COMPUTABLE GENERAL EQUILIBRIUM MODELS:
Theory and Applications

Mário Fortuna Sameer Rege

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Centro de Estudos de Economia Aplicada do Atlântico - CEEAplA

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Foreword

The book is divided into two parts. The first part deals with the theory of general equilibrium and microeconomics necessary for any student to appreciate the derivation of the equations used in the models. It gives a brief overview of static and dynamic general equilibrium models with an objective of juxtaposing both to enable the novice reader to quickly grasp the benefits of different models. It also gives working programs in GAMS and R to play around with the parameters and appreciate the interplay between the parameters and model results and the limitations. We emphatically state that this is but a very very elementary introduction and meant only as a pre-cursor to the world of economic modelling.

The second part deals with the main objective of the book, the dynamic general equilibrium model for the Azores and the various policy simulations from the model. Initially it explains in detail the model equations and the Social Accounting Matrix and then outlines the simulation results obtained from the model.

Ali Bayar, Brussels, July 2010

Acknowledgment

Finalization of this book was only possible with many inputs that will not be visible in its contents but should be acknowledged for their importance.

At the forefront we thank the Regional Government of the Azores through its regional directorate for science and technology (Direcção Regional da Ciência e Tecnologia), responsible for promoting science and technology in the Azores. Their financing of the printing tasks was fundamental.

The content and context of the book has its foundations in the initial research project for the development of a model that would be capable of simulating the impacts of public policies. This project was promoted through a bilateral programme of the United States of America and Portugal associated with the use of military facilities in the Azores. The Regional Government of the Azores together with the Government of the United States of America, through its Department of Agriculture and the Luso-American Development Foundation, are the three entities that financed this project. We are truly grateful for their funding.

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Ali Bayar and his ECOMOD group was also a centrepiece in the development of the model and gave all the support that was necessary under all circumstances.

We are sincerely grateful to the support of Susana Sardinha who single handedly managed the administrative tasks of CEEAplA, and whose tenacity and patience has been crucial in overcoming most of the hurdles faced during application for funds, presenting reports and the publication of this book.

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Introduction

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Contents

1.1. Introduction

This chapter outlines the structure of the book. It traces the origins, development and growth of applications of Computable General Equilibrium (CGE) models and ties it to the application for the Azores. The literature in the area straddles theory, algorithms for finding the equilibrium price vector, different policy issues and different countries or regions.

1.2. Computable General Equilibrium Models

The origin of CGE models can be traced to the work of [Johansen (1974)] for Norway in the early 70s. [Dervis *et al.* (1982)] at the World Bank were the pioneers to apply this methodology to country specific issues with Korea as a prime example. The edifice has its foundations firmly based on a base data set of the economy and its various agents (firms, household(s), government(s), trade partner(s)) called as a Social Accounting Matrix or SAM.

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1.2.1. The advent of CGE models for policy analysis

The limited applicability of partial equilibrium models with a *ceteris paribus* [a] assumption was woefully inadequate for the economy wide issues like changes in tax policy regimes or trade quotas and tariffs, poverty eradication programmes, energy taxes to name a few. In response to such a criticism economy wide models were built with a firm footing in macro economic identities, with an objective of finding an equilibrium price vector and the corresponding set of equilibrium output. These models could be briefly described in a complementarity format as depicted in table 1.1. The complementarity problem (CP) is defined as find $z \in \mathbb{R}^n$ that solves $F(z) \geq 0$, $z \geq 0$ and $z^T F(z) = 0$, where z is a column vector of prices [with z^T as its transpose] and $F(z)$ is a vector of excess demand (demand minus supply) at price z. Equilibrium values are denoted with $*$ superscript. The model builder is concerned with finding the equilibrium price vector p^* in the benchmark and counter factual cases when the model responds to some policy shock.

