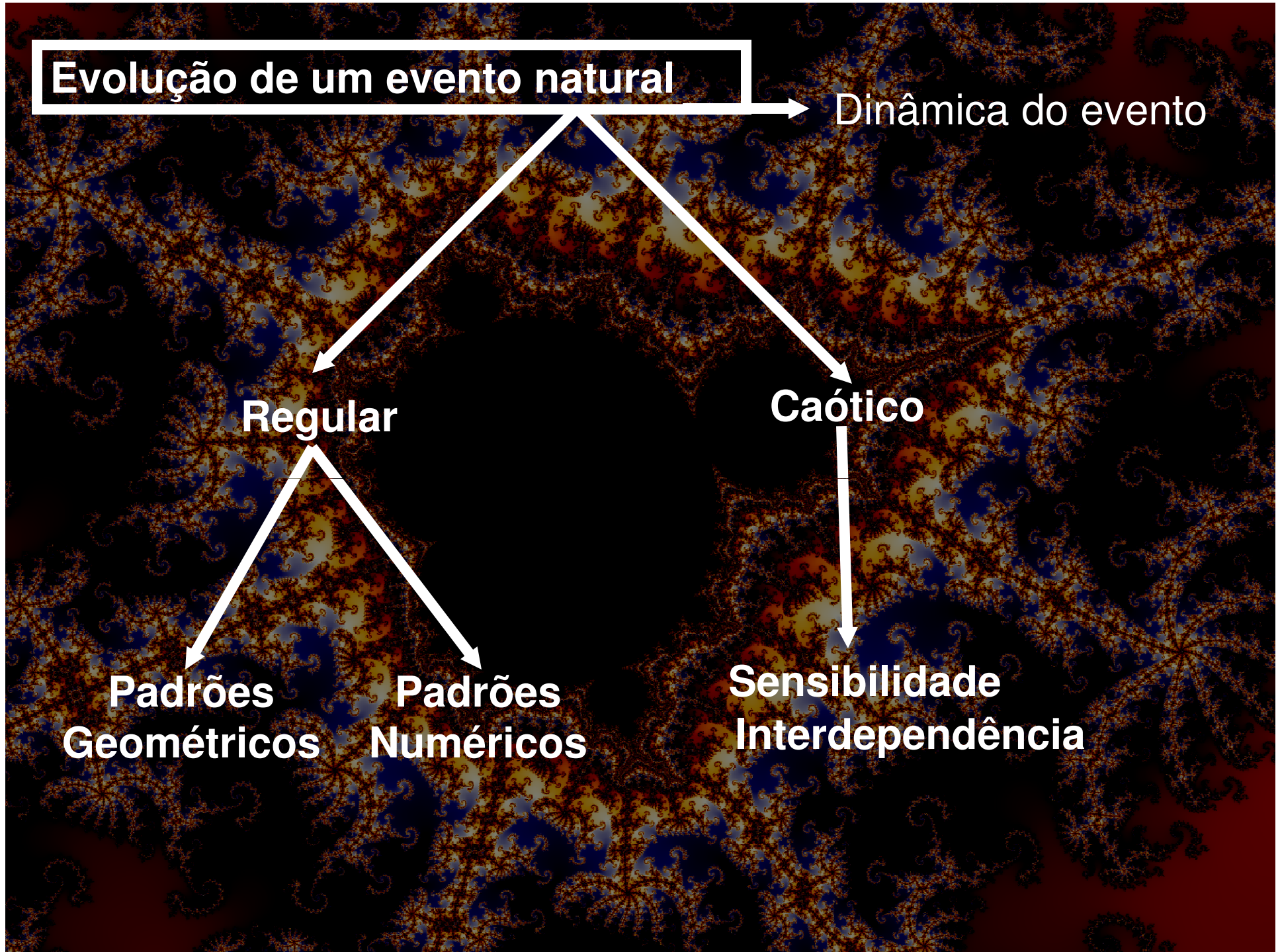
Regular

Caótico

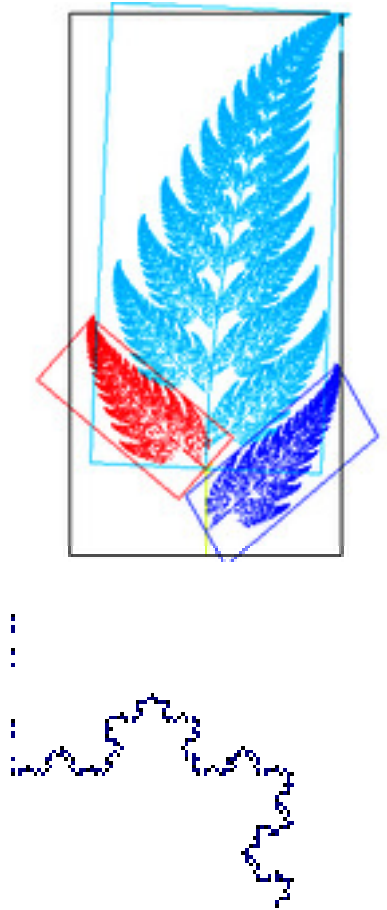
