MATHEMATICAL EDUCATION AND INTERDISCIPLINARITY: PROMOTING PUBLIC AWARENESS OF MATHEMATICS IN THE AZORES

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Abstract:

Mathematical literacy in Portugal is very unsatisfactory in what concerns international standards. Even more disturbingly, the Azores archipelago ranks as one of the worst regions of Portugal in this respect. We reason that the popularisation of Mathematics through interactive exhibitions and activities can contribute actively to disseminate mathematical knowledge, increase awareness of the importance of Mathematics in today's world and change its negative perception by the majority of the citizens. Although a significant investment has been undertaken by the local regional government in creating several science centres for the popularisation of Science, there is no centre for the popularisation of Mathematics. We present our first steps towards bringing Mathematics to unconventional settings by means of hands-on activities. We describe in some detail three activities. One activity has to do with applying trigonometry to measure distances in Astronomy, which can also be applied to Earth objects. Another activity concerns the presence of numerical patterns in the Azorean flora. The third activity explores geometrical patterns in the Azorean cultural heritage. It is our understanding that the implementation of these and other easy-to-follow and challenging activities will contribute to the awareness of the importance and beauty of Mathematics.

Keywords: Recreational Mathematics, Interdisciplinarity, Astronomy, Biology, Cultural heritage

Introduction

Mathematics is present in our daily life, even in the lives of those suffering from *math*-related phobias. However, too many people are unaware of this, which may be both a consequence and a cause of a poor mathematical literacy. Therefore, it is crucial to raise public interest towards Mathematics. We suggest that this goal can be achieved partly through an investment in the popularization of Mathematics so that a broad cross-section of the public can have contact with mathematical ideas and concepts and realise that Mathematics is present in our daily life.

The results obtained by the Portuguese students, and particularly the Azorean students, in Mathematics are unsatisfactory (Mullis *et al.*, 2012; OECD, 2014; Soares, 2015). These results are a reflection of the mathematical illiteracy and of the little importance the society as a whole gives to Mathematics. Too often there is a vicious cycle in which parents pay little attention to their children lack of success because they had a similar experience as students. The changes, therefore, must occur across the society.

Flewelling and Higginson (2001) suggest that students can overcome math anxiety and find learning mathematics to be a rewarding and successful experience when teachers change the typical classroom culture by having students experience rich learning tasks. Also, in *How the Brain Learns Mathematics*, Sousa (2008) argues “Information is most likely to get stored if it makes sense and has meaning”. Thus, we contend that activities related with popularization of Mathematics can also contribute to the mathematical education of students.

Here, we present very preliminary ideas of how popularization of Mathematics can be achieved in the Azores.

Popularization of Science and Mathematics

The popularization of Mathematics has a very long history and we shall only mention a few important contributions. The very famous mathematician David Hilbert gave popular lectures in 1921 for students returning to the university after the war and continued the series through the 1920’s (Reid, 1970). Lucas’ Tower of Hanoi game is an example with a much broader impact (Hinz, 1989). Many other examples throughout history can be given. A particularly relevant example is the work of a lifetime of Martin Gardner. Gardner wrote the famous “Mathematical Games and Recreations” column for *Scientific American* magazine for more than 25 years. This work has inspired hundreds of readers to delve more deeply into the large world of Mathematics.

Even though popularization has a long tradition, it is a relatively recent topic that motivated the fifth international study of the International

Commission on Mathematical Instruction (Howson & Kahane, 1990), and had seen important developments after the World Mathematical Year 2000.

Worldwide there are hundreds of science museums and science centres that provide interactive exhibitions, in many different fields of Science. A visit to these centres does not replace the formal instruction in school, but it can be extremely inspiring and engage a child or an adult in the process of discovery. Until recently there was a lack of evidence supporting the general view that these visits play an important role in promoting the science learning of the public. The International Science Centre Impact Study, a consortium of 17 science centres in 13 countries (including Portugal) addressed this question and its results strongly support the contention that individuals who used science centres were significantly more likely to be science and technology literate and engaged citizens (Falk *et al.*, 2014).