Table 1.1.: Linear Complementarity Formulation of General Equilibrium

No activity earns a positive profit	$-A^T p^* \geq 0$
No commodity is in excess demand	$b + Ay^* - d(p*) \geq 0$
No prices or activity levels are negative	$p^* \geq O$, $y^* \geq O$.
An activity earning a deficit is not run, and an operated activity runs at zero profits	$(A^T p^*)^T y^* = 0$
A commodity in excess supply has a zero price, and a positive price implies that supplies equal demands	$p^{*T}(b + Ay^* - d(p^*)) = 0$
vector of endowments	$b = b_i$
vector of prices	$p = p_i$
net market demand functions	$d(p) = d_i(p)$
vector of activity levels	$y = y_j$
technology matrix of input-output coefficients consistent with unit production	$A = A(p)$

Source: [Mathiesen (1985)]

1.2.2. The policy issues that are addressable through CGE models

CGE models being real sector models do not model money or assets. The complexity of the asset price process (often modelled as a Brownian motion which is continuous everywhere but differential nowhere) and the time horizon (often on a daily basis) makes the modelling of assets in real computable general equilibrium models highly erroneous. CGE models often work with an input-output table that is a compilation of annual flows of goods and services and is a snapshot of the aggregate technology used to produce commodities.

The issues plaguing government and non-government agencies often deal with public finance, trade, environment, poverty alleviation to name a few. All these issues are dynamic in nature and have implications across time. Large scale CGE

[a] latin for all other things being held constant

models with multi sector are mainly used to study detailed impacts of policy issues mentioned above in a myopic setting. Other models are dynamic in nature and model the investment-savings behaviour as a state variable obtained from a dynamic optimization of discounted separable lifetime utility function. This implies a very strong assumption on the ability of individuals to forecast perfectly the prices in the distant future.

1.2.3. Designing specific CGE models for specific problems

1.2.3.1. *Data availability and functional forms*

Just like CGE modellers use Leontief production function to model technology [$a = \min(x, y)$], often the data availability acts as a constraint on the modeller's ability to address various policy issues [model = min(data,functional forms)].

In the absence of any input-output table the modellers use Cobb-Douglas or CES forms for the production function. However when an input-output table is available, which nowadays is available for most countries, one uses the Cobb-Douglas/CES functional forms for value added which is fixed proportion of output in case one decides to use a Leontief production function. For modelling energy issues wherein the final output electricity is fungible between different technologies of generation, a KLEM production function is normally used. The functional forms can be Cobb-Douglas or CES with CES being more common on account of a constant but non unit elasticity of substitution.

1.2.3.2. *Trade Policy*

Trade is an integral part of all economies and availability of import data determines the functional form used to model imports. In case of a single country with an export and import vector in final demand and a vector of imports in inputs, one models trade with an export supply and import demand function. In real life at an aggregate level of data availability as exhibited in input-output tables it is difficult to reconcile the import and domestic production of the same commodity especially when traditional trade theory propounds the fact of specialisation and factor price equalisation. In face of the facts modellers circumvent the theory by resorting to the Armington function assumption that allows for imports and domestic production to coexist. This approach was first proposed by [Armington (1969)]. To incorporate mangoes (commodity i) produced in Brazil, Argentina, Egypt, India etc. (country j) for consumption in Portugal, Spain, Belgium etc. (destination country k) as available in national statistics, one uses the Armington function. It is a 2 level function with the top level as an aggregate demand for a homogeneous commodity such as mango and the second tier distinguishing between the different types of mangoes from place of origin. Trade liberalisation or trade sanctions that expand or curtail the choice set of commodities from different sources cannot be modelled via the Armington function as it assumes a pre-defined availability of varieties. This criticism is however extendable to any argument put forth in economics because there is nothing in the theory that endogenously determines the expansion or contraction of the choice set. Innovation is not modelled endogenously and hence there is no way of knowing ex-ante whether a particular product would ever be invented and would find its way into successful innovations of mankind like the internet. External deficit is a cause of concern especially when countries are unable to service external debt. One option is to devalue the currency in the hope that it will stem imports and boost exports and eventually reduce the trade gap. CGE models limit themselves to a real exchange rate, which is defined as the price index of tradeables to non-tradeables, which can be manipulated to keep the benchmark trade deficit constant in foreign currency terms. This has an effect of controlling the extent of foreign transfers and thus leading to a more appropriate evaluation of the welfare effects of any policy.