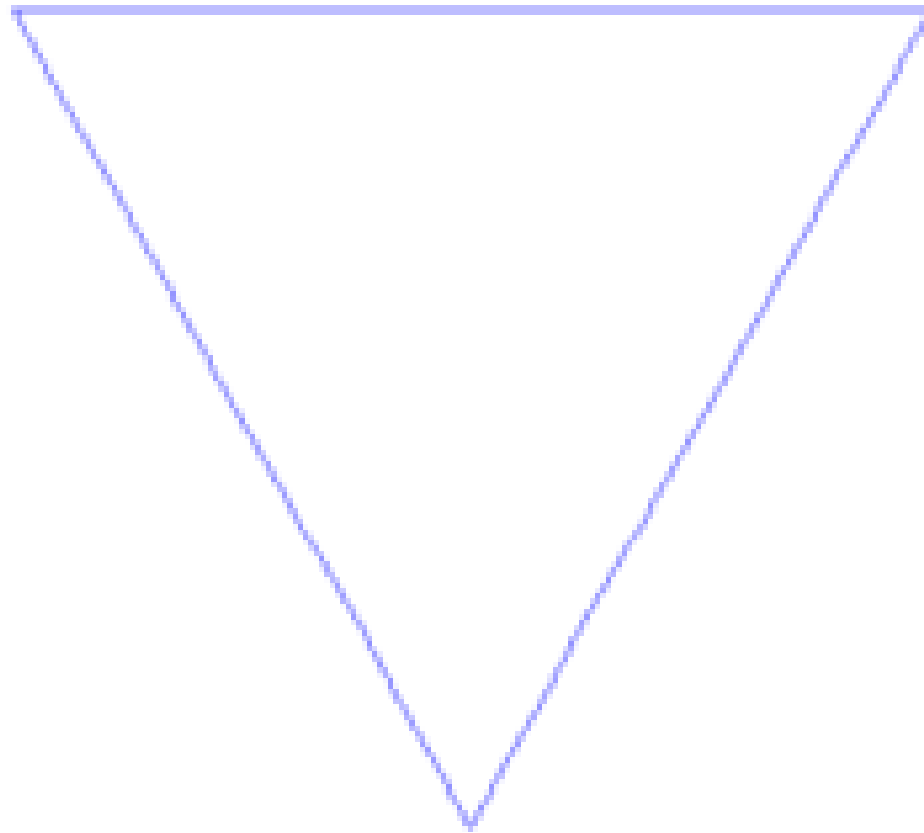
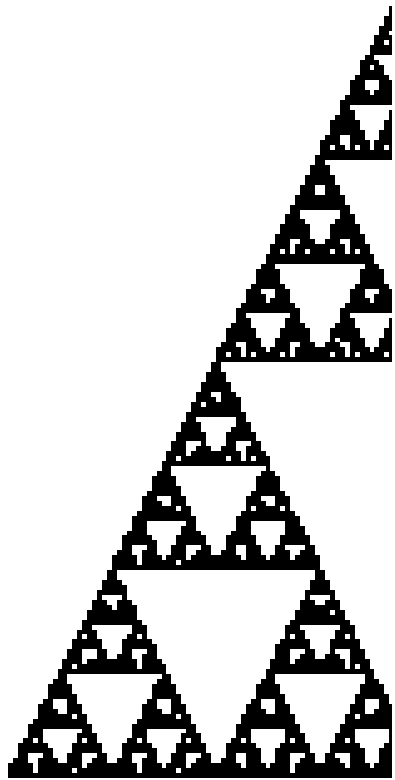
Padrões  
Geométricos