Unfortunately, there are very few museums or centres dedicated to the popularization of Mathematics. There are a few exceptions, like *Mathematikum*, in Gieβen (Germany), or the recent *MoMath*, in New York (USA). Some large science museums have permanent or regular mathematics exhibitions, like the *Cité de Sciences et L'industries,* in Paris (France), the *Pavilhão do Conhecimento,* in Lisbon (Portugal), or the announced new mathematics gallery in the London Science museum (UK), due to open in 2016. Popularization of Mathematics, like the popularization of Science, is in the razor's edge: the difficult balance between the rigorous language of Mathematics and the need to make the message accessible to a large audience.

Popularization of Science and Mathematics in Portugal

In the last two decades there has been a boom of science centres in Portugal[13]. Also, the involvement of scientists and academics in activities dedicated to the popularization of Science is now extremely common (e.g. Russo & Christensen, 2010). Yet, there is still no museum or centre specifically dedicated to Mathematics. Also, it is relevant to notice that in 1185 science news published in one of the most respected national newspapers, *Público*, in the year 2005, Mathematics was the area of knowledge least present, in 23rd place with only 11 articles (Mourão, 2007).

Fortunately, things appear to be slowly improving. The *Ludus Association*, created less than a decade ago, has given a seminal contribution to the popularization of Mathematics in Portugal. This association aims the promotion and dissemination of Mathematics, particularly Recreational Mathematics and Abstract Games in its various forms, such as educational,

[13] Ciência Viva Centres Network's website: http://www.cienciaviva.pt/centroscv/rede.

cultural, historical and competitive. Another important example is the *Atractor Association* (Arala Chaves, 2006). The television series *Isto é Matemática*, that consisted in short episodes dedicated to the popularization of Mathematics, promoted by the Portuguese Mathematical Society, achieved a considerable success in recent years. For other examples, see Eiró *et al.* (2012).

This important work of the last years must be adapted and reproduced to reach the different parts of the country and the widest possible audience in a regular and consistent way, so that there is a positive change in the society towards Mathematics.

Popularization of Science and Mathematics in the Azores

Presently, the Azores archipelago has six science centres. Four of these centres are located in the main island of S. Miguel and are dedicated to Astronomy, Geoscience and Volcanology, Microorganisms, and to the Natural Sciences and Technology. The others are located in the islands of Terceira and Faial and are dedicated to the Environment and Climate, and to Marine Life, respectively. The other six islands, although only representing 15% of the population, have no centre or museum dedicated to Science. Again, there is no centre or museum dedicated to the popularization of Mathematics in the archipelago, nor there are plans in this direction.

Although there is no centre for promoting Mathematics in the Azores, in the recent past there have been several initiatives dedicated to this purpose organised by the Department of Mathematics of the University of the Azores. Some of the most relevant initiatives were two series of talks *A walk through science: Mathematics in or lives*[14] and *Mathematics in the Afternoon – Azores*[15]. In 2013, in collaboration with the *Ludus Association*, this department organised two international meetings: *Board Game Studies Colloquium XVI*[16] and *Recreational Mathematics Colloquium III*[17]. Also, there have been frequent presences in the local television and articles in the local newspapers aiming to promote Mathematics[18].

Before considering how to develop further the popularization of Mathematics in the region, let us first mention one particular difficulty the Azorean science centres face in trying to achieve their goals successfully. If a school or a family want to learn about Astronomy or Volcanism and they do not live in S. Miguel Island they cannot realistically have access to the science centres created for these purposes – the geography and the cost of

[14] See http://www.ciencia.uac.pt.
[15] See http://www.tmacores.uac.pt.
[16] See http://ludicum.org/ev/bgs/13.
[17] See http://ludicum.org/ev/rm/13.
[18] See http://sites.uac.pt/rteixeira/divulgacao.

