Philosophical Notes

Concerning

Extension Intension

Extensional Measure

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AUTHOR

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E. J. MOREIRA DA SILVA

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Contents

The identity of quality and quantity present in measure is only implicit *at first, and not yet* posited. *This implies that each of the two determinations, whose unity is measure, also claim validity on its own account. In this way, on the one hand, quantitative determination of what is there can be altered, without its quality being affected thereby, but, on the other, this indifferent increase and decrease also has a limit, the transgression of which alters the quality.[…] When a quantitative alteration takes place, it appears, to start with, to be something quite innocent; but something quite different lurks behind it, and this seemingly innocent alteration of the quantitative is like a ruse with which to catch the qualitative.*

G. W. F. Hegel
The Encyclopaedia Logic

Introductory Note

AS THE EPIGRAPH THE AUTHOR CHOSE for this slim book of his obliquely indicates, he wrote the following Notes not thinking of them as an end in itself, but as a means—a *tool*, if you like—to achieve a specific purpose. Namely, better to understand a subject that he had often dealt with falsely convinced he needed no further enlightenment concerning it; a subject which, nonetheless, kept cropping up in the course of his philosophical, literary, and culture studies—in particular in connection with the strictly related topics of the Kantian REGULATORY IDEAS OF REASON (*regulative Ideen der Vernunft*) and the concept of CULTURE (*cultura homini, Bildung*), both of which, in their turn, are by nature indelibly bound with the (rational) Ideas of STATE and SOCIETY,—and which, thus, gradually forced on him the contrary conviction that he, in fact, needed to consider it afresh.

That subject is, of course, the one which the pages that follow try to place for him in a new

and more satisfying perspective: the relation between EXTENSION, INTENSION, and MEASURE, the latter viewed as numerically-determined, and numerical determination of, extensional magnitude.

The author embraced that task by first looking around for some guiding principles. That is to say, in short, by first looking around for help. It is possible, however, that, at the start, he did not look in the most promising places: those most prone to make justice to his expectation to find the above-mentioned topics construed in a light favourable to his specific interest in them and to his particular purposes.

Indeed, look in whatever direction he look, he saw himself faced, most of the times, either with propositions written in that kind of shorthand-language which is called 'equation' or 'formula'—a type of language which, unfortunately, he could not master, and thus understand properly, and which, as a result, not infrequently left him with the impression that to become addicted to it would ever be the most convenient way to unlearn how to write in longhand—or with propositions intently written in plain language, for plain people, like him, but which, nonetheless, little could appease his stubborn demand for logically-correct and accurate statements. That is to say, his strong addiction to genuinely plain statements, genuinely formulated by a simple mind trying to achieve the feat of addressing itself successfully to genuinely plain minds, like his. Just to give the hypotheti-

cal—and perhaps by now discontented—reader an instance, his labours once procured him the trouble of seeing himself faced with this (not at all rhetorical) question: "Are there concepts in physics that can be measured by the use of simpler schemas?"

Now, a concept is and will ever be... Yes, a concept. That is to say, the designation 'concept' signifies the concept (the signified) CONCEPT, just like, for instance, the designation 'man' signifies the concept (the signified) MAN.

Like all other concepts, the concepts CONCEPT and MAN are abstractions, and, therefore, are, like all abstractions, ideal (psychic) entities. The concept MAN, it is true, is an abstraction whose instances invariably are real (physical) entities, whereas the concept CONCEPT is an abstraction whose instances invariably are themselves, on the contrary, ideal entities: *e.g.*, the concepts EXTENSION and INTENSION. Nonetheless, all concepts are abstractions of either attributive or substantive qualities—such as *Extension* and *Intension*, again, or as the real substantive quality *Potatoness* and the ideal substantive quality *Conceptness*. And, of course, when they are viewed in respect to their (conceptual or formal) subjectuality and objectuality, as opposed to their (material) objectivity, all qualities are ultimately ideal entities. For, when you encounter a potato, for instance, you most certainly do not encounter *Potatoness*, but, when you think POTATO, you may very well encounter it.

it is not definitely possible to measure substantive qualities, therefore, not because they are conceptual entities instead of real entities, but simply because, *qua* conceptual identities—and therefore like the concept CONCEPT,—they all are unmeasurable.

Of course, you can measure the magnitude in extensity or in intensity (which are themselves attributive qualities or qualities of qualities) of uncountable qualities of qualities; in particular, the magnitude in extensity and intensity of those qualities which are real (physical) attributive qualities.

But, again, you cannot measure this or that substantive quality: even a substantive quality whose conceptual (and therefore ideal) extension comprehend, as it in fact happens with the concept QUANTITATIVE CONCEPT, an infinitude of real (physical) entities.

Indeed, an Angel—could he just be found *qua* real quality—would not in the least be quantitatively different from any human Devil: due to the sober fact that, looked at in qualitative terms, he would remain absolutely different from the most immense (in intensity) human Devil. And therefore would not in the least be (quantitively) measurable in comparison to any human Devil that might be taken as epitome of immensity in degree (intensity) concerning human *Devilishness*.

In a qualitative sense, all men—be they *angels* or *devils*—are, indeed, equal (in their es-

sential attributes); although, in a quantitative sense (and in regard both to their essential and accidental attributes), they all are (the gods be praised for that) immensely different from each other.

The same, you see, applies to all concepts. *Qua* substantive quality, the concept MEASURE presently in the author's mind may indeed be quantitatively less (in intension) a concept than the concept MEASURE in the hypothetical reader's mind. Nonetheless, both these concepts will ever be equally (qualitatively) instances of the universal concept MEASURE. And, as such, as concepts or notions (*notiones*), as ideal substantive qualities which in themselves are abstractions of ideal or real qualities, will ever, likewise, be unmeasurable.

Concepts may be considered either as to their form or as to their content. In what regards their form or logical identity, they all are substantive qualities; in regard to the content of their sphere of notions (*sphaera notionum*), they are abstractions either of real substantive qualities (like the concept POTATO), ideal substantive qualities (like the concept CONCEPT), real substantive qualities quantitively determined (like the concept H_2O), or real or ideal attributive qualities (like, respectively, the concepts TEMPERATURE and INTELLIGENCE).

It is not simply a matter of physics, then, whether there are or there are not, "in physics," "concepts… that can be measured".

Consequently, the author must be allowed the following question: *Should he, should any one indeed, look for guiding principles regarding the attributive qualities* Extension *and* Intension *where manifestly the (unmeasurable) concept of (unmeasurable)* SUBSTANTIVE QUALITY *is confused with the (likewise unmeasurable) concept of (measurable)* ATTRIBUTIVE QUALITY?

Well, the hypothetical reader may think in involuntary representing (*vorstellen*) his or her answer, *let's not be so picky, no matter how addicted we be to—well—genuinely plain statements.*

And he or she could not, indeed, but be in the right.—Were it not for the fact that shorthand-writing really does tend to become a mental habit, and, thus, to impair the type of thinking that naturally goes hand in hand with longhand writing.

Consider, as a second and last instance, the following statement: "The operations by which extensive magnitudes are combined vary enormously from magnitude to magnitude."

Do we not encounter here, again, confusion between quality (identity) and quantity (magnitude)?

What those words do mean is that, for instance, the method necessarily used to combine, say, extensive magnitude *2 m* with extensive magnitude *3 m* may vary "enormously" from the method necessarily used to combine, say, extensive magnitude *300 m* with extensive

magnitude *200 m.* And yet it is manifest that this was not the meaning that the person who wrote such words intended to convey. Such a person, no doubt, intended to say something like this: *The operations by which extensive magnitudes are combined vary enormously, according to the nature of the (attributive) qualities* (e.g., Length or Weight) *that determine the qualitative character* (e.g.,m or kg) *of the extensive magnitudes to be combined, instead of their quantitative character* (e.g., 3 m or 3 kg).

Of course, we all are liable to intend to say something like "none, essence," and end up by saying something like "nonsense". Not to take into account the factor which invariably is the most determinant one for the ascertaining of meaning: the intentionality of the person who may happen to listen to, or to read, us. Will he, will she, be capable of intentionalizing *nonsense*, in spite of our having meant, and in fact having conveyed, *none, sense?* Will he, will she, as a result, mentally represent: *yes, you are definitely talking nonsense?* Or will he or she, on the contrary, mentally represent: *yes, you are definitely right: none, sense!*

Be that as it may, the fact is that the author of the Notes that follow finally took sense.—And consequently started to look for guidance in himself, instead of looking for it where despair had mostly been the wages of his trouble (not to say, *sin*).

One day, Galatians 6:6 (Galatians indeed, not the ratio *6 : 6*) came to his mind: *The*

one who is taught the word is to share all good things with the one who teaches him (or something to an effect simillar to this effect).

Who, most of all others, has been teaching me?—the author then asked himself: for all the fruitless trouble he had put himself through had not yet, in truth, reached the point of despoiling him of endophasia. And it so happened that the immediate effect of such self-questioning was the sudden reappearance in his mind of these other (alas! not famous) words: *The most important revolution from within the human being is "his exit from his self-incurred immaturity". Before this revolution he let others think for him and merely imitated others or allowed them to guide him by leading-strings. Now he ventures to advance, though still shakilly, with his own feet on the ground of experience.*

Well, we all know that *words are like sherries:* they keep up coming to you— until, alas, that most fateful moment comes to you: the moment for *last words*.

It is no wonder, than, that the above-quoted words ended up by convoking the appearance in the author's mind of these other (quite famous) words: *...it is custom and example that persuade us, rather than any certain knowledge. And yet, a majority vote is* worthless *as a proof of truths that are at all difficult to discover; for a single man is much more likely to hit upon them than a group of people. I was, then, unable to choose anyone whose opinions struck me as*

preferable to those of all others, and I found myself as it were forced to become my own guide.

Now, to find oneself "forced to become [one's]...own guide" does not, certainly, entitle one with the right to decide (to quote Matthew Arnold) "how things really are". However, when the supposed truths one tries to discover while playing the part of one's "own guide" indeed are "truths that are" "difficult to discover", with the consequence that it is likewise "difficult" to discover how the things they concern "really are"— then, should one not to think himself or herself entitled with the right to discriminate the impression produced in oneself by those same supposed truths? That is to say, should one then not abide by the stricture of Walter Pater, who stated (in oblique response to Arnold): *the first step towards seeing one's object as it really is, is to know one's own impression as it really is, to discriminate it, to realise it distinctly.*

Well, that was basically what the present author tried to achieve, as he went on writing and rewriting the Notes that follow. And, if he has just said "basically", instead, for instance, of "exactly", this was so because, in fact, the task he constantly kept in his mind was not at all— could not be— the task of ascertaining for himself what or "how" the (attributive) qualities *Extension, Intension*, and *Extensional measure* "really are" in their ideal relation to each other, but, on the contrary, the task of indeed ascertaining "how" in fact such real qualities *ideally are*.

Indeed, one is not at the liberty to discriminate what and how certain *things* "really are". No, one is not—for the simple reason that they either are not at all real (physical) things, but ideal (psychic) *things*, or, being real (physical) ones, are such that their *human* truth would not be subjectable to experimental investigation even by means of one of the so-called *ideal machines:* were one of them, one day, to become real, and, as a result, to invalidate the ideal principle of non-contradiction (as if all principles were not by nature epistemic entities and therefore ideal entities!)

It remains to acknowledge that, in the end, the present author did obtain great help from two distinguished thinkers, without whose knowledge and teaching he never would have reached the understanding of *Extension*, *Intension*, and E*xtensional measure* that—be the reader's judgment of it whatever it be—pervades the following Notes.

Those two—well known—thinkers are Immanuel Kant and G. W. F. Hegel. The debt the author owes to them is well documented in the final section of the book ("Notes and References"), as well as in the somehow necessarily scant list of Bibliography which follows it.

The Absolute Idealism of the latter (Hegel) is not, it is true, thoroughly compatible with the Subjective Idealism of the former (Kant). However, did not the most distinguished modern supporters of Absolute Idealism (Fichte,

Schelling, and Hegel himself), after all, fight Kant's idealism by turning his own weapons against him? Besides, the ideas they expressed concerning *Extension*, *Intension*, and *Measure* must be seen more as tools more or less common to them all than as swords and picks—or, still more invidious, peer-reviews—pitted against each other.

The present author is quite aware of the many shortcomings of the Notes that follow, as well as of the possibility that, here and there, they contain erroneous assertions that he was unable to pinpoint and therefore to correct—or even deficiencies of a quite different order.

Having put so much effort on writing them, he nonetheless decided that they somehow might be of use to anyone who, having chanced upon this slim book of his, might consider them worth his or her attention: even if only to learn from their specific shortcomings *the importance of being earnest*, and therefore of no longer remaining long in coming—to a decision *to know one's own impression as it really is, to discriminate it, to realise it distinctly*.

Philosophical Notes
Concerning
Extension, Intension
and
Extensional Measure

1. PROPERTIES, OR QUALITIES, HAVE BEEN traditionally divided into (i) complex ones (*e.g.*, the qualities *Horseness*, and *Tableness*) and (ii) simple ones (*e.g.*, the generic qualities *Colour* and *Flavour*, or the specific qualities *Pinkness* and *Strawberry Flavour*).

2. This distinction, however, is nothing more than a conceptual short-cut determined by the need to conceive a *whole* as a subordinating (therefore, complex) unit and each of its *parts* as a coordinated (therefore simple or less complex) unit or subunit.

3. Consider, for instance, the supposedly simple quality *Pinkness*. What is it, after all, *qua* quality, but a specific subordination of the qualities (the co-ordinates) *Red, Green, and Blue* (RGB), or of the qualities (the co-ordinates) *Cyan, Magenta, Yellow, and Black* (CMYK)?

4. Considered from a logical point of view, and therefore in abstraction from any of their infinite instances, qualities, or properties, are called PREDICATES, in distinction from the SUBJECT, the entity they may be predicated of.

5. Considered from an ontological point of view, and therefore considered *qua* real qualities (*e.g.*, 'the *colour* of this car') or ideal ones (*e.g.*, 'the *profoundness* of this idea'), qualities, or properties, are called ATTRIBUTES—since it is possible to *attribute* them to a subject (*e.g.*, 'this car is *red*'), but it is not possible to *attribute* a subject to them (*e.g.*, 'this *red* is car').

6. Qualities have also been divided, by physicists such as Galileo and philosophers such as Descartes and Locke, into (i) primary ones and (ii) secondary ones.

7. PRIMARY QUALITIES, Locke declares, are "[i] such as are utterly inseparable from the body, in what state so ever it be; [ii] such as in all the alterations and changes it suffers, all the force can be used upon it, it constantly keeps; and [iii] such as sense constantly finds in every particle of matter which has bulk enough to be perceived, and the mind finds inseparable from every particle of matter, though less than to make itself singly be perceived by our senses". |[1]

8. PRIMARY QUALITIES are then "*solidity, extension, figure, and mobility*"—for, "Take a grain of wheat, divide it into two parts, each part… [will] still [have] *solidity, extension, figure, and mobility*; divide it again, and it… [will] still [retain] the same qualities; and so divide it on till the parts become insensible, they must retain still[,] each of them all[,] those qualities."