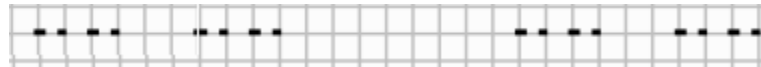
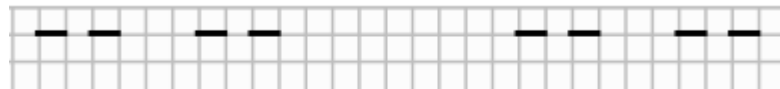
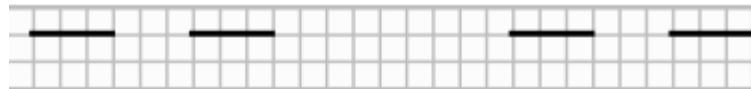
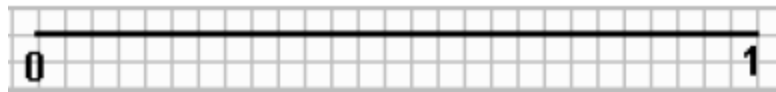
Padrões  
Numéricos

Sensibilidade  
Interdependência



# Padrão Geométrico - Auto semelhança





Conjunto de Cantor

Comprimento

1

$$\frac{2}{3} \quad \left(\frac{2}{3}\right)^1$$

$$\frac{4}{9} \quad \left(\frac{2}{3}\right)^2$$

$$\frac{8}{27} \quad \left(\frac{2}{3}\right)^3$$

$$\frac{16}{81} \quad \left(\frac{2}{3}\right)^4$$

$$C_n = \left(\frac{2}{3}\right)^{n-1}$$

# Comprimento



1