traveling are very real obstacles. It is true that the astronomy centre, OASA, does occasionally visit schools in other islands with their portable planetarium and telescopes, and this somewhat minimizes the problem, but this is not the case for the other science centres. What this implies is that the creation of another centre, in this case for the popularization of Mathematics, is probably not the best option as our goal is to reach most of the population. Also, the economic difficulties the country and the region are presently facing also contribute to make this option unviable. We therefore propose to popularize Mathematics based on the following ideas:

- Conceive and build interactive activities and portable exhibitions;
- Present some of these activities in a "mathematical corner" in the different science centres. If possible, some of the activities should be related with Mathematics that is relevant to the particular area of knowledge of that centre. Those islands that do not have a science centre could receive these activities in a local public library, museum or school;
- Develop contents that reach a large audience, which is not limited to schools and to those that visit museums, science centres and other institutions alike. One possibility is to present mathematical concepts and ideas in guided tours around towns (see Sections 4 and 5). Another possibility is to offer games and activities, where Mathematics is actively present, in local fairs or festivities.

Next, we present some activities to be implemented in this context.

Exploring methods to measure distances

One of the most important problems in Astronomy is the measurement of distances to other celestial bodies. This question arises in determining the distance to the Sun or the Moon but also in determining the distance to a nearby star or a very distant galaxy. There are many different methods in Astronomy to measure distances and, in general, each method is limited to a particular range of distances, originating the concept of a cosmic distance ladder: one method can be used to measure nearby distances, a second can be used to measure nearby to intermediate distances, and so on. At the base of this ladder is the method of parallax that gives a direct distance measurement to nearby stars.

The method of parallax is based on simple trigonometry and, because it can also be used to determine terrestrial distances, we propose an activity to experimentally determine distances, which perhaps could be implemented in the OASA centre for Astronomy. This aims to illustrate concepts of high school mathematics in real life problems and also contribute to a better understanding of this important method in Astronomy.

The different steps required to implement this activity can be summarized as:

▪ One starts by understanding the parallax concept by closing one eye and moving one's head until obtaining an alignment of two objects at different distances. After switching the closed eye with the open eye, one observes the effect of parallax – the two objects are no longer aligned.

▪ One observes that to detect this effect the closer object cannot be too distant – for very distant objects the distance between our eyes is not enough for the parallax to be noticeable.

▪ One finds a target object for which one wants to measure the distance, call it D, like a tree or a cross in a church. One must make sure there are other objects in the background at a much larger distance.

▪ One observes the target object from some place A so that the object is aligned with a background object referred as X. One moves in a perpendicular direction to the object a sufficient distance so that the object is now aligned with another background object Y and refer to this place as B (cf. Figure 1).

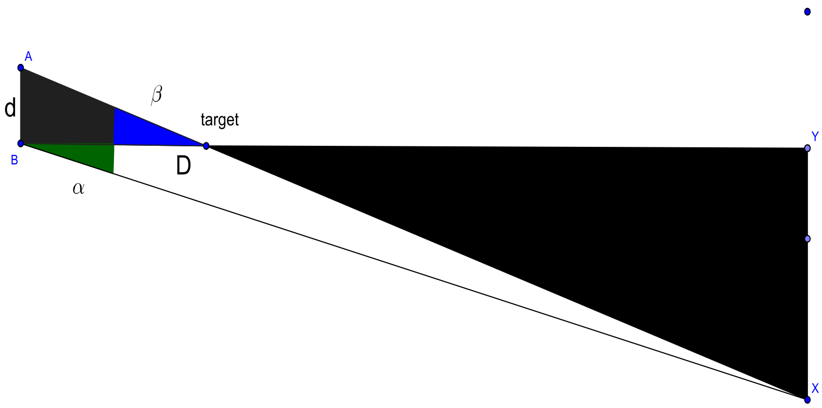

Figure 1: A schematic representation of how to use parallax to measure distances.

▪ With an appropriate device, to be described ahead, one measures the angle between the direction B to Y and B to X and denote it by α. With a tape measure the distance between A and B, call it d. Notice that this angle is small and it gets smaller and harder to measure as the chosen target object is further away.