1.2.3.3. *Environment Policy*

Increasing dependence on fossil fuel and a possible link to global warming and climate change and the need to have more sustainable technologies has brought carbon emissions into center stage of policy debate. Whether it translates into policy action or not is a function of the political choices that are available at the disposal of government. However there is an unequivocal demand for models that will be able to tabulate the welfare implications of taxes on use of fossil fuels. This brings modellers to model carbon dioxide (CO_2) and the value of fossil fuel consumption is converted using the existing price to obtain the average physical consumption in terms of barrels of fuel or crude used. This is then used to obtain the heat content and thermodynamic relationships that are immutable as per nature's laws and insusceptible to political interference or religious beliefs give the amount of carbon emitted per ton of fossil fuel used. [b] Carbon, molecular weight 12 and Oxygen, molecular weight 16 implies Carbon Dioxide [CO_2] has weight 44 ($12 + 16 \times 2$). Thus 12 tonnes of carbon emit 44 tons of carbon dioxide in the air. The modellers then may use the armington function (bituminous, anthracite, lignite coal from region A, B etc, crude oil from Saudi Arabia, Kuwait, Iraq, Nigeria, Iran (currently under sanctions), natural gas, hydro power, nuclear(controlled technology),

[b] To put very crudely the each element is made up of atoms and each atom is made up of neutrons (no charge), protons (+ve charge) and electrons (-ve charge). The atomic number is the number of protons Z found in the nucleus of the atom that is made of neutrons and protons. The neutron number N is the number of neutrons in the nucleus. Atomic mass number $A = Z + N$. Elements having same Z but different N are called isotopes. Elements exist as a mixture of isotopes and the average weight determines the atomic weight. The atomic weights of elements are defined relative to the most abundant isotope of Carbon, which is arbitrarily assigned a number 12. One mole of any substance is that number of mass units [kg, pounds, grams, tons etc.] equal to the molecular weight of the substance. One gram mole of an element equals 6.023×10^{23} atoms irrespective of the element (Avogadro's Number). What changes is the weight of that mole of atoms and thus 1 gram mole of carbon weighs 12 grams and 1 gram mole of oxygen weighs 16 grams and 1 gram mole of uranium 238 weighs 238 grams. For further details refer [Fenn (2003)]

wind etc.) to produce a homogeneous commodity called electricity that is used in intermediate and final consumption.

1.2.3.4. *Tax Policy*

Taxes are the backbone of any economy and primarily responsible for reducing income inequality. The sustainability of a welfare state, health care, education, infrastructure all largely dependent on the ability of the state to raise taxes to fund them. Leakages from the system that lead to transfers into private hands instead of the state coffers greatly reduce the ability of the state to implement programmes that enhance welfare of its citizens. Investment in infrastructure without a commensurate levy to recover costs can lead to bankruptcy, especially if the debt is foreign or in case of domestic debt, lead to inflation. CGE modellers have typically analysed the welfare impacts of changes in tax regimes like implementing a Value Added Tax (VAT) or changes in tariffs or implementing quotas etc. Since the ramifications of fiscal policy are dynamic, models that incorporate explicit dynamics have been built to address some of the issues. To model inter generational issues, typically one uses Overlapping Generations (OLG) models.