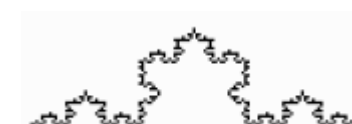
$$4 \times \frac{1}{3}$$



$$16 \times \frac{1}{9} \quad \left(\frac{4}{3}\right)^2$$



$$64 \times \frac{1}{27} \quad \left(\frac{4}{3}\right)^3$$



$$256 \times \frac{1}{81} \quad \left(\frac{4}{3}\right)^4$$

Curva de Koch

$$k_n = \left(\frac{4}{3}\right)^{n-1}$$

1



$$\left(\frac{20}{9}\right)^1$$



$$\left(\frac{20}{9}\right)^2$$



$$\left(\frac{20}{9}\right)^3$$



$$\left(\frac{20}{9}\right)^4$$

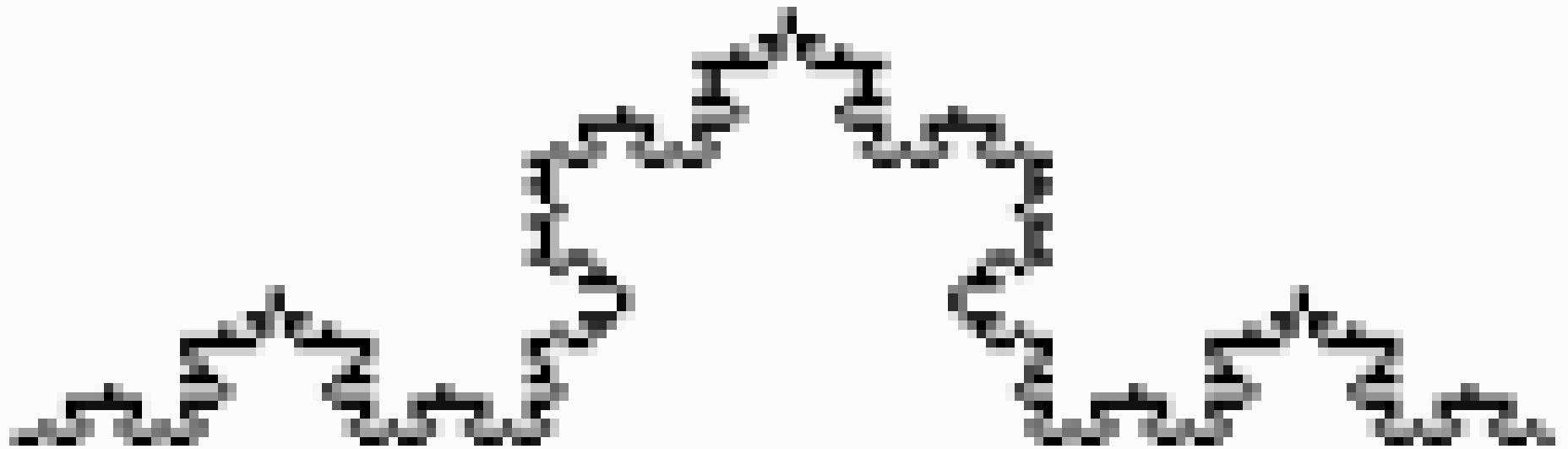
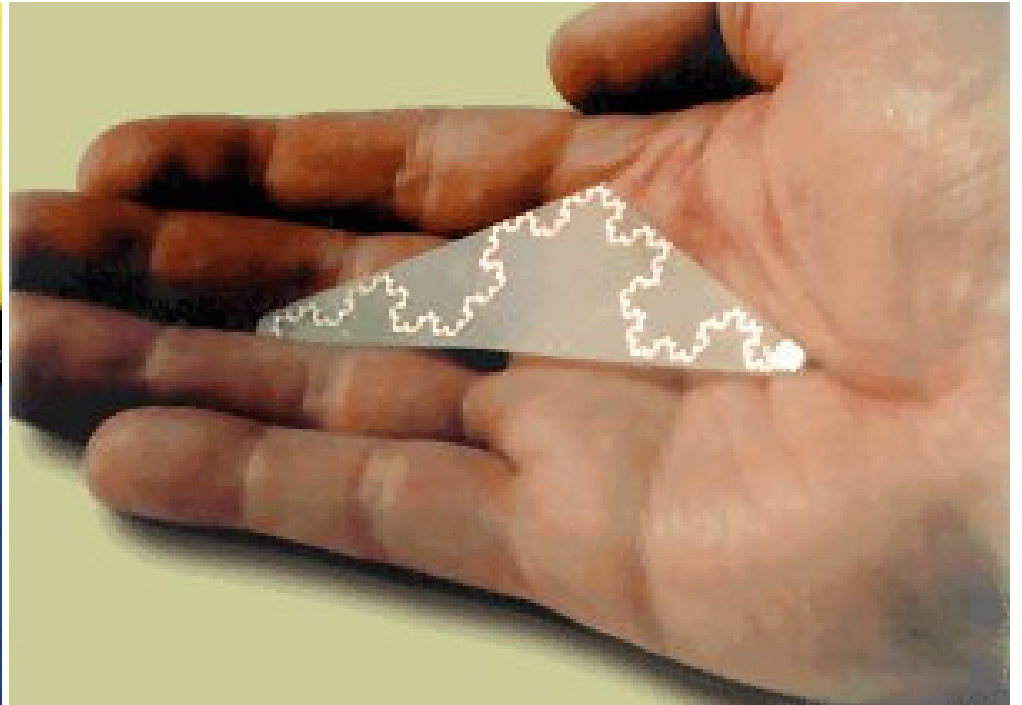
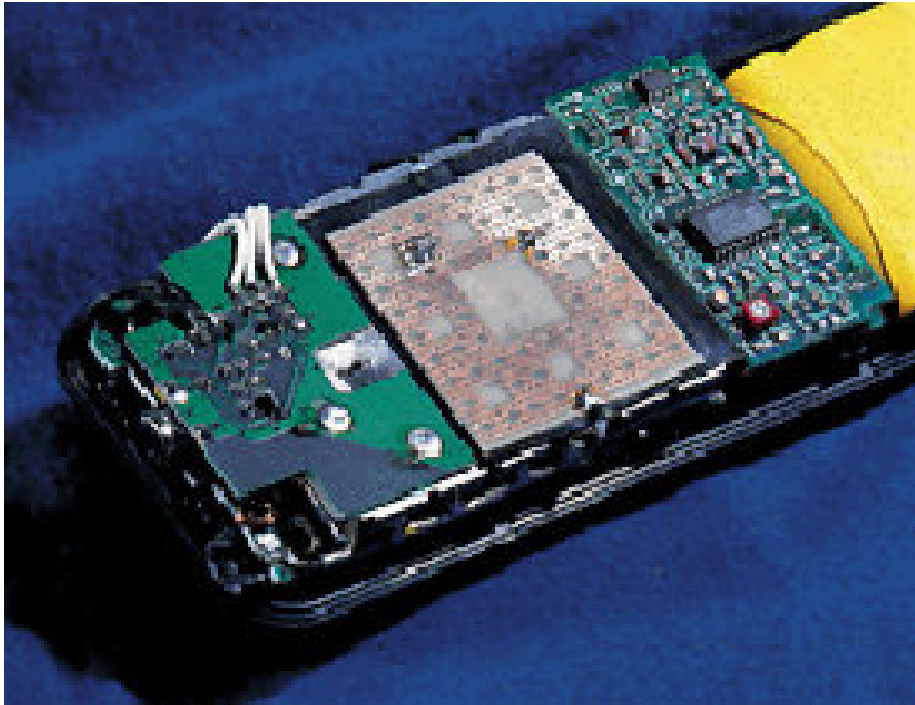