▪ All the necessary information to determine the distance is now in your hands! As the distance to the target object is much smaller than the distance to the background objects we have $\tan \alpha \approx \tan \beta = d/D$, or $D \approx d \cot \alpha$.

▪ After one or several measurements, the application of this method in Astronomy becomes easier to explain and understand.

To measure the angle we propose a simple although not very rigorous method using a protractor (obviously if a theodolite is available it can be used with much better results). An image of a protractor can be printed and glued on cardboard or templex and the angles can be marked on this surface using pins.

Exploring numerical patterns in the Azorean flora

The cultivation of pineapple (*Ananas comosus*, Cayenne variety) is a tradition on the island of S. Miguel. Originating in South America, pineapples reached the Azores in mid-nineteenth century. Initially no more than an ornamental plant, it became an industry after some years. With time, and due to their unique aroma and flavour, pineapples ended up being an emblematic product of the Azores.

There are also other ways of looking at pineapples and these are connected with an interesting mathematical pattern. To better understand the connection between pineapples and Mathematics, let us call to mind Leonardo of Pisa (circa 1170-1240). Known as Fibonacci, Leonardo of Pisa was an important medieval mathematician. In 1202, Fibonacci wrote a treaty called *Liber Abaci*. One of the chapters dwelled on problem solving; there, Fibonacci presented the problem: "A man placed a pair of rabbits, male and female, in a place with walls all around. How many pairs can be bred in one year, bearing in mind that, every month, each pair generates another one, which from the second month becomes productive?"

Let's try and solve this problem. In order to do that, we need to analyse what happens at the beginning of each month. Let's start with a pair of newborn rabbits. In the second month, this pair becomes adult, so in the following month they give birth to the first pair of rabbits. Thus, in the third month there are already two pairs. In the fourth month, the initial pair gives birth to another pair, while the first pair becomes adult. On the whole, there are three pairs of rabbits. In the fifth month, both the initial pair and their first offspring, now adults, have new descendants. If to these we add the pair of bunnies from the previous month, there is a total of five pairs of rabbits. If one carries on with the problem, the numbers obtained shall be 1, 1, 2, 3, 5, 8, 13, 21, … The series of numbers obtained is known as the *Fibonacci sequence* and obeys to a most interesting pattern: each number is the result of the addition of the two previous ones (for example, 8=5+3 and 13=8+5).

But what is the connection between these numbers and pineapples? If we really look at a pineapple, we notice that the diamond-shaped markings that make up its skin (called bracts) are organized in spirals (cf. Figure 2). A closer look allows us to conclude that there are two families of parallel spirals, some whirling to the right and others to the left. What is astonishing is that if we count the total number of spirals of each family, we always get the same numbers: 8 and 13! What an amazing mathematical pattern! In the hustle and bustle of every day life we sometimes do not realise that Nature is full of numerical patterns, and pineapples are no exception.

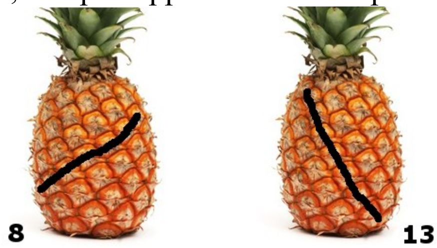

Figure 2: The numerical patterns of the Azorean pineapple.

There are many other captivating ways of exploring the Fibonacci sequence. Next, we propose exploring this sequence in an itinerary on the island of Faial.

Stop One: the Capelo Park. The Natural Forest Reserve of the Capelo Park has an area of 96 hectares and a significant diversity of endemic plants. The Maritime Pine (*Pinus pinaster*) can also be found there. Given the abundance in pines, it is quite easy to find pine cones.

Figures 3A and 3B show a common pine cone photographed from the base up, with the connecting stem in the middle. A closer look reveals that there are two sets of spirals: one whirls to the left (counter-clockwise) and the other to the right (clockwise). One of the sets has 8 spirals and the other has 13, two consecutive Fibonacci numbers.