9. Consequently, it is an incontrovertible fact, that "division (which is all that a mill, or pestle, or any other body does upon another, in reducing it to insensible parts) can never take away either solidity, extension, figure, or mobility from any body, but only makes two or more distinct separate masses of matter, of that which was but one before". [2]

10. SECONDARY QUALITIES, in their turn, are "such qualities which in truth are nothing in the objects themselves, but powers to produce various sensations in us by their *primary qualities*, i. e. by the bulk, figure, texture, and motion of their insensible parts, as colours, sounds, tastes, etc." [3]

11. In short, PRIMARY QUALITIES are, according to Locke, objective qualities—since they are constitutive of "objects themselves" and, as such, cannot but subsist apart from whatever determinations they may impose on sense perception, as well as, of course, apart from those

determinations sense perception itself may impose on them.

12. In short, SECONDARY QUALITIES are, according to Locke, subjective qualities ("colours, sounds, tastes, etc.")—since, inversely, they "are nothing in the objects themselves" beyond the "powers" these exert on our senses "by their *primary qualities".* That is to say, since secondary qualities themselves are nothing more than an effect—a subjective effect—objectively produced in us by primary qualities.

13.. Kant, for whom our knowledge of the external world is determined both formally (*a priori*) and materially (*a posteriori*), and for whom all our representations (*Vorstellungen*) are (subjective) appearances (*Erscheinungen*), with the consequence that he denies us access to "things in themselves" (*Dinge an Sich*), either via sensibility (*Sinlichkeit*) or the understanding (*Verstand*) |[4]—Kant, of course, could not endorse Locke's notion of primary qualities as constituents of "objects themselves" and, consequently, as *qualities in themselves*. |[5]

14. As it was seen above, Locke declared: "division... can never take away either solidity, extension, figure, or mobility from any body, but only makes two or more distinct separate masses of matter, of that which was but one before".

15. Kant riposted: "Long before *Locke's* time, but assuredly since him, it has been generally assumed and granted without detriment to the actual existence of external things, that many of their predicates may be said to belong not to the things in themselves, but to their appearances, and to have no proper existence outside our representation. Heat, colour, and taste, for instance, are of this kind. Now, if I go farther, and for weighty reasons rank as mere appearances the remaining qualities of bodies also, which are called primary, such as extension, place, and in general space, with all that which belongs to it (impenetrability or materiality, shape, etc.)—no one in the least can adduce the reason of its being inadmissible." |[6]

16. "No one", indeed. But then, if the problem of the ontological status of qualities is left aside—and here lies the reason why the difference between Locke's and Kant's views are dealt with here,—no one, too, will contest that, as predicates, secondary qualities such as "colour" and "taste" logically presuppose a subject in which to inhere. And, consequently, that, even if all qualities "have no proper existence outside our representation", the distinction between (i) **PRIMARY** or **SUBSTANTIVE QUALITIES** and (ii) **SECONDARY** or **ATTRIBUTIVE QUALITIES**—which is the ground of the dependence of the latter on the former—stands.

17. As Kant himself was well aware of, |[7] perception of physical entities forces us to cognize them in terms of the conceptual difference between 'substance' and 'property', and, as a result, imposes on us the epistemic conviction (i) that they *are* substances, (ii) that, as such, they *have* certain qualities, and (iii) that these, in turn, *have* a certain degree or intensity.

18. Viewed in conceptual terms, properties are, however, either (i) essential (*e.g.*, *Liquidness*, in the case of the complex quality *Water*) or (i) accidental (*e.g.*, *Transparency*, again in the case of the complex quality *Water*). And what, then, can *substances* be, if not bundles of essential (primary and secondary), as well as structural, qualities?

19. Consequently, and to push now aside the indispensable intervention, the *glue*, of structural qualities, would one not be justified in conceiving *Water*, for instance, as a bundle of essential qualities such as *Liquidness* and of accidental qualities such as *Transparency*? As it is known, not a few among contemporary metaphysicians think so.

20. Furthermore, were it not for linguistic necessity, would one not be right in stating not only that 'the quality *water* is a complex quality', but also that, on the face of it being such, a structured complex of quantities or magnitudes, 'the quality *water* ***is*** quantity? |[8]

21. Indeed, did not Kant, for instance, think himself right in stating that "All appearances *are* extensive magnitudes" (*Alle Anschauungen* sind *extensive Größen*), |[9] instead of stating that "All appearances *have* extensive magnitude" ("Alle Anschauungen *haben* extensive Größen")?

22. Qualities are given to us, the perceivers, under the aspects of (i) FORM and (ii) MATTER.

23. In respect to FORM, all real qualities are perceived by us as determined by two *media:* (i) TIME and (ii) SPACE—both of which, considered in their infinity, are *quanta continua* or unlimited magnitudes*;* the former (time) being liable to ideal (conceptual) limitation and the latter (space) being liable to ideal (conceptual) as well as real limitation.

24. As a result, all qualities are necessarily conceived of both as *quanta continua* and *quanta discreta*, or, to put it in a different formulation, as discretely-continuous *quanta:* as *quanta* that, *qua* limitation, are *discreta*, and that, *qua* absence of limitation, are *continua*.

25. In respect to MATTER, and viewed in their conceptual generality (*e.g., Animality*, *Colour, Weight, Temperature*) or specificity (*e.g., Horseness*, *Pinkness*), qualities (the real as well as the ideal, the ones, among the real, which are mediately discovered by man as well as those that

are immediately given by Nature) cannot but be cognized by us, who intuit (*anschauuen*) them, as essential constituents (therefore as universal determinants) of being and becoming.

26. All qualities *are* indeed quantities or magnitudes: those that are primarily perceived through the senses (such as *Weight*, *Temperature,* and *Physical Beauty*) as well as those that are perceived through the mind alone (such as *Intelligence*, *Imagination*, and *Honesty*).

27. Qualities that are perceived through the senses *are* magnitudes that either can be numerically determined with precision (such as *Temperature*) or that cannot be so determined (such as *Beauty*).

28. Considered in their quantitative identity (*qua* magnitudes), as well in their qualitative identity (their universal and all-determinative uniqueness), SPACE and TIME, the two formal *media* of our knowledge of the external world, are absolutely opposite determinants of all qualities—which, as a result, are as much quantitative qualities as qualitative quantities. |[10]

29.. Qualities *are* simultaneously INTENSIONAL MAGNITUDES and EXTENSIONAL MAGNITUDES—INTENSION (*intentio*, in-tension) and EXTENSION (*extentio*, ex-tension) being not only formal determinations of magnitude in general, but also

determinations that absolutely oppose each other, and that, as a result, negate and affirm, necessarily presuppose, each other. (Just like, for instance, the absolutely opposed concepts LIFE (NO-DEATH) and DEATH (NO-LIFE), or the likewise absolutely opposed concepts SUBJECT and OBJECT, negate and affirm, and therefore presuppose, each other.)

30. *Qua* form, EXTENSION and INTENSION are reciprocal, and therefore relative, determinations (qualitative as well as quantitative) of SPACE and TIME: as becomes evident when it is considered that the unidimensional flow of time is capable of being intuited only in spatial terms (*qua* spatial succession, one moment after the other), and that, in its turn, space is capable of being intuited only in temporal terms (*qua* temporal succession, one *spot* latter, or sooner, than those contiguous with it).

31. *Qua* form, SPACE and TIME, EXTENSION and INTENSION, are therefore determinants, qualitatively as well as quantitatively, of all material qualities (*e.g., Temperature*). And, as such, are as much (i) formal qualities of material qualities and formal quantities of material quantities as (ii) formal quantities of material qualities and formal qualities of material quantities.

32. Since qualities are either indeterminate or determinate quantities, or magnitudes, EXTEN-

SION and INTENSION are, like the formal quantitative quality *Number,* either indeterminate (unlimited) or determinate (limited) formal quantities, or magnitudes.

33. *Qua* ***indeterminate*** quantity or magnitude, **EXTENSION,** the formal quality of material qualities, is unlimited spatial continuity, and consequently is (unlimited) *quantum continuum.* |[11]

34.. *Qua* ***determinate*** quantity or magnitude, **EXTENSION** is limited spatial continuity or *Length*, and consequently is likewise (limited) *quantum discretum.* |[12]

35. *Qua* ***indeterminate*** quantity or magnitude, **INTENSION,** the other formal quality of material qualities, is unlimited temporal continuity, and consequently is (unlimited) *quantum continuum.*

36.. *Qua* ***determinate*** quantity or magnitude, **INTENSION** is limited temporal continuity or *Duration*, and consequently is likewise (limited) *quantum discretum.* |[13]

37. Since, however, EXTENSION and INTENSION are reciprocal, and therefore relative, determinations of SPACE and TIME, *Length* is, considered qualitatively as well as quantitatively, *extensional duration*, and *Duration*, likewise considered qualitatively as well as quantitatively, is *temporal length*.

38. Considered as given in TIME, qualities are **DEGREE** or **LIMIT IN INTENSION** (intensity), and therefore each of them *is*, at time T^x, a specific INTENSIONAL MAGNITUDE. |[14]

39. Consequently, viewed in abstraction from any and all of its temporal determinations, from any and all of its (determinate) instances (degrees), INTENSIONAL MAGNITUDE is conceivable alone as an interminable *quantum continuum:* as unlimited but limitable (undetermined but determinable) temporal continuity.

40. Considered as given in SPACE, qualities *are* **CONTINUITY** and **LIMIT IN EXTENSION**, and therefore each of them *is*, at at time T^x, a specific EXTENSIONAL MAGNITUDE.

41. Consequently, viewed in abstraction from any and all of its spatial determinations, from any and all of its (determinate) instances (limits), viewed as SPACE itself, EXTENSIONAL MAGNITUDE is likewise conceivable only as an endless *quantum continuum:* as unlimited but limitable (undetermined but determinable) spatial continuity.

42. *Qua* DEGREE, each determinate INTENSIONAL MAGNITUDE, and, *qua* LIMIT, each EXTENSIONAL MAGNITUDE, is capable of being conceived only as UNIT, and consequently any series of INTENSIONAL or EXTENSIONAL magnitudes is as well

capable of being conceived only as a multiplicity of units.

43. Only undetermined quantity or magnitude, then, is capable of being conceived as the ultimate, insuperable, UNITY between quantitative (determinate) UNITY-*qua*-MULTIPLICITY and quantitative (determinate) MULTIPLICITY-*qua*-UNITY. That is to say, only undetermined quantity or magnitude, then, is capable of being conceived as the ultimate, insuperable, IDENTITY of (i) the QUANTITATIVE IDENTITY (UNITY) and the (ii) QUANTITATIVE NON-IDENTITY (MULTIPLICITY) of all determinate magnitudes.

44. Formally and generically, **TIME** determines all qualities *qua* **INTENSIONAL MAGNITUDE** (DEGREE); specifically, TIME determines each of them *qua* a specific temporal continuity or magnitude: a specific DURATION.

45. Formally and generically, **SPACE** determines all qualities *qua* **EXTENSIONAL MAGNITUDE**; specifically, SPACE determines each of them *qua* a specific spatial continuity or magnitude: a specific LENGTH.

46. Considered in abstract, **INTENSIONS** are *quanta continua* (*e.g.*, the internally unlimited *quantum continuum* that mediates between the absolute absence (0 %) and the absolute presence (100 %) of the colour 'black') which time

limits and, therefore, determines as *quanta discreta*. That is to say, which time determines as (i) a specific DEGREE (intensity) or (ii) a certain plurality of simultaneous DEGREES (intensities). |[15]

47. Considered in abstract, **EXTENSIONS** are externally limited, but internally unlimited, *quanta*. *Qua* externally limited *quanta* or quantities, they are *quanta discreta* or discrete quantities (*e.g.*, the extensional *quantum* we refer to when we say, for instance, 'the length of this rope'). *Qua* internally unlimited *quanta* or quantities, they (extensions) are *quanta continua* or continuous quantities *(e.g.*, again the extensional quantum we refer to when we say 'the length of this rope').

48. An **INTENSIONAL** *quantum continuum*, then:

> (1) is given at each (discrete) moment (*quantum discretum*) of the temporal (discrete) continuity;
>
> (2) is consequently given *qua quantum discretum* or DEGREE (intensity).

49. An **INTENSIONAL** *quantum continuum* IS **SYNCHRONICALLY *GIVEN***:

> **(1)** *qua* (discrete) DEGREE/INTENSITY (T^4 T^5 T^6 T^7 T^8 T^9 T^{10}), or

(II) *qua* (discrete) DEGREES or INTENSITIES ($T^1T^2T^3$).

50. An **INTENSIONAL** *quantum continuum* IS **DIACHRONICALLY *GIVEN*:**

(I) *qua* (temporally) ***discontinuous*** (discrete) DEGREE/INTENSITY ($T^4\ T^5\ T^6\ T^7\ T^8$), or

(II) *qua* (temporally) ***discontinuous*** (discrete) DEGREES/INTENSITIES ($T^1T^2T^3$), or

(III) *qua* (temporally) ***continuous*** (discrete) DEGREE/INTENSITY (T^{8-9-10}).

51. *Qua* **(I)** (temporally) ***discontinuous*** (discrete) DEGREE/INTENSITY, an **INTENSIONAL** *quantum continuum* is **SYNCHRONICALLY *PERCEIVED*** as **(A)** a **HOMOGENEOUS** (qualitative) **CONTINUITY–EXTENSION** ($T^4\ T^5\ T^6\ T^7\ T^8$).

52. *Qua* **(II)** (temporally) ***discontinuous*** (discrete) DEGREES or INTENSITIES, an **INTENSIONAL** *quantum continuum* is **SYNCHRONICALLY *PERCEIVED*** as **(B)** a **HETEROGENEOUS** (qualitative) **CONTINUITY-EXTENSION** ($T^1T^2T^3$).

53. *Qua* **(I)** (temporally) ***discontinuous*** (discrete) DEGREE/INTENSITY—*qua* temporal discreteness,—an **INTENSIONAL** *quantum continuum* is **DIACHRONICALLY *PERCEIVED*** as **(A)** a temporal-

ly-discontinuous **HOMOGENEOUS** (qualitative) **CONTINUITY-EXTENSION** (as temporal discreteness) (T^4 T^5 T^6 T^7 T^8).

54. *Qua* **(II)** (temporally) ***discontinuous*** (discrete) DEGREES/INTENSITIES, an **INTENSIONAL** *quantum continuum* is **DIACHRONICALLY *PERCEIVED*** as **(B)** a temporally-discontinuous **HETEROGENEOUS** (qualitative) **CONTINUITY-EXTENSION** ($T^1T^2T^3$).

55. *Qua* **(III)** (temporally) ***continuous*** (discrete) DEGREE/INTENSITY—*qua* temporal discrete continuity,—an **INTENSIONAL** *quantum continuum* is **DIACHRONICALLY *PERCEIVED*** as **(B)** a temporally-discontinuous **HETEROGENEOUS** (qualitative) **CONTINUITY-EXTENSION** ($T^1T^2T^3$).