$$\left(\frac{20}{9}\right)^5$$



$$\left(\frac{20}{9}\right)^6$$







Triângulo Sierpinski

Perímetro

3

Área

1

$\frac{9}{2}$

$$2\left(\frac{3}{2}\right)^2$$

$\frac{3}{4}$

$\frac{27}{4}$

$$2\left(\frac{3}{2}\right)^3$$

$\left(\frac{3}{4}\right)^2$

$\frac{81}{8}$

$$2\left(\frac{3}{2}\right)^4$$

$\left(\frac{3}{4}\right)^3$

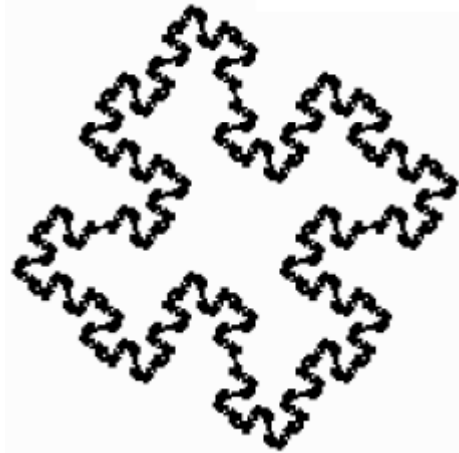
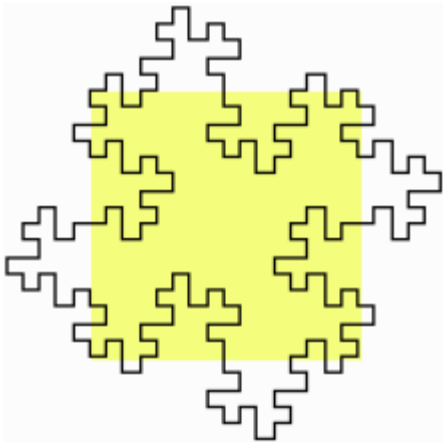
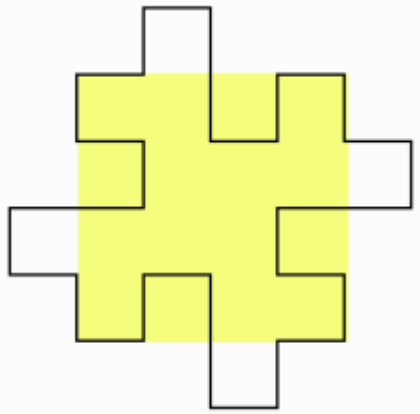
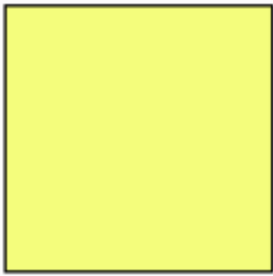
$\frac{241}{16}$

$$2\left(\frac{3}{2}\right)^5$$

$\left(\frac{3}{4}\right)^4$

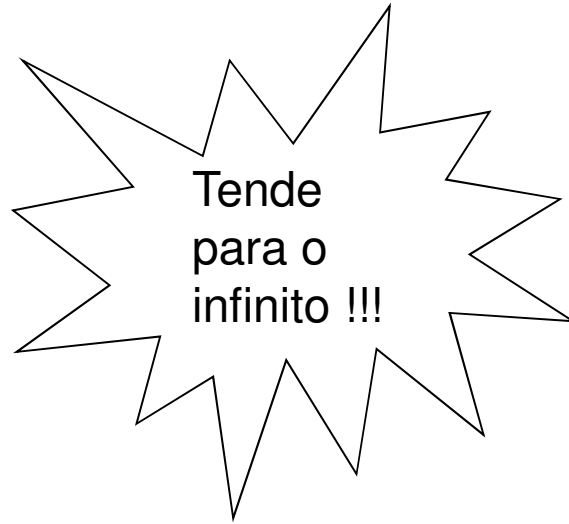
$$p_n = 2\left(\frac{3}{2}\right)^n$$

$$a_n = \left(\frac{3}{4}\right)^{n-1}$$



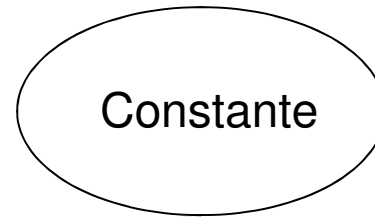
Ilha de Minkowski

Perímetro



Tende  
para o  
infinito !!!