Stop Two: the Seaside Avenue of Horta. It is possible to find several palm trees in the green areas of the avenue, namely Canary Island Date Palms (*Phoenix canariensis*).

In Figure 3C we can see the trunk of one of these palm trees. Again we can find two families of spirals: one whirling to the left, the other to the right. In the image only one spiral of each family is identified. The spirals that whirl clockwise had a steeper climb than the others (looking at the photo we can see that those spirals do not whirl around the trunk completely, though the counter-clockwise spirals whirl around more than twice). If we count the number of spirals in each family, we come up again with the numbers 8 and 13!

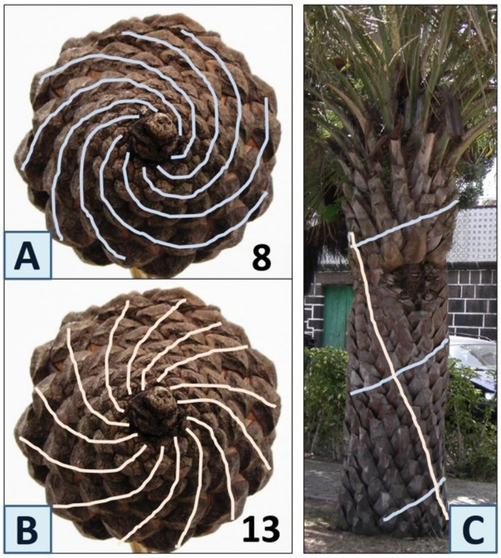

Figure 3: The numerical patterns of a pine cone and of a palm tree.

But why can we frequently find Fibonacci numbers in plants? The answer lies in the way they grow. If we look at the tip of an offshoot, we can see little bumps, called *primordia*, from which all the plants characteristics are going to evolve. The general disposition of the leaves and petals is defined right at the beginning, when the primordia are being formed. Thus, we just have to study the way the primordia appear (Stewart, 1995).

What we can observe is that the consecutive primordia are distributed along a spiral, called the generating spiral. The significant quantitative characteristic is the existence of a single angle between the consecutive primordia that are, thus, equally spaced along the generating spiral. The angle's amplitude, called the golden angle, generally has 137.5 degrees. In the nineties of the twentieth century, Stephane Douady and Yves Couder identified a possible physical cause for this phenomenon (Douady & Couder, 1992). The physical systems evolve normally to states that minimise energy. What the experience by the two physicists suggests is that the golden angle that characterises plants growth represents simply a minimum energy state for a sprout system mutually repelled.

But what connection is there between the golden angle and Fibonacci's sequence? If we take pairs of consecutive numbers of the sequence and divide each number by the previous one, we get 1/1=1; 2/1=2; 3/2=1.5; 5/3=1.666…; 8/5=1.6; 13/8=1.625; 21/13= 1.615…; 34/21=1.619… As we continue with this calculation, the results approach a specific value, sometimes by default, others by excess. This value is a non-periodic infinite tithe, approximately equal to 1.618, represented by the Greek letter *phi* (Φ) and also known as the *golden number*. By dividing 360 degrees by *phi* we get an angle of approximately 222.5 degrees. Since this is a value higher than 180 degrees, we should consider the corresponding convex angle and, then, subtract from 360 degrees the value obtained. The result is 137.5 degrees!

In 1907 the mathematician Iterson proved that by joining consecutively a set of dots separated by 136.5 degrees along a generating spiral, it is possible to identify two families of spirals, some whirling clockwise, others counter-clockwise. In the overwhelming majority of the times, the total number of spirals in each family is a Fibonacci number (Livio, 2012). Even more, the values are two consecutive Fibonacci numbers, this is so because the ratio between these numbers is close to the golden number.

Stop Three: Faial's Botanical Garden. The Botanical Garden covers an area of 8000 square meters and plays an important scientific and educational role, mainly due to its considerable plant collection, mostly endemic.