56. Change in **INTENSIONAL MAGNITUDE** may or may not result in perceivable change in **EXTENSIONAL MAGNITUDE**. For instance, a pot over the heat will not be perceived to expand, but the mercury in a thermometer is clearly seen to expand when it is heated.

57. On the other hand, change in the **INTENSIONAL MAGNITUDE** of quality *x* (*e.g.*, *Temperature*) up from a certain intensity (*e.g.*, 0^0 or 100^0) may cause quality *y* (*e.g.*, *Solidity* or *Vaporosity*) to take the place of another (*e.g.*, *Liquidness*), and therefore to result in change in **EXTENSIONAL MAGNITUDE**.

58. Scientists as well as philosophers of science usually separate qualities into (i) **EXTENSIONAL** and (ii) **INTENSIONAL** qualities.

59. By the designation 'extensional (or extensive) qualities', they mean those qualities whose magnitudes are absolutely cumulative—such as *Weight*, for instance, since it is possible to obtain absolute increase in weight by combining two or more bodies with equal or different weight).

60. By the designation 'intensional (or intensive) qualities', they mean those qualities whose magnitudes, on the contrary, are not absolutely cumulative—such as *Temperature*, for instance, since it is not possible to obtain absolute increase in temperature (say, increase from 100°C to 200°C) by combining two bodies at equal or different temperature (say, two separate litres of a liquid at 100°C).

61. Besides, scientists as well as philosophers of science tend to use the designations 'extensional (or extensive) qualities' and 'additive qualities' more or less interchangeably. |[16]

62. However, all qualities are extensional as well as intensional (are extension as well as intension), as it has been noted above based on the authority of Hegel—who made the point that (qualitative) extensional and intensional magnitudes cannot be differentiated on account of a

specific difference: "like continuous [unlimited] and discrete [limited] magnitude, extensive and intensive magnitude are not two species (each of which would contain a determinacy that would be lacking in the other); whatever has extensive magnitude has intensive magnitude, and vice versa." |[17]

63. It follows that the capability or incapability of each two or more (qualitative) magnitudes to be *added* to each other cannot be grounded on the distinction between (i) extensional qualities or (qualitative) extensional magnitudes and (ii) intensional qualities or (qualitative) intensional magnitudes.

64. Furthermore, the magnitudes of at least some of the so-called "intensional qualities", of the 'non-additive' qualities, such as *Density*, are measured in units (*e.g.*, *kg* and *m*) which are by nature directly associated with extensional magnitudes, and, therefore, with 'extensional qualities' (such as *Weight* and *Length*). This shows, of course, that units of measure such as *kg* and *m* are as much units of extensional as of intensional magnitude, and that—Hegel thus being right—it cannot but be, to say the least, *awkward* to distinguish between intensional and extensional qualities on the basis of the additiveness or non-additiveness of their magnitudes and, nonetheless, go on to measure the

magnitudes of the former in terms of the qualitative identity of the magnitudes of the latter.

65. It thus seems best:

(i) to regard the numerical determinations of all (qualitative) magnitudes (intensional and extensional), not the magnitudes themselves, as (cognitively 'additive'—since:

(A) each unit of measure (*e.g.*, *m*) consists, in conceptual or formal terms, in the ascription of the number ONE (1) to the physically determined magnitude (*e.g.*, 2 *m*) corresponding to it;

(B) as a result, and *qua* form, all units of measure are indifferent to any and all material determinations;

(C) likewise as a result, it is, for instance, conceptually possible not only to add the formal (*Weight*) magnitude '100 kg' to itself, and conceptually obtain the formal (*Weight*) magnitude '200 kg', but also to add the formal (*Temperature*) magnitude '100^{0}C' to itself, and conceptually obtain the formal (*Temperature*) magnitude '200^{0}C';

(ii) to regard some (qualitative) magnitudes (intensional and extensional) as **ABSO-**

LUTELY CUMULATIVE *in rem:* since, again, it is possible, for instance, to obtain absolute increase in weight by accumulating the different weights of any two, or of more, bodies;

(iii) to regard some (qualitative) magnitudes (intensional and extensional) as **RELATIVELY CUMULATIVE** *in rem:* since, after all, when two bodies with a different temperature are joined, heat will flow from the warmer body to the cooler body, and the two bodies will gradually reach the same temperature (thermal equilibrium)—so that, for instance, the mixing of two separate litres of a liquid, one litre with a temperature of 60°C and the other litre with a temperature of 100°C, will cause not only an ABSOLUTE INCREASE in the (intensional and extensional) magnitudes of the quality *Volume* (from one litre to two litres), but also a RELATIVE INCREASE (from 60°C to 80°C), and a RELATIVE DECREASE (from 100°C to 80°C), in the (intensional and extensional) magnitudes of the quality *Temperature*. (There would not be a perceivable change in the magnitude of the quality *Temperature* if the whole extension of the combined water, or at least part of it, were not an *extension* (in space) of the (intensional) *Temperature* degree (in time) 80°C.)

66. Now, from what has come to light above, it is to be concluded: (i) that the quality or property *Extension is* magnitude of space, and that, as result, the entities which (intensionaly) exhibit it are entities possessed of extensional magnitude; (ii) that the quality or property *Extension* is, then, a *quantum;*

67. Now, extension may be thought of either as infinite or finite extension.

68. If, *qua* quantity or magnitude, all extension is a *quantum*, infinite extension is an infinite *quantum*, finite extension is a finite *quantum*.

69. Again, finite extension may be thought of both as (i) *quantum continuum* and (ii) q*uantum discretum*.

70. *Qua quantum continuum*, extension is internally unlimited, but limitable, quantity or magnitude: is (extensional) quantity conceived of as plurality of indiscrete units. |[18]

71. *Qua quantum discretum*, extension is externally (doubly) limited quantity or magnitude: is (extensional) quantity conceived of as determinate unit.

72. *Qua quantum discretum*, and therefore *qua* limited magnitude, extension is either (i) numer-

ically undetermined magnitude or (ii) numerically determined magnitude. |[19]

73. *Qua* (i) numerically undetermined magnitude, extension is unascertained (extensional) MEASURE; *qua* (ii) numerically determined magnitude, extension is ascertained (extensional) MEASURE.

74. *Qua* numerically determined magnitude, EXTENSIONAL MEASURE can be thought of (i) in material or (ii) in formal terms.

75. Thought of primarily (i) in material as well as in quantitative terms and *qua* numerically determined magnitude, EXTENSIONAL MEASURE is objective magnitude both as (materially) given or determined *quantum*, sensation, and as (formally) posited (*gesetzt*) *quantum*.

76. Thought of primarily in (ii) formal (conceptual) as well as in qualitative terms and likewise *qua* numerically determined magnitude, EXTENSIONAL MEASURE is objective magnitude simply as (formally) posited *quantum:* as definite quantity of extension (formally) posited as a standard *for* the numeration and calculation of indefinite quantity (continuity) in extension.

77. As a result, EXTENSIONAL MEASURE *qua* numerically determined objective magnitude or numerical determinacy becomes definable as

'material extensional magnitude (*e.g.*, length *3 m*) determined *by* formal extensional magnitude (*e.g.*, the unit *m*)'.

78. *Qua* numerically determined objective magnitude or numerical determinacy, quantity in extension ***has*** a measure of *x* when 'measure' is understood as formally (conceptually) limited continuity (*e.g.*, the representation *in mente* of the extension *m*), and thus as both (i) quantity (extensional magnitude) and (ii) quality (*e.g.*, the extensional magnitude *m*, instead of the extensional magnitude *ft*).

79. Again *qua* numerically determined objective magnitude or numerical determinacy, quantity in extension ***is*** measure *x* when 'measure' is understood as material and limited continuity (as measure *in rem*) and thus as only (i) quantity.

80. When MEASURE is thought of in ***qualitative terms***, and therefore *qua* formal numerical determination, the designation 'extensional measure' becomes synonymous with the designation 'unit of extensional measure', since qualitative as well as quantitative (formal) measure (magnitude) always consists in a specific unit of measurement: *metre, foot, inch*, etc.

81. When MEASURE is thought of in ***quantitative terms*** and *qua* material numerical determinacy, the designation 'extensional measure' signifies

'extensional continuity which materially equals either (i) a formally specific unit of measurement (*e.g.*, 'one (1) metre'), (ii) a formally specific fraction of a formally specific unit of measurement (*e.g.*, 'half (1/2) a metre'), (iii) so many times a formally specific unit of measure (*e.g.*, 'three times one (1) metre'), or (iv) a formally specific ratio (*e.g.*,'2 : 4 m').

82. Now, viewed as numerical determination, all units of extensional measure are concepts or conceptual entities. Therefore, *qua* conceptual form, *qua* concepts considered in general or as to their subjectual (cognitive) determinacy, all extensional units of measure are qualitative *notae* or marks; *qua* conceptual content, *qua* concepts considered as to their objectual (cognitive) determination, all of them (all units of extensional measure) are quantitative concepts and, therefore, quantitative *notae* or marks.

83. Quantitative concepts cognitively objectualised as specifically concepts of measurement in extension are formed, in Kantian terms, |[20] by means of the imagination (*Einbildungskraft*), which schematizes them (in space and time) and thus transforms them in abstract representations (*Vorstellungen*) of an absent concrete representation: the intuition (*Anschauung*) of a definite extensional *quantum* (or magnitude). That is to say, quantitative concepts cognitively objectualised as specifically concepts of meas-

urement in extension are obtained by cognizing a definite extensional *quantum* (or magnitude) as a representative or paradigmatic *quantum*, to be used as a standard for the measuring of extensional magnitudes (for *measurement in extension*).

84. *Qua* quantitative concepts, all units of extensional measure are, therefore, materially determined *ab initio*—which is the same as to say that, considered as form, all units of extension measurement are capable of numerically determining extensional magnitude *qua* intuited (and therefore objective) magnitude only as a result of themselves subjectively consisting in the image (*Bild*) of a specific intuited extensional magnitude, thought (*vorgestellt*) in abstract, intentionalised, as representative measure. That is to say, as one of several specific extensional-visual correlatives of the purely conceptual determinacy *One*.

85. In itself, the concept UNIT OF MEASURE of course presupposes the concept UNIT (in fact, the concepts WHOLE UNIT and FRACTIONAL UNIT) as much as it presupposes the concept LIMIT.

86. In its turn, the concept UNIT presupposes the concept UNITY as much as the latter presupposes the equally quantitative concepts MULTIPLICITY and TOTALITY—which, of course, entail

number as much as number and annumeration entails them.

87. As a result, and *qua* specific quantitative or mathematical concepts (*metre, mile, inch, foot,* etc.), all units of measure are, if we follow Kant, |[21] determined *a priori* by the (conceptually) apodictic correlation between the concept NUMBER and the concepts of the understanding (*Verstandesbegriffe*) of quantity (*Quantität*): the mathematical (*mathematische*) categories (*Kategorien*) of (i) UNITY (*Einheit*), (ii) MULTIPLICITY (*Vielheit*), (iii) and TOTALITY (*Allheit*). In the absence of the latter—which correspond, respectively, to the concepts (i) MEASURE (*das Maß*), (ii) MAGNITUDE (*das Größe*), and (iii) ALLNESS (*das Ganze*)—the (conceptual) opposition between (A) PART (FRACTIONAL UNITY and therefore MULTIPLICITY) and (B) WHOLE (INTEGRAL UNITY and therefore TOTALITY) would remain incognizable, as well, then, as the concepts NUMERIC SERIES ('1, 2, 3…'), ADDITION, SUBTRACTION', MULTIPLICATION, DIVISION, etc.

88. In fact, the concepts UNITY and MULTIPLICITY—as well as the concepts PART (FRACTION) and WHOLE (TOTALITY)—are concepts absolutely opposed to each other. This much becoming evident as soon as it is realised that UNITY perforce conveys the notion NON-MULTIPLICITY (and vice-versa), and that PART (FRACTION) perforce conveys the notion NON-TOTALITY (and vice-versa).

89. When it is not construed in absolute terms (as 'absolute unity'), the concept UNITY is therefore entailed by the concept MULTIPLICITY (and vice-versa) as much as the concept TOTALITY is entailed by the concept FRACTIONALITY (and vice-versa). And indeed the concept UNITY, ONE (1), is entailed by the concept MULTIPLICITY OF THREE (3), for instance, as much as the concept TOTALITY OF THREE (3) entails, for instance, the correlative concept FRACTION ONE THIRD (1/3): ONE PART (1/1) OF THREE EQUAL PARTS (3/3).

90. As a result, the concept (of unity) UNIT OF MEASURE is entailed by the corresponding concept (of multiplicity) SO MANY TIMES A UNIT OF MEASURE (and vice-versa) as much as the concept (of totality) THE WHOLE OF A UNIT OF MEASURE is entailed by the corresponding concept (of fractionality) HALF A UNIT OF MEASURE (and vice-versa).

91. Though, in logical terms, they are specific concepts, when they are considered as lower concepts (*conceptus inferiores*)—and therefore in relation to the higher or general concepts (i) UNITY, (ii) MULTIPLICITY, (iii) TOTALITY, and (iv) FRACTIONALITY,—the concepts (i) UNIT OF MEASURE, (ii) SO MANY TIMES A UNIT OF MEASURE, (iii) THE WHOLE OF A UNIT OF MEASURE, and (iv) HALF A UNIT OF MEASURE are themselves, of course, no less higher or general concepts (*conceptus superiores*), when, now, they are considered in

relation to lower or specific concepts such as (i) ONE (1) METRE, (ii) SO MANY TIMES ONE (1) METRE, (iii) ONE (1) WHOLE METRE, and (iv) HALF A METRE.

92. This leads to the distinction between (i) generic signification" (*significatio generalis*) and (ii) specific signification (*significatio specialis*)—without which what follows would not be conceptually graspable.

93. The signifier **EXTENSIONAL MEASURE** |[22] must needs be understood in relation both (I) to its SIGNIFIEDS and (II) to any of its possible REFERENTS. That is to say, the import of the designation EXTENSIONAL MEASURE has necessarily to be understood both **(I) FORMALLY**, or in **SUBJECTIVE (CONCEPTUAL) TERMS**, and **(II) MATERIALLY**, or in **OBJECTIVE (PHYSICAL) TERMS**. |[23]

94. Now, "quantum is determined as *number* (*das Quantum [ist] als Zahl bestimmt*). |"[24] And consequently, when it is understood **(I) FORMALLY** or in **SUBJECTIVE TERMS**, the designation EXTENSIONAL MEASURE expresses both **(A) QUALITATIVE MEANING** and **(B) QUANTITATIVE MEANING**.

95. Understood in its (FORMAL) **(A) QUALITATIVE MEANING**, the designation EXTENSIONAL MEASURE acquires **(i) GENERIC MEANING** as well as **(ii) SPECIFIC MEANING**: its SIGNIFIED—the concept

to which it relates *qua* SIGNIFIER—is (i) GENUS as well as (ii) SPECIES.