Área



Constante

# Padrão numérico - Sucessão

Fibonacci 1, 1, 2, 3, 5, 8, 13, 21, 34,...



$$1^2+1^2=1 \times 2$$

$$1^2+1^2+2^2=2 \times 3$$

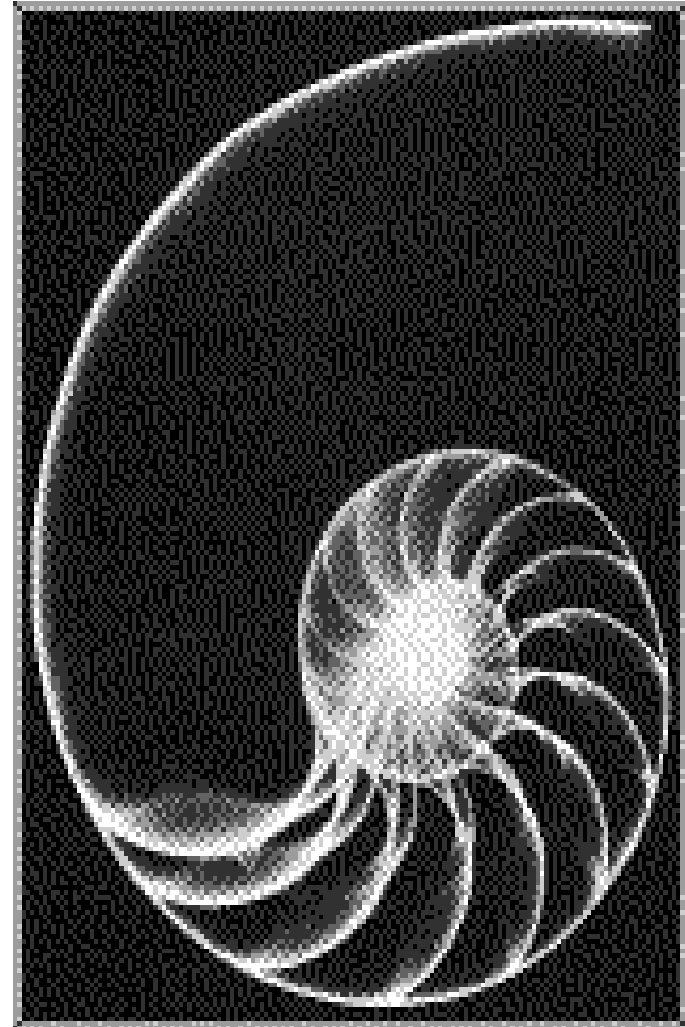
$$1^2+1^2+2^2+3^2=3 \times 5$$

$$1^2+1^2+2^2+3^2+5^2=5 \times 8$$

$$1^2+1^2+2^2+3^2+5^2+8^2=8 \times 13$$

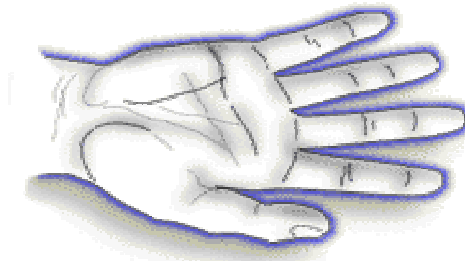
$$1^2+1^2+2^2+3^2+5^2+8^2+13^2=13 \times 21$$

$$1^2+1^2+2^2+3^2+5^2+\dots+(F(n))^2=F(n).F(n+1)$$



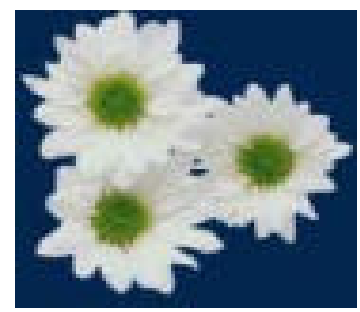
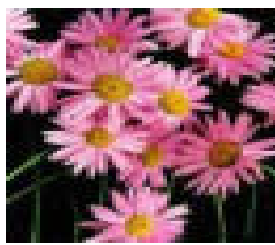
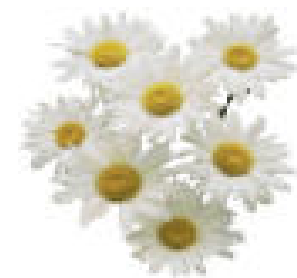