1.2.4. Approaching policy issues in a small island economy through CGE models

Small island economies, especially separated from their parent countries or independent countries with huge distances from any large body mass with much bigger market sizes are always dependent on some sort of assistance. The assistance may be in form of daily existence like essentials food, water, energy supplies or like defence needs. The limited ability of these islands to generate a sustainable source of income that can sustain life at the current levels of choice offered makes them highly susceptible to vagaries of economic cycles, besides emigration. Models have been built for ultra-peripheral regions like Azores. The production economy in case it exists is hardly of any noticeable size and trade, mostly imports forms a large part of the economy. As far as employment is concerned, the public sector will be a major source of economic activity, collecting taxes and granting subsidies. The ability of these islands to independently sustain infrastructure development through taxes and tolls may also be limited and would be dependent on external subsidies. Possible export and employment generating sectors like tourism are also affected by business cycles to a much greater extent than other regions of the country. CGE models have to incorporate all these structural characteristics. The models also cannot have a real exchange rate incorporated as these islands do not have an independent trade policy wherein they can impose tariffs or levy subsidies on exports as they are heavily dependent on imports.

1.3. Structure of the book

The book is divided into the following three parts: first, CGE Models: Theory and Literature; second, CGE Modelling of a Small Island Economy: the Azores and finally the third, Policy Simulations.

The first deals briefly with the theory of general equilibrium, both static and dynamic. It also gives stylised data and programs that can be used as a basic building block for more complex models.

The first part has two chapters numbered 2 and 3. Chapter 2 titled *Microeconomics of General Equilibrium Models* covers both, the static simple exchange model and existence of price equilibrium besides solution to a numerical problem to reinforce the concepts, and, the dynamic part that gives an introduction to the finite and infinite horizon Ramsey model along with the 2-period and multi-period overlapping generations model. A short tour of the bare essential of microeconomics necessary for computable general equilibrium models concludes.

The third chapter titled *Programs for a Stylised CGE Model and Ramsey Model*, describes in detail a simple data set for a 4 commodity × 4 sector × 3 consumer economy with multiple taxes and a simple demand system. It outlines the calibration procedure and gives a GAMS [Rosenthal (2006)] program to obtain the benchmark and counter factual simulations. The second part of the chapter deals with the tools necessary for solving dynamic models like the finite and infinite horizon Ramsey problems and gives codes in R [R Development Core Team (2009)].

The second part *CGE Modelling of a Small Island Economy: the Azores*, consists of three chapters numbered from 4 to 6.

The fourth chapter titled *The Azores: A Succinct Introduction* gives an overall view about the politics, economics, demographics of the Azores and highlights the key issues that confront the island. The objective is to link the simulations with the policy issues.

The fifth chapter, *Dynamic General Equilibrium Model of the Azorean Economy*, is one of the two main chapters of the book, outlining in detail the description of the model structure replete with model equations and variable descriptions.

The sixth chapter, *The 2001 SAM* is the other main chapter outlining the sources of the data for constructing the benchmark data set for 2001. In addition it tabulates various data used in the model in addition to calibration of the base parameters and their values.

The third part titled *Policy Simulations* comprises of three chapters numbered from 7 to 9 and deals with the results and explanations of the simulation results obtained for policy simulation carried out with the model.

Chapter seven, *Road Construction Project Under Public-Private Partnership* deals with the welfare impacts of the construction of the new road on the island of São Miguel, termed in Portuguese as *Sem Custos para UTilizadores* (SCUT), which means *without costs to users.*

Chapter eight, *Impacts of Closure of a Military Base on a Small Island Open Economy* delves into the economic impacts of the American military air base at Lajes on the island of Terceira. It is an important source of foreign revenue, due to the construction activities that were ongoing at one point in time. However as of today the level of economic activity has been drastically curtailed on account of many operations now being done by the Americans. The chapter evaluates the economic impacts of shutting the air base.

The ninth and final chapter, *Impacts of Tax Cuts on a Small Island Open Economy* analyses the impacts of fiscal changes possible due to the limited autonomy granted to the Azores by the Portuguese government. The Azores is termed as the *Região Autónoma dos Azores* because it can within certain limits alter the taxes and subsidies that are in the national statutes. In one such case the region decided to reduce corporate taxes by 30% and income taxes by 20%. This chapter evaluates the welfare impacts of these cuts as there was no corresponding compensation for shortfall in revenue from the mainland.