96. To understand the designation EXTENSIONAL MEASURE in its (QUALITATIVE) **(i) GENERIC MEANING** is to understand it in **ABSOLUTE TERMS**.

97. To understand the designation EXTENSIONAL MEASURE in its (QUALITATIVE) **(ii) SPECIFIC MEANING** is to understand it in **RELATIVE TERMS**.

98. Understood in its (QUALITATIVE) **(ii) SPECIFIC MEANING** (and therefore in **RELATIVE TERMS**), the designation EXTENSIONAL MEASURE signifies, for instance: *the extensional magnitude or interval referred to as* METRE, *whose first three submultiples are (i) the* DECIMETRE *(one tenth), (ii) the* CENTIMETRE *(one hundredth), and (iii) the* MILLIMETRE *(one thousandth), and whose first three multiples are (i) the* DECAMETRE, *(ii) the* HECTOMETRE, *and (iii) the* KILOMETRE.

99. In short, when it is understood in its (QUALITATIVE) (ii) SPECIFIC MEANING (and therefore in RELATIVE TERMS), the designation EXTENSIONAL MEASURE signifies, for instance, ***The extensional measure 'm'*** (*the measure-in-extension 'm'*). For it then expresses qualitative meaning which is determined as to species ***qualitatively*** (specifically–relatively determined: *The (unit of)* ***extensional measure 'm'***), but ***quantitatively*** undetermined (specifically–relatively undeter-

mined: ***How many units*** *of the extensional (unit of) measure 'm'?*)

100. Understood in its (QUALITATIVE) **(i) GENERIC MEANING** (and therefore in **ABSOLUTE TERMS**), the designation EXTENSIONAL MEASURE signifies: *any (extensional) magnitude or quantity conventionally chosen as a (physical and usually imaginatively representable) paradigm (a unit of measure) for the numerical determination (measurement) of undetermined extensional magnitude (or measure).*

101. In short, when it is understood in its (QUALITATIVE) (i) GENERIC MEANING (and therefore in ABSOLUTE TERMS), the designation EXTENSIONAL MEASURE signifies itself: ***Extensional measure***. For it then expresses meaning: (A) which is ***qualitatively*** determined as to gender (generically determined: ***Extensional*** *(unit of)* ***measure***), but ***qualitatively*** undetermined as to species (specifically–relatively undetermined: ***Which*** *extensional (unit of) measure?*); (B) which is ***quantitatively*** undetermined as to gender (generically undetermined: ***How many units*** *in whatever extensional (unit of) measure?*), and therefore also as to species (specifically–relatively undetermined: *e.g.*, ***How many units*** *in extensional (unit of) measure 'm'?*).

102. When it is understood in its (FORMAL) **(B) QUANTITATIVE MEANING**, the designation EX-

TENSIONAL MEASURE likewise acquires **(iii) GENERIC MEANING** as well as **(iv) SPECIFIC MEANING**.

103. To understand the designation EXTENSIONAL MEASURE in its (QUANTITATIVE) **(iii) GENERIC MEANING** is also to understand it in **ABSOLUTE TERMS**.

104. To understand the designation EXTENSIONAL MEASURE in its (QUANTITATIVE) **(iv) SPECIFIC MEANING** is also to understand it in **RELATIVE TERMS**.

105. When it is understood in its (FORMAL) **(B) QUANTITATIVE MEANING** as well as in its **(iii) GENERIC MEANING** (and therefore in **ABSOLUTE TERMS**), the designation EXTENSIONAL MEASURE acquires signification that overlaps with that of the hypernym MEASURE, in relation to which it becomes hyponymic (or specific). It is, therefore, only as a result of the specific difference which separates it from the designation INTENSIONAL MEASURE, that it signifies:

> (1) ***The number* ONE *(1)***, *formally considered* qua a priori *unit of specifically-indeterminate extensional measure, because constitutively entering (as generic determinant* a posteriori*) into all specific (determinate) extensional measures (metre, foot, inch, etc.) And therefore formally considered* qua ***absolute de-***

terminant *(in potency) of any numerically-undetermined extensional* quantum.

(2) ***Any number greater than the number*** **ONE** *(2, 3, 4, etc.), formally considered* qua a priori *aggregation of the* (a priori) *numerical unit of specifically-indeterminate extensional measure.* |[25] *And therefore formally considered* qua ***absolute determinant*** *(in potency) of any numerically-undetermined extensional* quantum, *because constitutively entering* a posteriori *(as generic determinant) into all specific (determinate) extensional measures (metre, foot, inch, etc.)*

(3) ***The number*** **ONE** ***(1)***, *the* a priori *unit of specifically-indeterminate extensional measure,* ***as numerator*** (1/2, 1/3, 1/4, *etc.*), *formally considered* qua *determinacy imposed by the* a priori *possibility of its fractioning into* ***any*** *of its aggregations (2, 4, 8, etc.), and of the exclusion from such an aggregation, viewed as total of fractions, of a unitary fraction. And therefore formally considered* qua ***absolute determinant*** *(in potency) of any numerically undetermined extensional* quantum, *because of its potentiality to enter* (a posteriori) *into the fractioning of the magnitudes ascribed to any specific extensional measure (metre, foot, inch, etc.)*

(4) ***Any* numerator greater than the number ONE** (2/1, 3/1, 4/1, *etc.*), *formally considered* qua *determinacy imposed by the* a priori *possibility of the fractioning of the number* ONE *(1)—the* a priori *unit of specifically-indeterminate extensional measure—into any of its aggregations (2, 4, 8, etc.), each of them viewed as a total of fractions of itself. And therefore formally considered* qua ***absolute determinant*** *(in potency) of any numerically undetermined extensional* quantum, *because of its potentiality to enter* (a posteriori) *into the fractioning of the magnitudes ascribed to any specific extensional measure (metre, foot, inch, etc.)*

(5) ***Any* numerator greater than the number ONE** (2/4, 4/6, 6/8, *etc.*), *formally considered* qua *determinacy imposed by the* a priori *possibility of the exclusion from any of the aggregations (2, 4, 8, etc.) of the number* ONE *(1)—the* a priori *unit of specifically-indeterminate extensional measure—of another such aggregation, both of them viewed as a total of fractions. And therefore formally considered* qua ***absolute determinant*** *(in potency) of any numerically undetermined extensional* quantum, *because of its potentiality to enter* (a posteriori) *into the fractioning of the magnitudes ascribed to any specific extensional measure (metre, foot, inch, etc.)*

(6) ***The number* ONE *(1)***, *the* a priori *unit of specifically-indeterminate extensional measure,* ***as denominator*** (2/1, 3/1, 4/1, *etc.*), *formally considered* qua *determinacy imposed by the* a priori *possibility of its fractioning into* ***any*** *of its aggregations (2, 4, 8, etc.), each of them viewed as total of fractions of itself. And therefore formally considered* qua ***absolute determinant*** *(in potency) of any numerically undetermined extensional* quantum, *because of its potentiality to enter* (a posteriori) *into the fractioning of the magnitudes ascribed to any specific extensional measure (metre, foot, inch, etc.)*

(7) ***Any denominator greater than the number* ONE** (1/2, 1/3, 1/4, *etc.*), *formally considered* qua *determinacy imposed by the* a priori *possibility of the fractioning of the number* ONE *(1)—the* a priori *unit of specifically-indeterminate extensional measure—into any of its aggregations (2, 4, 8, etc.), and of the exclusion from them, considered as a total of fractions, of a unitary fraction. And therefore formally considered* qua ***absolute determinant*** *(in potency) of any numerically undetermined extensional* quantum, *because of its potentiality to enter* (a posteriori) *into the fractioning of the magnitudes ascribed to any specific extensional measure (metre, foot, inch, etc.)*

(8) ***Any* denominator greater than the number ONE** (2/4, 4/6, 6/8, *etc.*), *formally considered* qua *determinacy imposed by the* a priori *possibility of the division of the number* ONE *(1)—the* a priori *unit of specifically-indeterminate extensional measure—into any of its aggregations (2, 4, 8, etc), and of the subtraction from any of them of a specific number of units or fractions. And therefore formally considered* qua ***absolute determinant*** *(in potency) of any numerically undetermined extensional* quantum, *because of its potentiality to enter* (a posteriori) *into the fractioning of the magnitudes ascribed to any specific extensional measure (metre, foot, inch, etc.)*

(9) ***Any integral ratio*** (3 : 4 : 5), *formally considered* qua *determinacy imposed by the* a priori *possibility of specifically-indeterminate extensional magnitudes—whose formal determination, of course, cannot be the immediate generic value of each of them individually (3, 4, 5), but the value which conceptually results from their reciprocal relation to one another* (*e.g.*, 2 : 4). *So that this latter value* (*e.g.*, the ratio $2 : 2^2$) *is in itself an absolute magnitude, and, as such, distinct from the (in relation to it) relative magnitudes which express it—whose alteration, therefore, cannot but be an alteration of one of its*

(specific) instances and, as such, have no effect at all over its generality. |[26]

(10) ***Any integral ratio*** (3 : 4 : 5), *consequently, also formally considered* qua ***absolute determinant*** *(in potency) of any numerically undetermined extensional* quantum, *because of its potentiality to enter* (a posteriori) *into the proportioning of any two or more magnitudes ascribed to any specific extensional measure (metre, foot, inch, etc.*

106. In short, when it is understood in its (FORMAL) (B) QUANTITATIVE MEANING as well as in its (iii) GENERIC MEANING (and therefore in ABSOLUTE TERMS), the designation EXTENSIONAL MEASURE signifies either (A) ***Extensional magnitude numerically determined*** qua ***INTEGRAL magnitude*** (*e.g., magnitude* 3 *of extensional measure*), (B) ***Extensional magnitude numerically determined*** qua ***FRACTIONAL magnitude*** (*e.g.,* 3/4 *of extensional measure* 4/4), or (C) ***Extensional magnitude numerically determined*** qua ***RATIO*** (*e.g., ratio* 3 : 4 : 5 *in extensional measure*).

107. For it (the designation EXTENSIONAL MEASURE) then expresses quantitative meaning which is ***quantitatively*** determined as to gender (generically determined: *e.g.*, *Any* ***three*** **(3)** ***units****, of whatever extensional (unit of) measure*), but ***qualitatively*** undetermined as to species (specifically–relatively undetermined: *Three* (3)

units ***of which extensional (unit of) measure****?*), and therefore also ***quantitatively*** undetermined as to species (specifically–relatively undetermined: *e.g.*, ***How many*** *units of extensional (unit of) measure 'm'?*)

108. When it is understood in its (FORMAL) **(B) QUANTITATIVE MEANING** as well as in its **(iv) SPECIFIC MEANING** (and therefore in **RELATIVE TERMS**), the designation EXTENSIONAL MEASURE signifies:

> (1) ***The number ONE (1)****,* ***instead of any other number****, formally considered* qua *numerical magnitude subjectively (formally) posited and* ***ascribed to a specific (determinate) extensional measure*** *(metre, foot, inch, etc.) And therefore formally considered* qua ***relative determinant*** *(in potency) of any numerically-determined extensional* quantum *subjectively (formally) posited*.
>
> (2) ***This, instead of that, aggregation of the number ONE*** *(2, 3, 4, etc.), formally considered* qua *numerical magnitude subjectively (formally) posited and* ***ascribed to a specific (determinate) extensional measure*** *(metre, foot, inch, etc.) And therefore formally considered* qua ***relative determinant*** *(in potency) of any numerically-determined extensional* quantum *subjectively (formally) posited*.

(3) ***The number* ONE *(1) as numerator*** (1/4, 1/6, 1/8, *etc.*), ***instead of any other numbe**r*, *formally considered* qua *unitary fraction to be excluded from a* ***denominator greater than* ONE *(1)*** *likewise subjectively (formally) posited and* ***ascribed to a specific (determinate) extensional measure*** *(metre, foot, inch, etc.). And therefore formally considered as numerator potentially entering,* qua ***relative determinant***, *in the fractioning of any numerically-determined extensional* quantum *subjectively (formally) posited*.

(4) ***This, instead of that, numerator greater than* ONE** (2/1, 3/1, 4/1, *etc.*), *formally considered* qua *fractional total of a* ***denominator* ONE *(1)*** *likewise subjectively (formally) posited and* ***ascribed to a specific (determinate) extensional measure*** *(metre, foot, inch, etc.). And therefore formally considered as numerator potentially entering,* qua ***relative determinant***, *in the fractioning of any numerically-determined extensional* quantum *subjectively (formally) posited*.

(5) ***This, instead of that, numerator greater than* ONE** (2/4, 4/6, 6/8, *etc.*), *formally considered* qua *number of fractions to be excluded from a* ***denominator greater than* ONE *(1)*** *likewise subjectively (formally) posited and* ***ascribed to a specific (determinate) extensional measure*** *(metre, foot, inch,*

etc.). And therefore formally considered as numerator potentially entering, qua ***relative determinant****, in the fractioning of any numerically-determined extensional* quantum *subjectively (formally) posited*.

(6) ***The number* ONE *(1) as denominator*** (2/1, 3/1, 4/1, *etc.*), ***instead of any other numbe****r, formally considered* qua *fractional whole equal to a* ***denominator greater than* ONE *(1)*** *likewise subjectively (formally) posited and* ***ascribed to a specific (determinate) extensional measure*** *(metre, foot, inch, etc.). And therefore formally considered as denominator potentially entering,* qua ***relative determinant****, in the fractioning of any numerically-determined extensional* quantum *subjectively (formally) posited*.

(7) ***This, instead of that****,* ***denominator greater than* ONE** (2/4, 4/6, 6/8, *etc.*), *formally considered* qua *fractional whole of a* ***numerator greater than* ONE *(1)*** *likewise subjectively (formally) posited and* ***ascribed to a specific (determinate) extensional measure*** *(metre, foot, inch, etc.). And therefore formally considered as denominator potentially entering,* qua ***relative determinant****, in the fractioning of any numerically-determined extensional* quantum *subjectively (formally) posited*.

(8) ***This, instead of that***, ***integral ratio*** (3 : 4 : 5), *formally considered* qua *ratio subjectively (formally) posited and* ***ascribed to a specific (determinate) extensional measure*** *(metre, foot, inch, etc.) And therefore formally considered* qua ***relative determinant*** *(in potency) of any numerically-determined extensional* quantum *subjectively (formally) posited*.

109. In short, when it is understood in its (FORMAL) (B) QUANTITATIVE MEANING as well as in its (iv) SPECIFIC MEANING (and therefore in RELATIVE TERMS), the designation EXTENSIONAL MEASURE signifies either (A) ***Extensional magnitude numerically determined***, qua ***INTEGRAL magnitude***, ***by a SPECIFIC unit of measure*** (*e.g., magnitude* 3 *of extensional measure 'm'*), (B) ***Extensional magnitude numerically determined***, qua ***FRACTIONAL magnitude***, ***by a SPECIFIC unit of measure*** (*e.g.*, 3/4 *of extensional measure* 4/4 m), or (C) ***Extensional magnitude numerically determined***, qua ***RATIO***, ***by a SPECIFIC unit of measure*** (*e.g., ratio* 3 : 4 : 5 *in extensional measure 'm'*).