Algumas flores têm **5** pétalas.  
O ser humano tem **2** mãos, cada uma com **5** dedos  
e cada dedo dividido em **3** partes.



O ananás tem **8** diagonais num sentido e **13** no outro

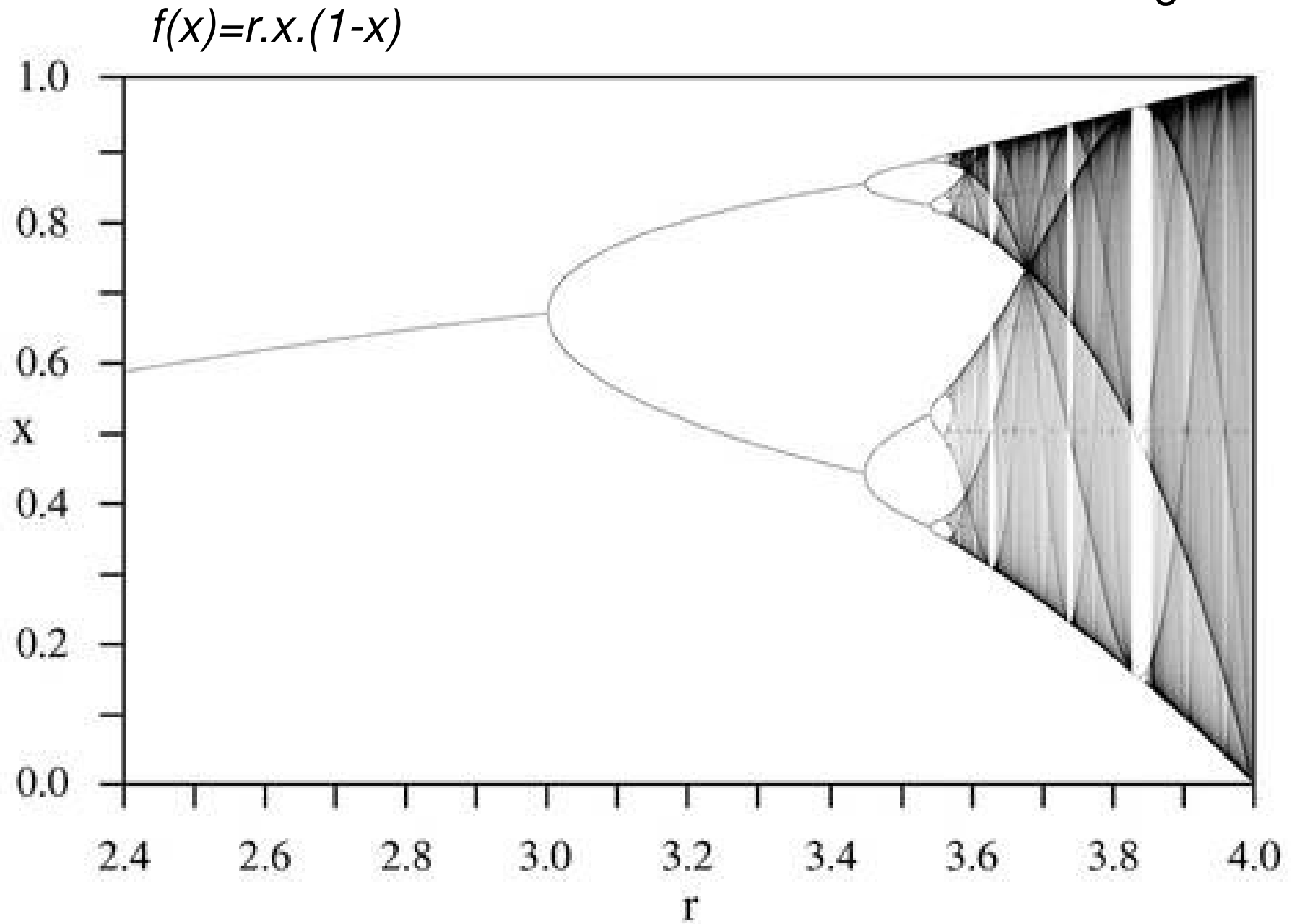
Porque será que as margaridas têm geralmente **34**, **55** ou **89** pétalas?