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PART 1
CGE Models: Theory and Literature

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2

Microeconomics of General Equilibrium Models

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Contents

2.1. Introduction

Partial equilibrium analysis deals with the impacts of price or demand change on a single market assuming everything else remains constant or *ceteris paribus*. In real life there are many markets functioning simultaneously in time and across time. Markets at a given point in time are called spot markets and across time (inter temporal) are called futures markets.

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This chapter gives a succinct and elementary introduction to microeconomics from the perspective of enabling the reader to appreciate general equilibrium modelling. It also lays the ground for building a simple computable general equilibrium models. Economic issues straddle the entire gamut of questions relating to birth and death rates, pensions, employment and unemployment, immigration, trade, environment, taxes, subsidies just to name a few. All the issues mentioned have implications across time and the effects of policy decisions are often felt with a lag. This implies that any model should have time as an indispensable component. However the modeller and the reader or user of the models also is faced with the cost and tractability of the models. Any model that professes to address all issues cannot do it satisfactorily and hence a plethora of models, each addressing the specific issue at hand. Models analyse a vast range of issues and methodologies from large scale static models of countries to analyse specific sector oriented policies to dynamic overlapping generations models with endogenous fertility that attempt to model the cost of children and the implications for pensions.

The chapter can be broadly split into two parts. The first deals with the static multi commodity equilibrium problem with some theory and illustrative examples. The second deals with the different types of dynamic models. A caveat for the reader that this is just a brief introduction and in no way can substitute a detailed review of literature.

2.2. Brief Theoretical Background

One seeks the answer to the following questions. Given an economy with $i = 1, 2, ..., \mathbb{N}$ individuals and $k = 1, 2, ..., \mathbb{K}$ commodities, with each individual having endowments of some or all commodities,

(1) Is there an equilibrium where supply and demand for each of the k commodities is equal? If yes, is it unique or there are many such equilibria?
(2) Is there a unique price vector at which the markets clear (supply equals demand) and how does one find it?
(3) Can this allocation of commodities after an equilibrium be improved upon or does the equilibrium imply that it is the best possible outcome? What is meant by best possible outcome and how is it defined?

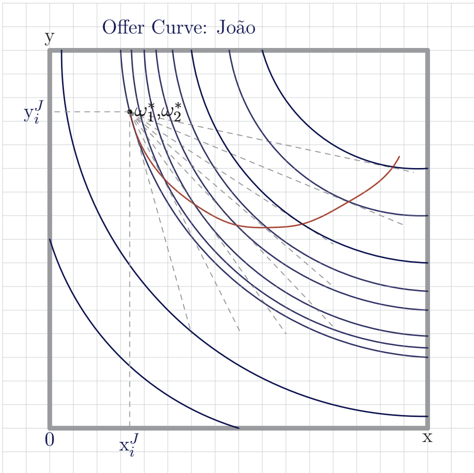
Each individual i has a utility function $\mathbb{U}_i(x_1, x_2, ..., x_{\mathbb{K}})$ over commodities k which $\in \mathbb{R}_k^+$ (the positive orthant of the k dimensional euclidean space). The consumers also have endowments e_i^k of some or all of the k commodities. Individuals aim to maximise their utility subject to their budget constraint. Their budget or income arises from the endowments that they hope to sell at a price that will be determined by the market. One implicitly assumes that there is a demand for all commodities or endowments and no commodity is in excess supply. There is no production in the system.

The amount of good k available in the market is sum over all individuals who own good k, given by $\sum_{i=1}^{\mathbb{N}} e_i^k = e^k$. The demand for k cannot exceed the available supply and individuals indulge in trade such that they are at least as better off after trade as before trade.

2.2.1. Offer Curve

The initial endowments of the individuals are known and given. What is not known is the price that the market is going to pay for them. The offer curve is the curve that shows the amount of endowment that an individual is willing to sell at a specified price. So it is the locus of the price-demand pair. A negative demand implies supply.