110. For it then expresses quantitative meaning which is ***quantitatively*** determined not only as to gender (generically determined: *e.g.*, *Any* ***three*** **(3)** ***units***, *of whatever extensional (unit of) measure*), but also as to species (specifically–relatively undetermined: *e.g.*, ***Three*** **(3)** ***units*** *of* ***extensional (unit of) measure 'm'***).

111. Now, when the designation EXTENSIONAL MEASURE is understood **(II) MATERIALLY** or in **OBJECTIVE (PHYSICAL) TERMS**, it likewise acquires **(A) QUALITATIVE MEANING** and **(B) QUANTITATIVE MEANING**.

112. Understood in the terms of the material reference of its **(A) QUALITATIVE MEANING**, the designation EXTENSIONAL MEASURE gives us 'measure' *qua* QUALITATIVELY DETERMINED QUANTITATIVE DETERMINATION; understood in terms of the material reference of its **(B) QUANTITATIVE MEANING**, it, on the contrary, gives us QUANTITATIVELY DETERMINED QUALITATIVE DETERMINATION.

113. It so happens, because, in its reference to ideal or conceptual entities (considered *qua objecta* of *cognitio*, and, therefore, in their objectuality, by opposition to the objectivity of material or physical entities), intension and extension are capable of being considered quantitatively only in analogical terms.

114. That this is so, is shown, for instance, by the following professorial statements of Kant, which indeed reveal the liberty he takes to speak in them of conceptual quantities qualified by the expression 'as it were' (*gleichsam*). Kant begins thus: "If a thing can be cognized through a certain mark without the mediation of a mark different from this mark, then such a mark is *im-*

mediate. A *mediate* mark, on the other hand, is a mark of a mark: e.g. Perishability is a mark of man. But an *immediate* one, for perishability is a mark of a mortal being, i.e., of a body[;] man is mortal, however, and has a body. ...among all marks, one of which is the mark of the other, a relation of subordination is always to be found. But there is also a relation of coordination, and this occurs among immediate marks, where every particular mark is a new ground of cognition of the thing."

115. Kant then goes on to utter the words that really are of interest here: "Through the subordination of marks I attain a deep, i.e., INTENSIVE QUANTITY. But through the coordination of marks[,] of *notae*[,J one achieves an EXTENSIVE QUANTITY[;] i.e., cognition that is worked out according to the ground and is extensive. The series of marks that are coordinated with one another is, as it were, a line without limits, which is infinite.... The EXTENSIVE QUANTITY of a cognition is, as it were, its volume, while its INTENSIVE QUANTITY is its density. In certain cases the extensive quantity of cognition is more valuable but in others the intensive. In morals, e.g., the intensive quantity of a cognition is more necessary, but in physics and medicine the extensive quantity of cognition is quite frequently more indispensable." |[27]

116. So, when the designation EXTENSIONAL MEASURE (MEASURE OR MAGNITUDE *qua* SPATIAL CONTINUITY OF DEGREE (OF QUALITY)) is understood in the terms of the material reference of its **(A) QUALITATIVE MEANING**, it gives us, (i) in a DIRECT OR IMMEDIATE WAY, the following qualitatively objective, and ***UNIVERSAL–ABSOLUTE***, determination: ***Physical extension as numerically determinable magnitude or measure (as spatial 'duration' or continuity of* LENGTH*), by opposition to physical intension as, likewise, numerically determinable magnitude or measure (as temporal duration or continuity of* DEGREE*).***

117. When it is so understood, the designation EXTENSIONAL MEASURE gives us, (ii) in an **INDIRECT OR MEDIATE WAY**, the following qualitatively objective, and no less ***UNIVERSAL–ABSOLUTE***, determination: ***All members of the class* PHYSICAL ENTITY *are extensional magnitudes, and therefore all of them are,* QUA *measure, numerically determinable.***

118. When, on the other hand, the designation EXTENSIONAL MEASURE (MEASURE OR MAGNITUDE *qua* SPATIAL CONTINUITY OF DEGREE (OF QUALITY)) is understood in the terms of the material reference of its **(B) QUANTITATIVE MEANING**, it gives us, (i) in a **DIRECT OR IMMEDIATE WAY**, for instance, the following quantitatively positional, and **PARTICULAR–RELATIVE**, determinations:

- ***This magnitude, this extensional continuity***

of degree x, y, z, as the UNIT-OF-MEASURE *METRE*; • *This magnitude, this extensional continuity of degree x, y, z, as the* UNIT-OF-MEASURE *INCH*; • *This magnitude, this extensional continuity of degree x, y, z, as the* UNIT-OF-MEASURE *FOOT, etc.*

119. Finally, when it is so understood, the designation EXTENSIONAL MEASURE gives us, (ii) in an **INDIRECT OR MEDIATE WAY**, quantitatively positional, and ***PARTICULAR–RELATIVE***, determinations such as the following: • *This flagpole (this particular member of the class 'physical entities') is length-magnitude 4 m;* • 2/4 *of the length of this flagpole (of this particular member of the class 'physical entities') are magnitude 2 m;* • *These two flagpoles (these two particular members of the class 'physical entities') are length-magnitude in the ratio 2 : 4, etc.*

120. From what has come to light above, what follows can be concluded:

(1) That, when the designation EXTENSIONAL MEASURE is understood in **(I) SUBJECTIVE TERMS**, as well as **(A) QUALITATIVELY** and **(i) GENERICALLY**, it signifies: ***any extensional magnitude intersubjectively posited as paradigmatic extensional quantity and, therefore, intersubjectively posited as objective measure of extensional magnitude.***

(2) That, when the designation EXTENSIONAL MEASURE is understood in **(I) SUBJECTIVE TERMS** as well as **(A) QUALITATIVELY** and **(ii) SPECIFICALLY**, it signifies: • ***Measure of extensional magnitude (unit of measure) METRE;*** • ***Measure of extensional magnitude (unit of measure) INCH;*** • ***Measure of extensional magnitude (unit of measure) FOOT, etc.***

(3) That, when the designation EXTENSIONAL MEASURE is understood in **(I) SUBJECTIVE TERMS** as well as **(B) QUANTITATIVELY** and **(iii) GENERICALLY**, the cognition of its signified (the concept it relates to *qua* signifier) implies the knowledge:

> *(a) that, as Kant has shown in all-embracing terms and has been referred to above,* |[28] *all (specifically-determinate) extensional measures (metre, foot, inch, etc.) are ultimately determined* a priori *(and therefore formally) by the concepts of the understanding* (Verstandesbegriffe) *of Quantity* (Quantität): *the categories* (Kategorien) *of Unity* (Einheit), *Multiplicity* (Vielheit), *and Totality* (Allheit);
>
> *(b) that therefore all (specifically-determinate) extensional measures (metre, foot, inch, etc.) are equally determined* a

priori *by the formal constitutiveness of (i) the so-called Real Numbers: Rational (Natural-Counting, Fractional, Integers (Whole, Negative, and Counting) and Irrational;*

(c) that, qua *potential determinant of measure (in absolute terms), any real number is, qualitatively or subjectually considered, quantitative determinacy in abstraction from any of its possible (formal or material) instances, and that it therefore ever remains in potency non-determining subjectual determinacy: (i) this or that abstract (non-objectual) whole-quantity not yet conceived of as-measure, and consequently conceived of only as quantity for a measure-to-be; (ii) this or that abstract (non-objectual) fractional quantity (the numerator) of an abstractly (non-objectually) fractioned whole-quantity (the denominator) likewise not yet conceived of as-measure, and consequently also conceived of only as quantity for a measure-to-be.*

(4) That, when the designation EXTENSIONAL MEASURE is understood in **(I) SUBJECTIVE TERMS** as well as **(B) QUANTITATIVELY** and **(iv) SPECIFICALLY**, the cognition of its signified (the concept it relates to *qua* signifier) implies the knowledge:

(a) that, understood as species, the concept 'measure' entails two distinct (qualitative as well as as quantitative) moments: (i) that of its ***formation****, of its positing as this or that specific unit (of measure), and (ii) that of its* ***application****, either formally,* qua *conceptual and imaginative cognitive objectuality (as it happens when someone calculates length, area, or volume in abstraction from any of their possible instances), or materially,* qua*, at the same time, objective standard of measurement and intuited* (angeschaut) *measure;*

(b) that, when any number is cognized not only in itself or in its immediate difference (heterogeneity) and isolation, not only from the point of view of its subjectual inequality to and consequent neutrality towards all other numbers, but also from the point of view of its indifference (homogeneity) and of the resulting possibility of its participation into one of the specific conceptual synthesis called 'calculation'—that, when any number is so cognized, mediately cognized, its formal cognitive configuration is altered, since it then inevitably acquires objectual character (this and its intrinsic subjectual character being reciprocally sublated);

(c) that, as a result, when any numerical magnitude is so cognized, mediately cognized, it formally becomes (i) an absolute as well as (ii) a relative numerical magnitude: (i) absolute, both in itself (qualitatively) and in its reciprocal relation with other numerical magnitudes (with the result, for instance, that, considering calculation exclusively as multiplication, only by multiplying 4 by 2 is 8 cognitively produced); (ii) relative, exclusively in its reciprocal relation with other numerical magnitudes (with the result, for instance, that 8 can be produced by adding 5 and 3 as well as by multiplying 4 by 2);

*(d) that, being numerical magnitudes in themselves (*qua *form) as well as numerical magnitudes empirically conditioned (*qua *matter or material* quanta*), |*[29] *all (specifically-determinate) measures of extensional magnitude cannot but be both absolute magnitudes (considered exclusively* qua *form) as well as doubly relative magnitudes (considered both* qua *form and* qua *matter)—as much when they are considered as 'mere cognition' (objectuality) as when they are considered as cognition exhibited by intuition (objectivity);*

(e) that, when they are thought of in **(B) QUANTITATIVE** *as well as in* **(iv) SPECIFIC** *terms, all (specifically-determinate) measures of extensional magnitude are necessarily represented* (vorgestellt) *both (I) under the simple aspect of materially unconditioned objectuality (conceptual form), simply as numerical magnitudes in themselves, and, inversely, (II) under the no less subjective, but synthetically complex, aspect of: (i) objectuality (conceptual form), (ii) intuitively (imaginatively) determined (limited) extensional* quantum, *and (iii) positing of an empirically derived vinculum between the two.*

(f) that, qua *representation* (Vorstellung), *all (specifically-determinate) measures of extensional magnitude are necessarily, therefore, both* cognitio originaria, *as immediate subjectuality and mediate objectuality, and* cognitio derivative, *as intuition objectively given and imaginatively exhibited;*

(g) that it is indeed as a direct result of this, that, again, they cannot but be cognized both as absolute magnitudes, considered exclusively qua *form, and as doubly relative magnitudes: considered both* qua *form and* qua *matter;*

(h) that the roots of their intrinsic relativity are, then, immersed as much in their original or a priori *formality (conceptuality) as in their derivative or* a posteriori *materiality (intuitionality);*

(i) that, it being so, such relativity is, in what concerns its originality, constitutive or grounded a priori, *and is therefore apodictically determined, whereas, in what concerns its derivativeness, it is non-constitutive or grounded* a posteriori relativity, *and thus cannot but be contingent or accidental.*

(*j*) *that, as a result, whenever (i) the correspondence between any numerical magnitude* (e.g., 3) *and (ii) this or that standard extensional magnitude or measure* (*e.g.*, the measure METRE) *is intentionalised, the nexus thus posited between (i) the former and (ii) the latter cannot but be contingent—that is to say, ever dependent on the historicity of its (intersubjective) grounding.*

(5) Finally, that, when the designation 'extensional measure' is understood **(II) MATERIALLY OR IN OBJECTIVE (PHYSICAL) TERMS**, as well as **(B) QUANTITATIVELY**, the cognition of its signified (the concept it relates to qua signifier) implies the knowledge:

(a) that, the quantitativeness it then expresses having objective character, it must correspond, in the first place, to a ***quantity objectively intuited*** (a posteriori) *and, therefore, to a* quantum discretum *(empirically) given by sensation; (b) that such a* ***quantity*** *must, perforce, be in itself the attribute of a specific quality, and, consequently, be qualitative quantity,* qua *extensional magnitude of a determinate degree of intension or (qualitative) intensity;*

(c) that, considered in its intuitional character, qua *empirical given, such a* ***quantity*** *must necessarily be objectively determined (limited) extensional magnitude or* quantum discretum*—must necessarily be, that is to say, the formal quality* Extension *as it is particularly actualized by the internal continuity of this or that individual complex quality (*qua*, of course, substantive quality);*

(d) that, thus considered, as discrete unit, such a ***quantity*** *must, perforce, be numerically determined quantity, but that, considered qua* quantum continuum (*internally* unlimited)*, it must, however, remain numerically undetermined magnitude;*

(e) that, in order for it to become numerically determined magnitude qua *measure in extension or extensional measure, such a* ***quantity*** *has to be measured by a specific unit (of measurement);*

(f) that this, however, implicates that the specific, but nonetheless purely subjective or positive, concept MEASURE X *be added to such a* ***quantity****, under the aspect of cognitive determinant, of the form of all intuition (space and time), which in itself is determined as much formally or* a priori *(by the concepts of the understanding of Quantity) as materially or* a posteriori (qua the intuition of *this or that determinate length*)*—with the consequence that,* qua *material determinacy, it cannot but be considered relative.*

121.. Now, imagine that you are wandering along a valley in the company of a stranger you have met recently, and that the two of you come across two isolated trees, one much higher than the other.

122. You immediately become aware, of course, that you are facing two trees, instead of none, one, or three—which is to say that you do, indeed, intuit the quality *Tree*, and therefore its

essential quality *Extension*, *qua* objectively grounded numerical determinacy.

123. Then you turn to your companion and exclaim: "Whoa, what a long tree! It must extend at least for ten or twelve metres—don't you think?" That is to say, by then you do, indeed, add to your intuition of the tree's extension, side by side with the concept TREE, the concept METRE in its imaginative or schematized form.

124. "Metres?!" exclaims your companion. "I know nothing of that. I am a true son of the Earth: I measure in feet: my measure stands. And I can pretty much assure you that that tree standing over there is—oh, yes, it is—no less then thirty feet high. Oh, yes. Metres! What a fancy stuff! Fine ladies and soft-spoken gentlemen, those do measure in metres, be that what it may. Not I. Neither, for that matter, Nature—or even God Himself. The Foot, the Foot—the Foot, not even Man, is the measure of all things."

125. Would you, on your part, consider such a tree—such a tree or any other member of the class PHYSICAL ENTITY—as much, objectively, the quality *Tree* as, again objectively, magnitude in *Extension* '10 m', or '32.8084 ft.'? Of course not. And maybe the reason for that is, that you have managed to read these Notes up to the end!