The growth of the stem and the branches of a plant produces leaves with a regular spacing between them. Yet, the leaves do not overlap when growing, for that would deprive the bottom ones from the sun light and humidity needed. Hence the reason the leaves tend to grow in positions that allow them to get both sun and rain. Many times, the leaves disposition is characterised by a spiral growth, which allows this type of optimising (cf. Figure 4A). *Phyllotaxis*, from the Greek and meaning "arrangement of leaves", studies precisely the distribution patterns of the leaves along the plants stem.

Let's have a look at the leaf distribution of some of the plants of the Botanical Garden. In the case of the *Viburnum treleasei* (Figure 4B), the *Hypericum foliosum*, the *Picconia azorica* and the *Veronica Dabneyi*, the leaves are in opposite pairs. Since the pairs of leaves present themselves in alternate 90 degrees angles (perpendicular planes), this is called decussate. And although 2 is a Fibonacci number, there is more to it.

Many of the plants at the Botanical Garden have alternate spiral leaves, and the distribution of this kind of leaves along the stem presents an interesting and easy way to spot the pattern (cf. Figure 4A). In order to do that, we should choose as reference a leaf located in the lower part of the stem or of a branch (marked by an X in the figure). Then we should count the following leaves until we reach a leaf with the same orientation as the reference leaf. We should also count the number of whirls around the stem resulting from the plant's growth. In the vast majority of times, the two values are alternate Fibonacci numbers (which means that there is one single Fibonacci number between them). In the example shown in Figure 4A, the values are 8 (the number of leaves) and 3 (the number of whirls). Remember that 3 and 8 are alternate Fibonacci numbers, for they have 5 in between them in the Fibonacci sequence. By knowing this, we also know the angle that separates two consecutive leaves. In this case, the angle is 3/8 of a complete whirl, that is, 3/8x360=135 degrees (an approximate value of the golden angle: 137.5 degrees). Then we can say the plant has a *phillotactic fraction* of 3/8, a value that characterises the angular separation of two consecutive leaves along the stem.

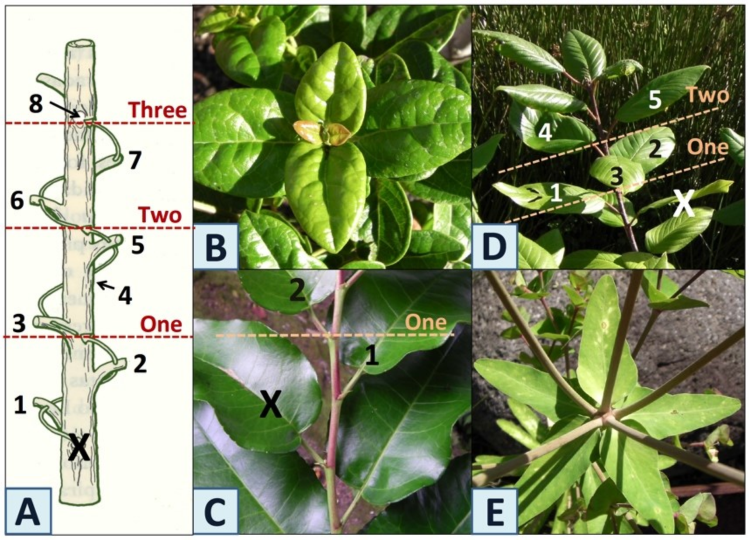

Figure 4: Examples of numerical patterns that can be found in Azorean flora.

Considering some examples together with their phyllotactic fractions, we have: *Prunus azorica* (Figure 4C), *Semele androgyna* and *Holcus rigidus* (1/2); *Ilex perado*, *Rumex azoricus*, *Frangula azorica* (Figure 4D) and the flower stems of *Azorina vidalii* (2/5); *Pericallis malvifolia*, *Myrica faya*, *Laurus azorica*, *Myrsine retusa* and *Vaccinium cylindraceum* (3/8); *Euphorbia azorica*, *Solidago sempervirens* and *Persea indica* (5/13).