Figure 2.1 shows the derivation of a hypothetical offer-curve of an individual called João. Assume that there are 2 commodities x and y. João owns (ω_x, ω_y) of x and y respectively. Also João has a Cobb-Douglas utility function given by $\mathbb{U}_J = x^{0.5}y^{0.5}$. If prices (to be determined) are given by p_x and p_y for x and y respectively then João's income is $\mathbb{I}_J = p_x\omega_x + p_y\omega_y$ and demand is given by $x = \frac{0.5\mathbb{I}}{p_x}$ and $y = \frac{0.5\mathbb{I}}{p_y}$. João's utility will be maximum where the budget line is tangent to the indifference curve and the slope of the budget line is given by the ratio of the relative prices $-\frac{p_x}{p_y}$. As the ratio of the prices varies, the demand will vary and we obtain the locus of the price-demand pair. All budget constraints irrespective of the prices will pass through (ω_x, ω_y) as that is the initial endowment. João's utility improves from the origin in the north-east direction. The curve that joins the points of tangency of the budget constraint with the indifference curve is the offer-curve. This curve shows what João is willing to offer given the prices prevailing in the market. One assumes that João does not dictate prices.



$(\omega_x, \omega_y) = (10, 10)$, $\mathbb{U} = x^{0.5}y^{0.5}$
$\mathbb{I} = p_x\omega_x + p_y\omega_y$
$\Rightarrow x = \frac{0.5\mathbb{I}}{p_x}$, $y = \frac{0.5\mathbb{I}}{p_y}$
Fix $p_y = 5$ and vary p_x from 1,2,..,10

p_x	p_y	$\mathbb{I}$	x	y
1	5	60	30.00	6
2	5	70	17.50	7
3	5	80	13.33	8
4	5	90	11.25	9
5	5	100	10.00	10
6	5	110	9.17	11
7	5	120	8.57	12
8	5	130	8.13	13
9	5	140	7.77	14
10	5	150	7.50	15

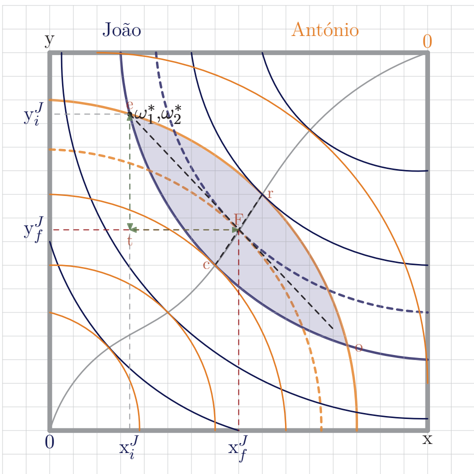
Fig. 2.1.: Offer Curve

2.2.2. Edgeworth Box

To explain the search for an equilibrium by graphical means, one often resorts to the Edgeworth Box. This box deals with 2 individuals and 2 commodities. Each individual has a unique utility function and endowments to begin with. Both are trying to improve on their initial endowments through trade with the amount of exchange determined by prices obtained via a market mechanism. This can be explained by the presence of a fictitious auctioneer who will call out prices and the individuals respond to them via net trades (supply-demand) by maximising their utilities at these prices.

Figure 2.2 shows the Edgeworth Box. Consider 2 individuals João and António. João's indifference curves are depicted in the normal way increasing from the origin in the north-east direction. António's however are inverted with the origin at the north-east corner of the box and increase in the south-west direction toward the origin. The dimensions of the box are calculated by summing the original endowments of João and António for each of the commodities x and y. If ω_J^x and ω_J^y denote João's endowments of x and ω_A^x and ω_A^y and denote those of António, then the x-dimension of the box is $e^x = \omega_J^x + \omega_A^x$ and the y-dimension is $e^y = \omega_J^y + \omega_A^y$. Note that the shape of the box need not be square but is depicted for ease of illustration. The x-axis denotes the supply of good x and the y-axis denotes the supply of good y. The origin for João is $(0, 0)$ and that for António is (e^x, e^y).