126. As a final curiosity, consider that the *Encyclopedia Britannica* assures its readers that: (i) the INCH was defined by King David I of Scotland, about 1159, as *the breadth of a man's thumb at the base of the nail;* (ii) the INCH was defined in the early the 14th century, during the reign of Edward II, as *three grains of barley, dry and round, placed end to end lengthwise;* (iii) the INCH has also been defined, at various times, as *the combined lengths of 12 poppyseeds*; (iv) the INCH has been officially defined, since 1959, as *2.54 cm*.

127. And, now, consider your companion from above: first, metamorphosed into yourself; secondly, metamorphosed from a worshipper of the natural FOOT into a worshipper of the natural INCH; thirdly, transported, one after the other, into the reign of David I of Scotland and into the reign of Edward II.

128. No doubt, he would remark to his Davidian *tree-inchness* fellow-contemplator: "of those two trees over there, the highest measures at least 400 inches"—the ground of his judgment of his being that, side by side with the sound-image *two* and the concept INCH, his imagination would have presented itself an approximate visual image of the two ticks limiting more or less the space a inch occupies nowadays in a modern inch-ruler. "Impossible!" his companion would exclaim. "I am definitely sure it measures

not more than about 170 inches"—the ground for this quite different judgment of his being that, side by side, perhaps, with the sound-image *two*, his imagination would have presented itself the visual image of the breadth of a man's thumb at the base of the nail.

129. And, again not having lost his *modern* stock of imaginative representations, he would also remark to his Edwardian *tree-inchness* fellow-contemplator: "of those two trees over there, the highest measures at least 400 inches." My God! How low is your highness! By the rood, I swear, that tree over there—let me see, let me see—that tree over there measures at least 1200 inches."—The ground of the judgment being now, of course, that his imagination would have presented itself, side by side with the concept INCH, the visual image of three grains of barley, dry and round, placed end to end lengthwise!

130. Are, then, the historical-technological determinants of our modern scientific units of measurement (and all units of measurement have once been modern) much different from those that undoubtedly would have much bewildered our time-travelling sycophant of the sacred inch, had he here been given the opportunity to contemplate in his cell-phone the pixel-picture of the imaginative picture in the mind of his Davidian and of his Edwardian inch-estimator?

131. In case the hypothetical reader know not yet the answer, he or she is here encouraged to query the Internet for information regarding, for instance, the history of the quantitative concept AMU.

132. If you take into consideration the determination of positive (*gesetzt*) unit of measure by reference to natural or objectively given measure, *Breadth of a man's thumb, grain of barley,* or *poppyseed*—its really indifferent, if you prefer one or the other and are in the habit of mentally representing this one or that one as *the measure* (assuming, of course, that your natural measure is constant): for all perceivable measures, objectively given or not, lack apodicticity, and, therefore, are relative.

133. If, on the other hand, you take into consideration the comparison of one positive (*gesetzt*) unit of measure (*e.g. inch*) to another (*e.g.cm*), you obviously go no further, since the measure playing the role of *comparatum* will itself have to be either (i) measure arbitrarily determined or (ii) positive measure determined by reference to natural or objectively given measure.

134. Besides, all units of measure, with no *scientific* exception, have been, have so far been, *human* units of measure.

135. It is a fact, it is true, that they all are not just *human* units of measure, but also constant or objective units of measure, as a result of the so-called 'objective world' ever to *seem* to *adapt* itself, or to *respond,* to them in a *meticulously* invariable (and therefore apodictic) way. And, as a consequence of that fact, they are indeed phenomenal *certainty (pistis)* or epistemic *conviction (doxa)*, although they most certainly are not *truth* or noumenal invariance.

136. Indeed, one would better never to prescind of, or conveniently tuck away from oneself, the pressing awareness that it is so. For—to end up going back to the above epigraph: although *an inch* may well be *just an inch*, no man can be *certain* that *this* cosmos *of Man*—which *modern* Man still doggedly represents (*vorstellt*) as cosmos *sub specie aeternitatis,* in spite of his scientific but grossly unphilosophical relativism—will indeed *adapt* itself, or objectively *respond,* to *just one more inch* in intensity in the "as cosmos" with the *invariance* it has *adapted* itself, or *responded*, to all the previous *pressing inches*.

137. No, no man.

138. Not even one of those most *modern* men who most unscientifically proffer to non-scientific people that *the* cosmos—which manifestly cannot *be* more than an Idea of reason (*eine Idee der Vernunft*), the paralogical representation

(*Vorstellung*) of a phenomenally limited phenomenal absence of noumenal limit—is expanding.

139. *Whereto?*—must not non-scientific people see themselves compelled to ask? *Is its limitless extension, in truth, 'adding' itself to the perforce mutually limiting extension of some other cosmos?*

140. Of course, not. Then, ought not scientific people to be concerned concerning the unscientific phraseology in which they proffer their scientific statements to non-scientific people?

Notes and References

1 John Locke. *An Essay Concerning Human Understanding: An Abridgement* (II, viii, 9). Ed. John W. Yolton, London, Everyman, 1991, p. 58.

2 *Idem. Ibidem.*

3 *Idem. Ibidem*, II, viii, 10.

4 The transcendental concept of appearances in space… is a critical reminder that nothing intuited in space is a thing in itself, that space is not a form inhering in things in themselves as their intrinsic property, that objects in themselves are quite unknown to us, and that what we call outer objects are nothing but mere representations of our sensibility, the form of which is space. The true correlate of sensibility, the thing in itself, is not known, and cannot be known, through these representations; and in experience no question is ever asked in regard to it."—Immanuel Kant. *Critique of Pure Reason,* A 30/B 45. Transl. Norman Kemp Smith, Houndmills, Macmillan, 1993, pp. 73–74.

> *…der transzendentale Begriff der Erscheinungen im Raume [ist] eine kritische Erinnerung, daß überhaupt nichts, was im Raume angeschaut wird, eine Sache an sich, noch daß der Raum eine Form der Dinge sei, die ihnen etwa an sich selbst eigen wäre, sondern daß uns die Gegenstände an sich gar nicht bekannt sind, und, was wir äußere Gegenstände nennen, nichts anderes als bloße Vorstellungen unserer Sinnlichkeit sind, deren Form der Raum ist, deren wahres Korrelatum aber, d.i.*

das Ding an sich selbst, dadurch gar nicht erkannt wird, noch erkannt werden kann, nach welchem aber auch in der Erfahrung niemals gefragt wird.—Immanuel Kant. *Werke*. Ed. Wilhelm Weischedel, Frankfurt am Main, Suhrkamp, 1956, vol. 3, pp. 77–78.

5 "...the power to produce any i*dea* in our mind I call *quality* of the subject wherein that power is. Thus a snow-ball having the power to produce in us the *idea*s of *white*, *cold*, and *round*, the powers to produce those *ideas* in us as they are in the snow-ball I call *qualities*" (John Locke. *An Essay Concerning Human Understanding*, II, viii, 8, p. 58).

6 Immanuel Kant. *Prolegomena to Any Future Metaphysics*, § 13, Note II. Transl. and ed. Gary Hatfield, Cambridge, Cambridge University Press, 2004, pp. 40–41.

Daß man, unbeschadet der wirklichen Existenz äußerer Dinge von einer Menge ihrer Prädikate sagen könne: sie gehöreten nicht zu diesen Dingen an sich selbst, sondern nur zu ihren Erscheinungen, und hätten außer unserer Vorstellung keine eigene Existenz, ist etwas, was schon lange vor Lockes *Zeiten, am meisten aber nach diesen, allgemein angenommen und zugestanden ist. Dahin gehören die Wärme, die Farbe, der Geschmack etc. Daß ich aber noch über diese, aus wichtigen Ursachen, die übrigen Qualitäten der Körper, die man primarias nennt, die Ausdehnung, den Ort, und überhaupt den Raum, mit allem was ihm anhängig ist (Undurchdringlichkeit oder Materialität, Gestalt etc.), auch mit zu bloßen Erscheinungen zähle, dawider kann man nicht den mindesten Grund der Unzulässigkeit anführen"*—Immanuel Kant. *Prolegomena zu einer jeden künftigen Metaphysik die als Wissenschaft wird auftreten können*, § 13, A 63. *Werke*, vol. 5, p. 152.

7 "...if we remove from our empirical concept of any object, corporeal or incorporeal, all properties which experience has taught us, we yet cannot take away that property through which the object is thought as substance or as inhering in a substance (although this concept of substance is more determinate than that of an object in general). Owing, therefore, to the necessity with which

this concept of substance forces itself upon us, we have no option save to admit that it has its seat in our faculty of *a priori* knowledge."—Immanuel Kant. *Critique of Pure Reason*, B 6, p. 45 (translation of Norman Kemp Smith).

> *...wenn ihr von eurem empirischen Begriffe eines jeden, körperlichen oder nicht körperlichen, Objekts alle Eigenschaften weglaßt, die euch die Erfahrung lehrt, so könnt ihr ihm doch nicht diejenige nehmen, dadurch ihr es als Substanz oder einer Substanz anhängend denkt, (obgleich dieser Begriff mehr Bestimmung enthalt, als der eines Objekts überhaupt.) Ihr müßt also, überführt durch die Notwendigkeit, womit sich dieser Begriff euch aufdringt, gestehen, daß er in eurem Erkenntnisvermögen a priori seinen Sitz habe*.—Immanuel Kant. *Werke*, vol. 3, p. 48.

8 The designation 'magnitude' (Latin *magnitudo*, from *magnus*, 'great') means 'greatness' in the sense of 'quantity'. All quantities, however, are quantities of quality *x* (of a determinate way of *being*), and therefore are always qualitative (quantities). As Hegel has shown, "magnitude (or, more correctly, quantity)—in distinction from quality—is a determination with respect to whose alteration this or that thing [this or that complex quality, such as *Water*] is indifferent [...] This implies that, in whatever direction the determination of magnitude is changed, the thing in question remains what it is." (*The Encyclopaedia of the Philosophical Sciences*, I ("The Science of Logic"), ii, B, § 99, "Addition." Trad. T. F. Geraets *et al.*, Indianapolis, Hacket, 1991, p. 158. One thing, indeed, is the abstraction (the conceptual entity) 'quality', other, quite different, any one of its (concrete) instances: the former *is* conceptual or formal qualitative quantity (*e.g.*, H_2O); the latter always is material qualitative quantity.

9 *Critique of Pure Reason*, A 162/B 293. (The emphasis was added by the quoter.).

10 "...in quantity we have something which is alterable, but which still remains the same in spite of its alteration. As a result, the concept of quantity turns out to contain a contradiction, and it is this contradiction that constitutes the dialectic of quantity. But the result of this dialectic is not a mere return to quality, as if the latter were what is true, and quantity on the contrary what is

untrue. Instead, the result is the unity and truth of the two of them: it is qualitative quantity or *measure*."—G. W. F. Hegel. *The Encyclopaedia of the Philosophical Sciences*, I ("The Science of Logic"), ii, B, § 106, "Addition". Trad. T. F. Geraets *et al*., Indianapolis, Hacket, 1991, p. 169.

> *...wir [haben] an der Quantität ein Veränderliches... welches ungeachtet seiner Veränderung doch dasselbe bleibt. Der Begriff der Quantität erweist sich hiermit als einen Widerspruch in sich enthaltend, und dieser Widerspruch ist es, welcher die Dialektik der Quantität ausmacht. Das Resultat dieser Dialektik ist nun aber nicht die bloße Rückkehr zur Qualität, so als ob diese das Wahre, die Qualität dagegen das Unwahre wäre, sondern die Einheit und Wahrheit dieser beiden, die qualitative Quantität—oder das* Maß.—G. W. F. Hegel. *Encyclopädie der philosophischen Wissenschaften im Grundrisse*. Ed. Leopold von Henning, Berlin, Dunker und Humblot, 1840, vol. I (*Die Logik*), p. 214.

11 "Quantity, *posited* essentially with the excluding determinacy that it contains, is *quantum* or limited quantity."—G. W. F. Hegel. *The Encyclopaedia of the Philosophical Sciences*, I ("The Science of Logic"), ii, B, § 101, p. 161 (Translation of T. F. Geraets).

> *Die Quantität wesentlich* gesetz***t*** *mit der ausschließenden Bestimmtheit, die in ihr enthalten ist, ist* Quantum*, begrenzte Quantität.*—G. W. F. Hegel. *Encyclopädie der philosophischen Wissenschaften im Grundrisse*. vol. I (*Die Logik*), p. 202.

12 "As the proximate result of being-for-itself, quantity contains within itself as ideal elements both sides of its process (repulsion and attraction). Hence it is both continuous and discrete. Each of these two moments contains the other within itself, so that there is no such thing as a merely continuous or a merely discrete magnitude. If we happen to speak of them as two particular and contrasting species of magnitude, that is just the result of our abstractive reflection."—G. W. F. Hegel. *The Encyclopaedia of the Philosophical Sciences*, I ("The Science of Logic"), ii, B, § 100, "Addition," pp. 160–161 (Translation of T. F. Geraets).

Die Quantität, als nächstes Resultat des Fürsichseins, enthält die beiden Seiten seines Prozesses, die Repulsion und die Attraktion, als ideelle Momente in sich und ist demnach sowohl kontinuierlich als auch diskret. Ein jedes dieser beiden Momente enthält auch das andere in sich, und es gibt somit weder eine bloß kontinuierliche noch eine bloß diskrete Größe. Wenn gleichwohl von beiden als zwei besonderen, einander gegenüberstehenden Arten der Größe gesprochen wird, so ist dies bloß das Resultat unserer abstrahierenden Reflexion....—G. W. F. Hegel. *Encyclopädie der philosophischen Wissenschaften im Grundrisse*. vol. I (*Die Logik*), p. 201.

13 "...the appearances, as mere intuitions that *fill a part of space and time*, are subject to the concept of magnitude, which synthetically unifies the manifold of intuitions *a priori* according to rules... the real in the appearances must have a degree, insofar as perception contains, beyond intuition, sensation as well, between which and nothing, i.e., the complete disappearance of sensation, a transition always occurs by diminution, insofar, that is, as sensation itself *fills no part of space and time*, but yet the transition to sensation from empty time or space is possible only in time, with the consequence that[,] although sensation, as the quality of empirical intuition with respect to that by which a sensation differs specifically from other sensations, can never be cognized *a priori*, it nonetheless can, in a possible experience in general, as the magnitude of perception, be distinguished intensively from every other sensation of the same kind...."—Immanuel Kant. *Prolegomena to Any Future Metaphysics*, § 26, pp. 60–61 (translation of Gary Hatfield).

...die Erscheinungen als bloße Anschauungen, welche einen Teil von Raum und Zeit einnehmen*, unter dem Begriff der Größe stehen, welcher das Mannigfaltige derselben a priori nach Regeln synthetisch vereinigt, ...so fern die Wahrnehmung außer der Anschauung auch Empfindung enthält, zwischen welcher und der Null, d. i. dem völligen Verschwinden derselben, jederzeit ein Übergang durch Verringerung stattfindet, das Reale der Erscheinungen einen Grad haben müsse, so fern sie nämlich selbst* keinen Teil von

Raum oder Zeit einnimmt*, aber doch der Übergang zu ihr von der leeren Zeit oder Raum nur in der Zeit möglich ist, mithin, obzwar Empfindung, als die Qualität der empirischen Anschauung, in Ansehung dessen, worin sie sich spezifisch von andern Empfindungen unterscheidet, niemals a priori erkannt werden kann, sie dennoch in einer möglichen Erfahrung überhaupt, als Größe der Wahrnehmung intensiv von jeder andern gleichartigen unterschieden werden könne*....—Immanuel Kant. *Werke*, vol. 5, pp. 176–77.