Besides these, there are other interesting patterns: the 5 flower stems of the *Euphorbia azorica* based on a 5 leaf set (Figure 4E); the flowers of the *Ranunculus cortusifolius*, *Pericallis malvifolia*, *Viburnum treleasei*, *Myosotis azorica*, *Daucus carota*, *Rubus hochstetterorum*, *Angelica lignescens*, *Lysimachia azorica* and *Spergularia azorica* all have 5 petals. Thus, one can conclude that many of the flowers of endemic plants of the Azores have 5 petals.

An interesting challenge would be to visit the Botanical garden and observe these properties *in loco*. Another possibility would be to find the phyllotactic fractions of other plants present in our daily lives.

Exploring geometrical patterns in the Azorean cultural heritage

The relationship between Mathematics and Art has deep roots in many cultures (Washburn & Crowe, 1988). Ornamental patterns, namely those existing in Azorean sidewalks and traditional crafts, can be studied from a mathematical point of view. The mathematical classification is based on the concept of symmetry, a unifying principle that involves regularity, equality, order and repetition, some important aspects of structure and form in Art. Mathematicians apply the Theory of Groups to study the ways that a motif repeats and the manner in which one part of a pattern relates to the others.

The possibilities turn out to be quite limited: there are only four types of symmetries (*reflections*, *translations*, *rotations* and *glide reflections*). We can systematically identify all symmetries of a given plane figure, and so establish a classification for that figure. There are three types of figures: *rosettes* (figures with rotational symmetries and, in some cases, mirror symmetries), *frieze patterns* (figures with translational symmetries in one direction), and *wallpaper patterns* (figures with translation symmetries in more than one direction, which leads to the paving of the whole plane). Quite surprisingly, there are exactly seven types of frieze patterns and seventeen types of wallpaper patterns. Although there are infinitely many types of rosettes, their symmetry is simple and quite easy to understand. For more details, see Martin (1982) or Teixeira (2015).

The sidewalks and squares paved in Portuguese Pavement are one of the most characteristic aspects of the heritage of many Portuguese towns. In the Azores, artistic paving dates from mid-twentieth-century. It is present in sidewalks, squares and private atriums and gardens, bearing different artistic patterns where the black basalt contrasts with the white limestone.

Figure 5A shows a rosette from Campo de S. Francisco, in S. Miguel. We immediately find rotational symmetries: if we rotate the figure around its center by a given amplitude, the figure obtained totally overlaps the initial figure. The amplitude to be used depends on the number of the motif's repetitions. In this case, we have 8 repetitions, for which the rotation angle should have an amplitude of 360/8=45 degrees (or any of its multiples) as to obtain a symmetry of that figure. We can also find mirror symmetries (a total of 8 axes of symmetry that pass through the rotation center; 4 of them cut in half opposite petals and the remaining 4 separate consecutive petals).

In Figures 5B to 5D, it is shown three examples of friezes from Horta, in Faial island. One should bear in mind that all friezes have a common property: the translational symmetries in one direction, which implies precisely the repetition of a motif along a strip. The frieze of Figure 5B also has vertical reflection symmetries. On the other hand, the friezes of Figures 5C and 5D share one common property: the half-turn symmetries. If one imagines each of these upside down, the result is that the configuration does not change. Besides the translational and half-turn symmetries, the first does not possess other symmetries; the second has horizontal and vertical reflection symmetries.

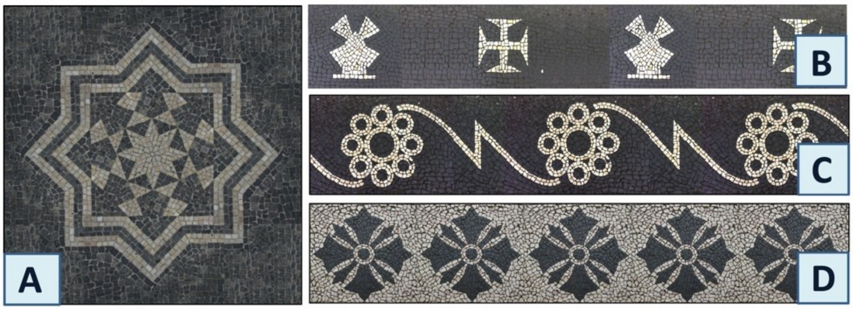

Figure 5: Examples of geometrical patterns that can be found in Azorean sidewalks.