The initial endowment point is shown as (ω_1^*, ω_2^*) and all budget constraints will pass through this point c. The figure also shows a shaded area $c-o-r-e$. This area is termed as the core wherein individuals can trade an improve their utility further. The dashed line $e-F$ is the relative price vector that is tangent to the indifference curves of both João and António which are also tangential to each other. Thus the point F is the point where one finds the equilibrium as it is not possible for either João or António to improve their utilities without decreasing the utility of the other. The point to note is that the outcome is a function of the relative prices of x and y and should anything happen like an external shock that would affect the relative prices, the point F would obviously move in either direction. In the final assessment João consumes at point (x_f^J, y_f^J) from the original (ω_1^*, ω_2^*) implying João traded $\omega_2^* - y_f^J$ for $\omega_1^* - x_f^J$. In António's case it is the reverse with his trades being $\omega_1^* - x_f^J$ for $\omega_2^* - y_f^J$.



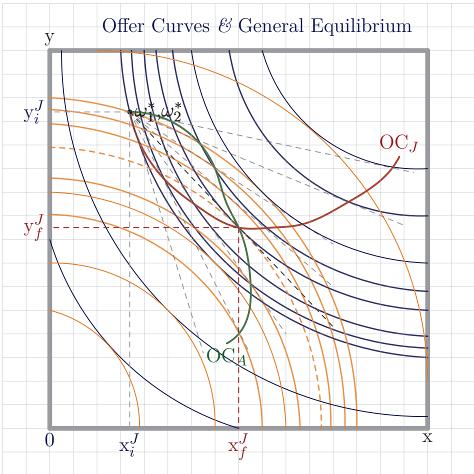
Two individual João and António.
João owns (ω_J^X, ω_J^Y)
and António owns (ω_A^X, ω_A^Y)
$x = \omega_J^X + \omega_A^X$
$y = \omega_J^Y + \omega_A^Y$
João: $\max_{x,y} \mathbb{U}_J(x, y)$
$p_X X_J + p_Y Y_J = p_X \omega_J^X + p_Y \omega_J^Y$
António: $\max_{x,y} \mathbb{U}_A(x, y)$
$p_X X_A + p_Y Y_A = p_X \omega_A^X + p_Y \omega_A^Y$
Budget Constraint
$Y = -\frac{p_X}{p_Y} X + \frac{1}{p_Y} I$

Fig. 2.2.: Edgeworth Box

2.2.3. Equilibrium

The problem is to find a price vector $\mathbf{p}^k$ $k = 1, 2$ such that each consumer maximises utility $\mathbb{U}(x_i)$ $s.t.$ $p_k x^k \le p_k e^k$. We need to find out k prices, one for each commodity. Later we will find that it is not possible to find k separate (and perhaps some prices being equal to each other), but only $k-1$ separate prices expressed as a ratio of the kth price. Thus we can **only** determine **relative prices**, where one is free to choose any of the k commodities as the base.

Figure 2.3 combines the Offer-curve and Edgeworth box principles to calculate the equilibrium price vector. Using the offer-curve approach and starting from the initial endowment vector (ω_1^*, ω_2^*) (this will be a k-dimensional box for k commodities) and obtain the offer-curves for João and António at each relative price vector. The offer curves intersect each other at a unique point and this point is the equilibrium point where no one has an incentive to trade and both have maximised their utilities. As long as the individual offer-curves do not intersect there is an incentive to trade and one is looking for a change in relative prices to maximise utility.