14 "The *limit* is identical with the whole of the quantum itself; as multiple *within itself* it is *extensive* magnitude, but as determinacy that is *simple* within itself, it it is *intensive* magnitude *or* degree."—G. W. F. Hegel. *The Encyclopaedia of the Philosophical Sciences*, I ("The Science of Logic"), ii, B, § 103, p. 163 (Translation of T. F. Geraets).

Die Grenze *ist mit dem Ganzen des Quantums selbst identisch; als* in sich *vielfach ist sie die* extensive*, aber als in sich* einfache *Bestimmtheit die* intensive *Größe oder der Grad.*—G. W. F. Hegel. *Encyclopädie der philosophischen Wissenschaften im Grundrisse*. vol. I (*Die Logik*), p. 205.

15 The parenthesis refer the reader to Figures 1.1–2.2, on pages 94–97 (T means TIME).

16 See, for instance, the seventh chapter ("Extensive Magnitudes") of Rudolf Carnap's *An Introduction to the Philosophy of Science* (ed. Martin Gardner, New York, Dover, 1995, pp. 70–77).

17 G. W. F. Hegel. *The Encyclopaedia of the Philosophical Sciences*, I ("The Science of Logic"), ii, B, § 103, p. 163 (Translation of T. F. Geraets).

Der Unterschied der kontinuierlichen und diskreten Größen von den extensiven und intensiven besteht darin, daß die ersteren auf die Quantität überhaupt *gehen, diese aber auf die* Grenze *oder Bestimmtheit derselben als solcher.—Gleichfalls sind die extensive und intensive Größe auch nicht zwei Arten, deren jede eine Bestimmtheit enthielte, welche die andere nicht hätte; was extensive Größe ist, ist ebensosehr als intensive, und um-*

gekehrt.—G. W. F. Hegel. *Encyclopädie der philosophischen Wissenschaften im Grundrisse*. vol. I (*Die Logik*), p. 205.

18 "...in its immediate relation to itself, or in the determination of self-equivalence posited by attraction, quantity is *continuous* magnitude; in the other determination which it contains—that of the *One*—it is *discrete* magnitude. But continuous quantity is also discrete, for it is only continuity *of the many;* and discrete quantity is also continuous, for its continuity is the One as that in which the many ones *are the same*, *unity*.—G. W. F. Hegel. *The Encyclopaedia of the Philosophical Sciences*, I ("The Science of Logic"), ii, B, § 100, p. 160 (Translation of T. F. Geraets).

Die Quantität zunächst in ihrer unmittelbaren Beziehung auf sich oder in der Bestimmung der durch die Attraktion gesetzten Gleichheit mit sich selbst ist kontinuierliche*, in der anderen in ihr enthaltenen Bestimmung des Eins ist sie* diskrete *Größe. Jene Quantität ist aber ebensowohl diskret, denn sie ist nur Kontinuität des* Vielen*; diese ebenso kontinuierlich, [denn] ihre Kontinuität ist das Eins als* Dasselbe *der vielen Eins, die* Einheit*.*—G. W. F. Hegel. *Encyclopädie der philosophischen Wissenschaften im Grundrisse*. vol. I (*Die Logik*), p. 201.

19 "...quantum... breaks up... into an indeterminate multitude of quanta or determinate magnitudes. Each of these determinate magnitudes, as distinct from the others, forms a unit, just as, on the other hand, considered all by itself, it is a many. And in this way quantum is determined as number. Quantum has its development and perfect determinacy in *number*, which contains the One within itself as its element."—G. W. F. Hegel. *The Encyclopaedia of the Philosophical Sciences*, I ("The Science of Logic"), ii, B, § 101, "Addition," § 102, p. 161 (Translation of T. F. Geraets).

...das Quantum [zerfällt] in eine unbestimmte Menge von Quantis oder bestimmten Größen. Eine jede dieser bestimmten Größen, als unterschieden von der anderen, bildet eine Einheit, so wie dieselbe andererseits für sich allein betrachtet ein Vieles ist. So aber ist das Quantum als Zahl

bestimmt. Das Quantum hat seine Entwicklung und vollkommene Bestimmtheit in der Zahl*, die als ihr Element das Ein... in sich enthalt.*—G. W. F. Hegel. *Encyclopädie der philosophischen Wissenschaften im Grundrisse*. vol. I (*Die Logik*), pp. 202–203.

20 Regarding the notion of 'schematism', consider the following statements of Kant: "No image could ever be adequate to the concept of a triangle in general. [...] Still less is an object of experience or its image ever adequate to the empirical concept; for this latter always stands in immediate relation to the schema of imagination, as a rule for the determination of our intuition, in accordance with some specific universal concept. The concept 'dog' signifies a rule according to which my imagination can delineate the figure of a four-footed animal in a general manner, without limitation to any single determinate figure such as experience, or any possible image that I can represent *in concreto*, actually presents. ...the *image* is a product of the empirical faculty of reproductive imagination; the *schema* of sensible concepts, such as of figures in space, is a product and, as it were, a monogram, of pure *a priori* imagination, through which, and in accordance with which, images themselves first become possible. These images can be connected with the concept only by means of the schema to which they belong. In themselves they are never completely congruent with the concept."—Immanuel Kant. *Critique of Pure Reason*, A 140/B 180–A 141/B 181, pp. 182–83 (translation of Norman Kemp Smith).

Dem Begriffe von einem Triangel überhaupt würde gar kein Bild desselben jemals adäquat sein. [...] Noch viel weniger erreicht ein Gegenstand der Erfahrung oder Bild desselben jemals den empirischen Begriff, sondern dieser bezieht sich jederzeit unmittelbar auf das Schema der Einbildungskraft, als eine Regel der Bestimmung unserer Anschauung, gemäß einem gewissen allgemeinen Begriffe. Der Begriff von Hunde bedeutet eine Regel, nach welcher meine Einbildungskraft die Gestalt eines vierfüßigen Tieres allgemein verzeichnen kann, ohne auf irgend eine einzige besondere Gestalt, die mir die Erfahrung darbietet, oder auch ein jedes mögliche Bild, was ich in concreto darstellen kann, eingeschränkt zu sein. ...das Bild

ist ein Produkt des empirischen Vermögen der [re] produktiven Einbildungskraft, das Schema *sinnlicher Begriffe (als der Figuren im Raume) ein Produkt und gleichsam ein Monogramm der reinen Einbildungskraft a priori, wodurch und wonach die Bilder allererst möglich werden, die aber mit dem begriffe nur immer vermittelst des Schema, welches sie bezeichnen, verknüpft werden müssen, und an sich demselben nicht völlig kongruieren.*—Immanuel Kant. *Werke*, vol. 3, p. 190.

21 Consider, for instance, the following statements of Kant: "...all mathematical cognition has this distinguishing feature, that it must present its concept beforehand in *intuition* and indeed *a priori*, consequently in an intuition that is not empirical but pure, without which means it cannot take a single step; therefore its judgments are always *intuitive*.... This observation with respect to the nature of mathematics already guides us toward the first and highest condition of its possibility; namely, it must be grounded in some *pure intuition* or other, in which it can present, or, as one calls it, *construct* all of its concepts *in concreto* yet *a priori*."—Immanuel Kant. *Prolegomena to Any Future Metaphysics*, § 7, pp. 32–33 (translation of Gary Hatfield).

...alle mathematische Erkenntnis dieses Eigentümliche babe, daB sie ihren Begriff vorher in der Anschauung, *und zwar a priori, mithin einer solchen, die nicht empirisch, sondern reine Anschauung ist, darstellen müsse, ohne welches Mittel sie nicht einen einzigen Schritt tun kann; daher ihre Urteile jederzeit* intuitiv *sind.... Diese Beobachtung in Ansehung der Natur der Mathematik gibt uns nun schon eine Leitung auf die erste und oberste Bedingung ihrer Moglichkeit: namlich, es muß ihr irgend* eine reine Anschauung *zum Grunde liegen, in welcher sie alle ihre Begriffe in concreto, und dennoch a priori darstellen, oder, wie man es nennt, sie* konstruieren *kann.*—Immanuel Kant. *Werke,* vol. 5, p. 142.

22 'Measure' is 'quantity', but, as it has been shown, intension is by nature 'qualitative quantity' or 'degree', whereas extension is by nature 'quantitative quality' or 'continuity of degree'—so that the designation

'extensional measure' first and foremost signifies 'quantity of quality' (a designation which, of course, is not synonymous, or at least does not overlap completely, with the designation 'qualitative quantity'). As the quotation which is given in note 10, above, shows, Hegel calls the indivisable unity of quality and quantity "qualitative quantity or *measure*". When he wrote these words, he did not bother to show that it is so, but the (conceptual) truth remains, that, if measure is "qualitative quantity", it is only because it cannot but be 'quantity of quality'. As it is used in these Notes, then, the designation EXTENSIONAL MEASURE is absolutely synonymous with the more accurate (but somewhat awkward) designation: MEASURE USED IN THE NUMERICAL DETERMINATION OF (THE QUALITY) EXTENSIONAL MAGNITUDE.

23 See Figures 3.1–3.2, on pages 98–99.

24 G. W. F. Hegel. *The Encyclopaedia of the Philosophical Sciences*, I ("The Science of Logic"), ii, B, § 101, "Addition," p. 161 (Translation of T. F. Geraets).

25 "In arithmetic the kinds of calculation are usually presented as contingent ways of treating numbers. If a necessity and hence a [matter for] understanding is to be found in them, then it has to lie in a principle; and this [in turn] can only be found in the determinations that are contained within the concept of number itself. [...] The determinations of the concept of number are *annumeration* and *unit;* and number itself is the unity of the two. But unity, when applied to empirical numbers, is only their *equality;* hence, the principle of the kinds of calculation has to be the positing of numbers in the relationship of unit and annumeration and the production of the equality of these determinations. [...] To calculate, therefore, is quite generally to count; the distinction between the kinds of calculation lies only in the qualitative character of the numbers which are counted together, and the principle of that character is the determination of unit and annumeration. [...] *Numbering*... [is] the making of numbers *generally*, which is the combining of as many *ones* as we want.—But it is the counting together of what are no longer merely *ones* but already numbers that is a kind of calculation."—G. W. F. Hegel. *The Encyclopaedia of the Philosophical Sciences*, I ("The Science of Logic"), ii, B, § 102, p. 162 (Translation of T. F. Geraets).

In der Arithmetik pflegen die Rechnungsarten *als zufällige Weisen, die Zahlen zu behandeln, aufgeführt zu werden. Wenn in ihnen eine Notwendigkeit und damit ein Verstand liegen soll, so muß derselbe in einem Prinzip, und dies kann nur in den Bestimmungen liegen, die in dem Begriffe der Zahl selbst enthalten sind.... Die Bestimmungen des Begriffs der Zahl sind die* Anzahl *und die* Einheit*, und die Zahl selbst ist die Einheit beider. Die Einheit aber, auf empirische Zahlen angewendet, ist nur die* Gleichheit *derselben; so muß das Prinzip der Rechnungsarten sein, Zahlen in das Verhältnis von Einheit und Anzahl zu setzen und die Gleichheit dieser Bestimmungen hervorzubringen. [...] Rechnen ist darum überhaupt* Zählen*, und der Unterschied der* Arten *zu rechnen liegt allein in der qualitativen Beschaffenheit der Zahlen, die zusammengezählt werden, und für die Beschaffenheit ist die Bestimmung von Einheit und Anzahl das Prinzip. [...]* Numerieren *ist... die Zahl* überhaupt *machen, ein Zusammenfassen von beliebig vielen Eins. – Eine Rechnungsart aber ist das Zusammenzählen von solchen, die schon Zahlen, nicht mehr das bloße Eins sind.*—G. W. F. Hegel. *Encyclopädie der philosophischen Wissenschaften im Grundrisse*. vol. I (*Die Logik)*, pp. 203–204.

26 What is here said paraphrases the definition of 'quantitative ratio' that Hegel gives in the "Addition" to I, ii, B, § 105 of his *Encyclopaedia of the Philosophical Sciences*,

27 Immanuel Kant. "The Blomberg Logic", § 116. *In:* —. *Lectures on Logic*. Transl. and ed. J. Michael Young, Cambridge, Cambridge University Press, 1992, pp. 83–85.

Wenn eine Sache durch ein gewißes Merckmahl ohne Vermittelung eines Merckmahls, das von diesem verschieden ist, erkannt werden kann, so ist solches ein Unmittelbahres *Merckmahl. Ein* Mittelbahres *Merckmahl aber hingegen ist ein Merckmahl vom Merckmahl: z. E. Die Verweslichkeit ist ein Merckmahl vom Menschen, aber ein Unmittelbahres, denn die Verweslichkeit ist ein Merckmahl eines sterblichen Wesens, d. i. eines*

Körpers, der Mensch aber ist sterblich, und hat einen Körper. Es ist aber unter allen Merckmahlen, davon eines das Merckmahl des anderen ist, allemahl ein Verhältniß der Subordination anzutreffen. Es giebt aber auch ein Verhältniß der Coordination, und diese findet bey unmittelbahren Merckmahlen statt, alwo ein jedes besonderes Merckmahl ein neuer Erkenntniß-Grund der Sache ist. [...] Durch die Subordination der Merckmahlen erhalte ich eine tiefe, d.i. INTENSIVE GRÖSSE*, durch die Coordination der Merckmahlen Notarum aber erlanget man eine* EXTENSIVE GRÖSSE*, d. i. nach dem Grund in ausgearbeitete und weitläuftige Erkenntniß. Die Reihe der einander Coordinirten Merckmahlen ist gleichsahm eine Linie ohne Grentzen, die unendlich ist.... Die* EXTENSIVE GRÖSS*e einer Erkenntniß ist gleichsam das volumen, die* INTENSIVE GRÖSSE *aber ist die Dichtigkeit derselben. In gewißen Fällen ist die* EXTENSIVE GRÖSSE *der Erkenntniß nützlicher, in anderen aber die intensive. In der Moral z. E. ist die* INTENSIVE GRÖSSE *einer Erkenntniß nöthiger, in der Physica aber, und Medicin ist sehr öfters die* EXTENSIVE GRÖSSE *der Erkenntniß unentbehrlicher.*—Immanuel Kant. *Gesammelte Schriften*. Ed. Deutschen Akademie der Wissenschaften zu Berlin, Berlin, Walter de Gruyter, vol. XXIV (*Vorlesungen*). 1966, pp. 108–110 (the emphasis was added by the quoter).