Weaving is one of the first and oldest traditional industries of the archipelago of the Azores. The art of weaving, using wool, linen or cotton, demands accuracy, time and perseverance. But the beautiful works are a living proof of all the craft and art of the Azorean weavers[19]. We shall now analyse the symmetries found in some works created by Joana Dias, an artisan from Santa Maria island. We begin by presenting the friezes printed in two bookmarks, both inspired by motifs that can be found in many traditional skirts of the Azorean culture (cf. Figures 6A and 6B).

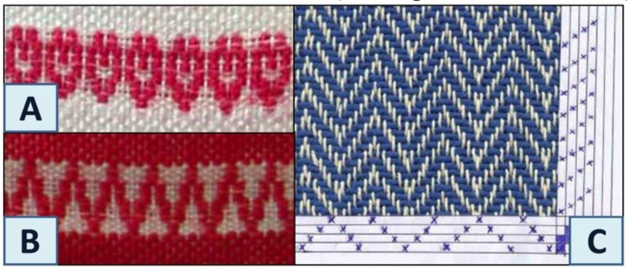

Figure 6: Some geometrical patterns that we can find in weaving works.

In the first, it is possible to identify translational symmetries in one direction and vertical reflections. In the second, we have the following types of symmetries: translational symmetries in one direction, half-turn symmetries (360/2 = 180 degrees rotations; if we imagine the strip upside down, its configuration is not altered); vertical reflections (the axes of symmetry are perpendicular to the frieze); and glide reflections (following the same direction as the frieze, these symmetries produce an effect similar to our footprints when walking barefoot on the sand). In Figure 6C, we have a 2-dimensional pattern (a wallpaper), with translational symmetries in more than one direction, half-turn symmetries, vertical reflections and horizontal glide reflections.

[19] The Regional Centre for Handicrafts's website: http://www.artesanato.azores.gov.pt.

This theme can be pursued by using symmetry itineraries for the pavements or by visiting local arts & crafts centres[20].

Another idea would be to implement an exhibition at a Science centre in which, for example, the photo of a frieze (from a pavement or a handcrafted object) would challenge visitors to go on an exploratory journey: each station would present a question, and the answer would lead to another station and another question, and so on; by answering correctly, the visitors in the end would be able to characterize the symmetries of the frieze and to classify it.

Conclusion

Mathematical literacy is important not only to produce better scientists and engineers but also to form better citizens – Mathematics is more than a body of knowledge, it is a way of thinking. There is an urgent need to change the perception of the relevance of Mathematics in the Azorean society. The purpose of popularization is to raise awareness, not just to educate, and the criterion of success is not only an increase of knowledge, but also a change of attitude (Howson and Kahane, 1990).

In this work we describe our initial efforts to develop interactive activities that can transmit the relevance and beauty of mathematical concepts and also their importance in science and in everyday life. Here we present in some detail two activities related with geometrical and numerical patterns in things that surround us and one activity addressing the use of trigonometry in Astronomy. These connections of Mathematics with Astronomy, Nature and Art can promote the awareness of Mathematics among young people and the general public.

Although the activities proposed here are almost cost-free, there will be other ones addressing other mathematical ideas that will inevitably be more elaborate and require substantial financial resources. Our plans are to first test and implement low cost activities and, after that, obtain financial support to develop more ambitious interactive exhibitions.

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[20] At http://sites.uac.pt/rteixeira/simetrias there are several symmetry itineraries available, as well as texts and media clippings on the subject.

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