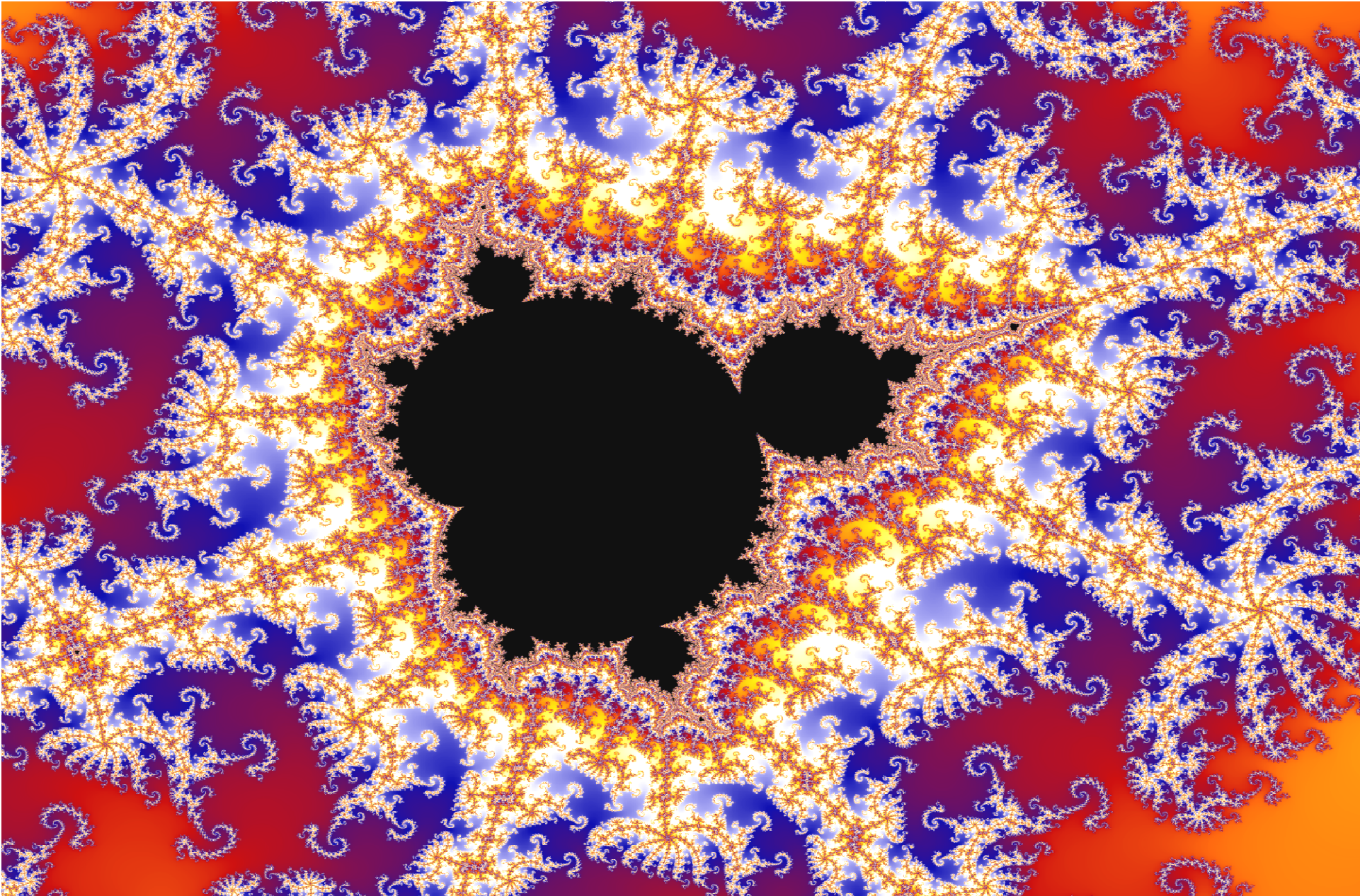
# Compreender os Eventos Caóticos

Diagramas



# Compreender os Eventos Caóticos

Diagramas





**Será a natureza caótica ?**