28 See above, Note 87; on pages. 46–47.

29 That is to say, *as well as numerical magnitudes externally conditioned (*qua *thought or* quantum *purely in its intellectual determinacy) by the positing of a vinculum between them and the intuition of extension as a* quantum *purposely (therefore, specifically) limited.*

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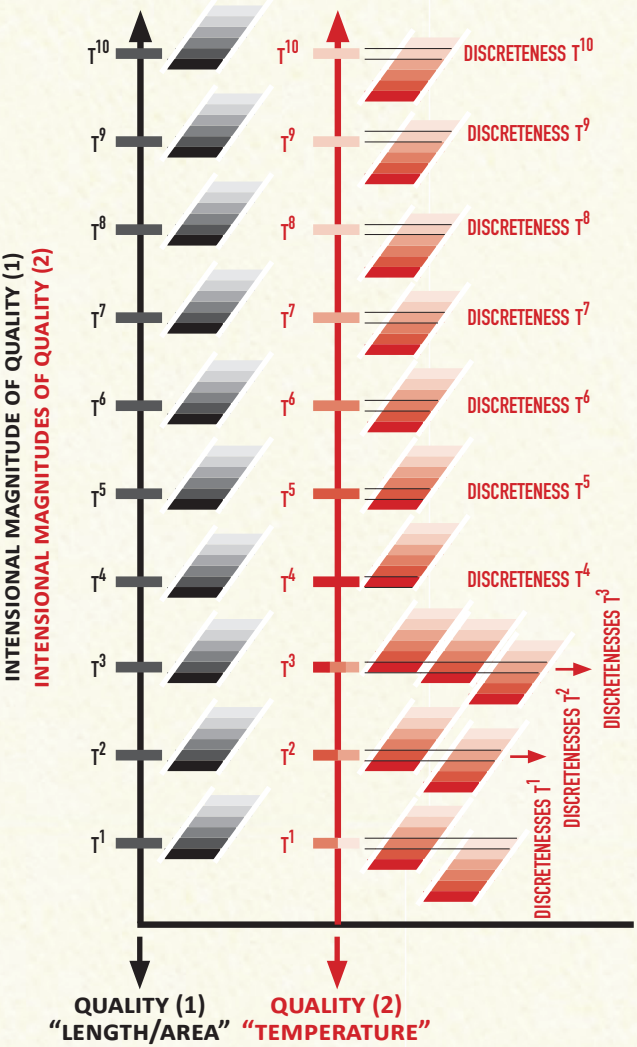


DETERMINED BY GIVENESS IN TIME
OF THE INTENSIONAL MAGNITUDES OF QUALITIES (1) AND (2)

FIGURE 1.1

EXTENSIONAL MAGNITUDE OF QUALITY (1)
EXTENSIONAL MAGNITUDES OF QUALITY (2)

PERCEPTION IN SPACE-TIME
OF THE EXTENSIONAL MAGNITUDES OF QUALITIES (1) AND (2)

FIGURE 1.2.

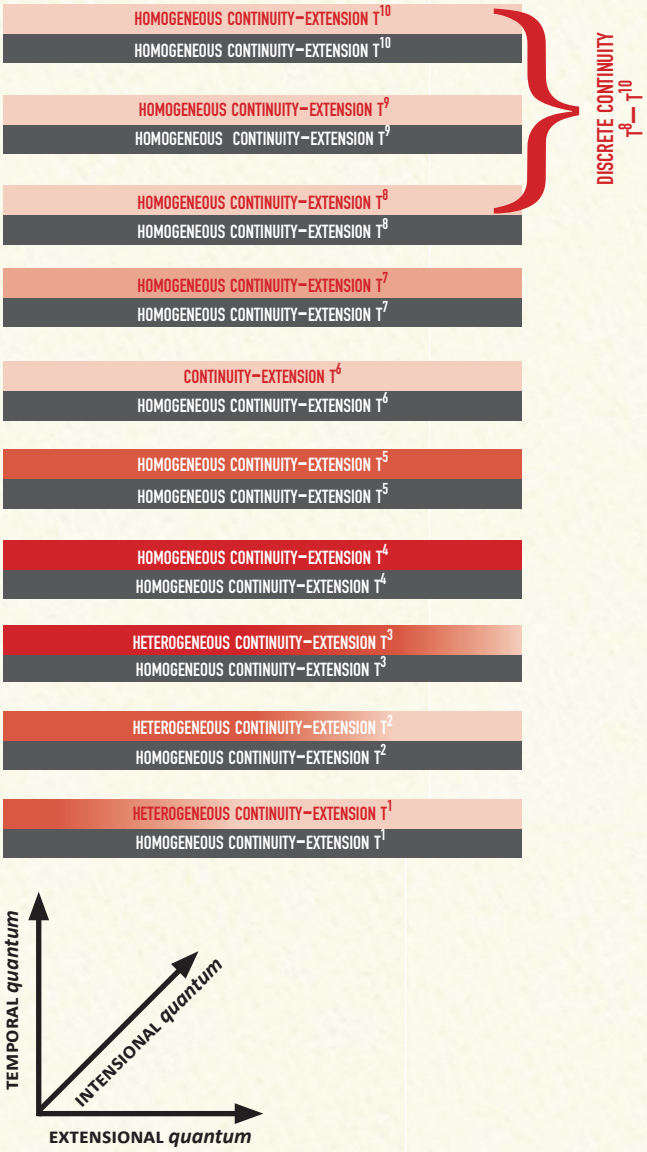
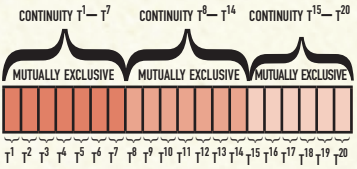

HOMOGENEOUS CONTINUITY–EXTENSION T10
HOMOGENEOUS CONTINUITY–EXTENSION T10
HOMOGENEOUS CONTINUITY–EXTENSION T9
HOMOGENEOUS CONTINUITY–EXTENSION T9
HOMOGENEOUS CONTINUITY–EXTENSION T8
HOMOGENEOUS CONTINUITY–EXTENSION T8
DISCRETE CONTINUITY
T8— T10
HOMOGENEOUS CONTINUITY–EXTENSION T7
HOMOGENEOUS CONTINUITY–EXTENSION T7
CONTINUITY–EXTENSION T6
HOMOGENEOUS CONTINUITY–EXTENSION T6
HOMOGENEOUS CONTINUITY–EXTENSION T5
HOMOGENEOUS CONTINUITY–EXTENSION T5
HOMOGENEOUS CONTINUITY–EXTENSION T4
HOMOGENEOUS CONTINUITY–EXTENSION T4
HETEROGENEOUS CONTINUITY–EXTENSION T3
HOMOGENEOUS CONTINUITY–EXTENSION T3
HETEROGENEOUS CONTINUITY–EXTENSION T2
HOMOGENEOUS CONTINUITY–EXTENSION T2
HETEROGENEOUS CONTINUITY–EXTENSION T1
HOMOGENEOUS CONTINUITY–EXTENSION T1
TEMPORAL quantum
INTENSIONAL quantum
EXTENSIONAL quantum

FIGURE 2.1

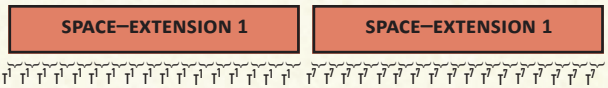
(A) TEMPORAL
DISCRETE CONTINUITY



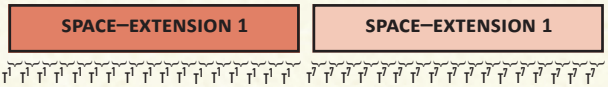
(B) TEMPORAL DISCRETENESS

GIVENESS IN TIME
OF THE INTENSIONAL MAGNITUDES OF QUALITY (2)

(A) SPATIAL HOMOGENEOUS CONTINUITY–EXTENSION



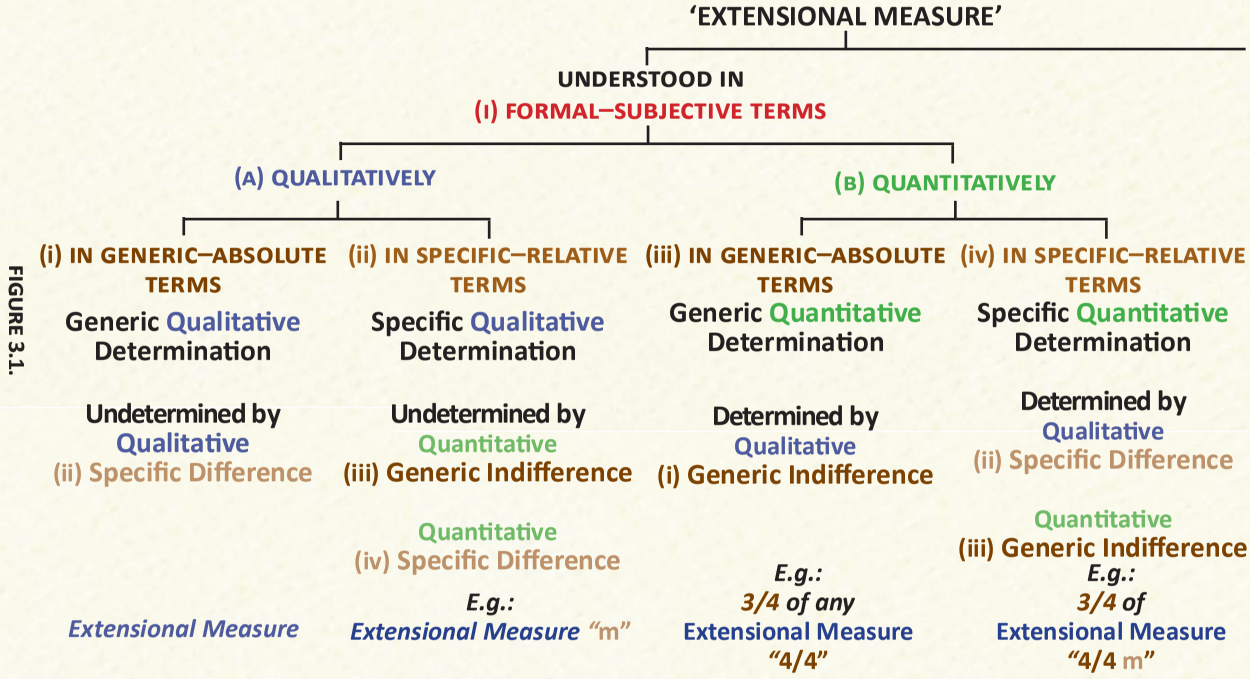
(B) SPATIAL HOMOGENEOUS CONTINUITY–EXTENSION



PERCEPTION IN SPACE-TIME
OF THE INTENSIONAL MAGNITUDES OF QUALITY (2)

FIGURE 2.2.

FIGURE 3.1.


'EXTENSIONAL MEASURE'
UNDERSTOOD IN
(I) FORMAL–SUBJECTIVE TERMS
(A) QUALITATIVELY
(B) QUANTITATIVELY
(i) IN GENERIC–ABSOLUTE TERMS
Generic Qualitative Determination
Undetermined by
Qualitative
(ii) Specific Difference
Extensional Measure
(ii) IN SPECIFIC–RELATIVE TERMS
Specific Qualitative Determination
Undetermined by
Quantitative
(iii) Generic Indifference
Quantitative
(iv) Specific Difference
E.g.:
Extensional Measure "m"
(iii) IN GENERIC–ABSOLUTE TERMS
Generic Quantitative Determination
Determined by
Qualitative
(i) Generic Indifference
E.g.:
3/4 of any
Extensional Measure
"4/4"
(iv) IN SPECIFIC–RELATIVE TERMS
Specific Quantitative Determination
Determined by
Qualitative
(ii) Specific Difference
Quantitative
(iii) Generic Indifference
E.g.:
3/4 of
Extensional Measure
"4/4 m"

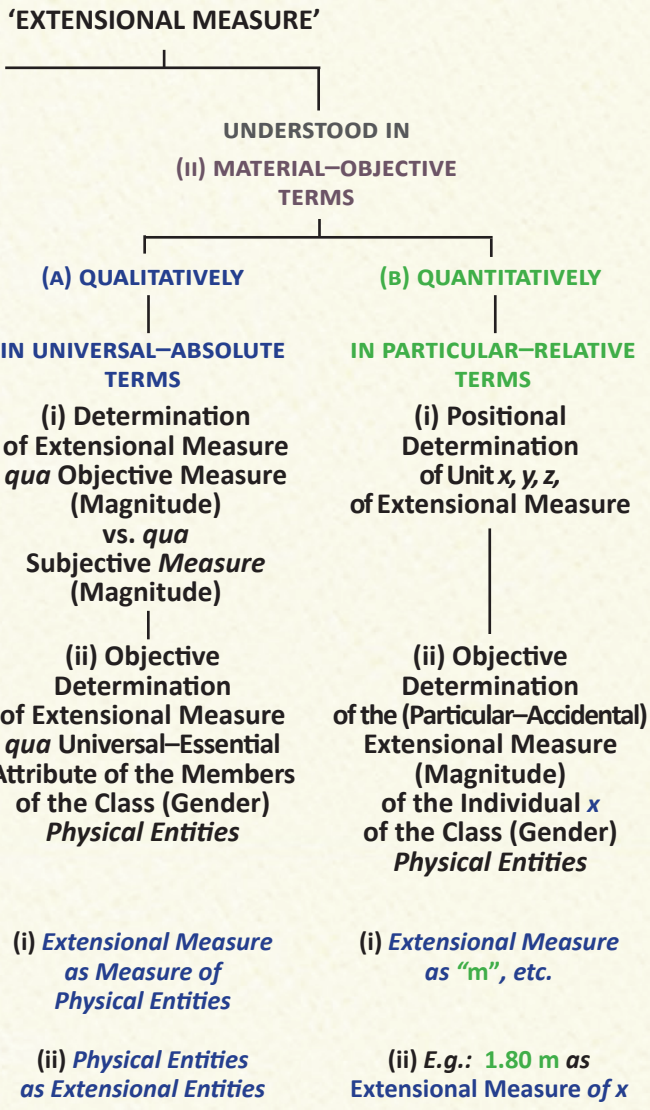

'EXTENSIONAL MEASURE'
UNDERSTOOD IN
(II) MATERIAL–OBJECTIVE TERMS
(A) QUALITATIVELY
(B) QUANTITATIVELY
IN UNIVERSAL–ABSOLUTE TERMS
IN PARTICULAR–RELATIVE TERMS
(i) Determination of Extensional Measure qua Objective Measure (Magnitude) vs. qua Subjective Measure (Magnitude)
(ii) Objective Determination of Extensional Measure qua Universal–Essential Attribute of the Members of the Class (Gender) Physical Entities
(i) Positional Determination of Unit x, y, z, of Extensional Measure
(ii) Objective Determination of the (Particular–Accidental) Extensional Measure (Magnitude) of the Individual x of the Class (Gender) Physical Entities
(i) Extensional Measure as Measure of Physical Entities
(ii) Physical Entities as Extensional Entities
(i) Extensional Measure as "m", etc.
(ii) E.g.: 1.80 m as Extensional Measure of x

FIGURE 3.2.

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