Two commodities x and y
Two individuals António and João
Initial Endowment João: (ω_1^*, ω_2^*)
Offer curve of João OC_J
Offer curve of António OC_A
João gives up $y_i^J - y_f^J$ to get $x_f^J - x_i^J$

Fig. 2.3.: Offer Curves & General Equilibrium

General Equilibrium is called a Walrasian Equilibrium for a given pure exchange economy if $\exists$ a price vector $\mathbf{p}$ and consumption bundles x_j^k for commodity k of consumer j such that

(1) At price-vector $\mathbf{p}$ each consumer maximises utility $\mathbb{U}$
(2) markets clear $\sum_{j=1}^{n} x_j^k \leq \sum_{j=1}^{n} e_j^k$ for each commodity k

The price vector passes through the original endowments (e^1, e^2) and is tangent to the indifference curves of both the consumers such that **excess demand=demand-supply** for each good is **0**.

2.3. Efficiency in General Equilibrium

Given a set of initial endowments $\boldsymbol{e}$ and a walrasian equilibrium $(\boldsymbol{x^*}, \boldsymbol{p^*})$, where $\boldsymbol{e}, \boldsymbol{x^*}, \boldsymbol{p^*}$ are endowments, final allocation and price vectors for commodities $k = 1, 2, .., K$ is there another set of allocation $\boldsymbol{x_a^*}, \boldsymbol{p_a^*}$ such that all consumers are at least as better off as $\boldsymbol{x^*}, \boldsymbol{p^*}$ and one consumer is strictly better off? If yes then the walrasian equilibrium process can be improved upon and the market is not the right mechanism to optimally allocate the initial bundle of resources.

2.3.1. Theorems of Welfare Economics

First Theorem of Welfare Economics A walrasian equilibrium always yields a pareto efficient allocation.

In other words the market forces will produce a final allocation of goods such that nobody can be made better off without making anyone worse off.

Proof:

(1) Let (x_a^*, p_a^*) be an alternate allocation that is pareto superior to the walrasian equilibrium (x^*, p^*).

(2) $1 \Rightarrow x_{a_i}^k \geq x_i^k \ \forall k$ and $x_{a_i}^k > x_i^k$ for at least one k

(3) $\sum_{i=1}^n x_{a_i}^k \geq \sum_{i=1}^n x_i^k$ and $\sum_{i=1}^n x_{a_i}^k > \sum_{i=1}^n x_i^k$ for at least one k.

(4) $\because$ (x^*, p^*) is a walrasian equilibrium, summing over all i consumers $\Rightarrow$ $\sum_{i=1}^n x_i^k = \sum_{i=1}^n e_i^k \ \forall \ k$

(5) 3 and 4 $\Rightarrow \sum_{i=1}^n x_{a_i}^k > \sum_{i=1}^n e_i^k$ for at least one k

(6) 5 $\Rightarrow$ demand $>$ supply for at least one commodity k which is not possible. The final allocation has to be such that the demand is $\leq$ supply of the total endowments of any commodity.

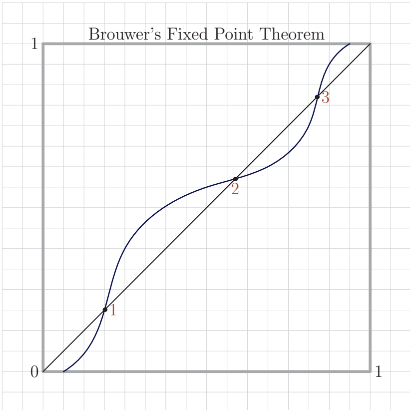
(7) Hence the assumption that (x_a^*, p_a^*) is an alternate allocation that is superior to the walrasian equilibrium (x^*, p^*) is **incorrect**

Second Theorem of Welfare Economics Given the original set of endowments $\boldsymbol{e}$ and walrasian equilibrium $(\boldsymbol{x^*}, \boldsymbol{p^*})$, to achieve a different (more equitable) pareto efficient allocation $(\boldsymbol{x_a^*}, \boldsymbol{p_a^*}) \neq$ the original $(\boldsymbol{x^*}, \boldsymbol{p^*})$ then an initial redistribution of the endowments to $\boldsymbol{e_a}$ is sufficient.

In other words the state can redistribute the original amongst the consumers and then allowing the market forces to operate and the final allocation of resources obtained will be pareto optimal and more equitable.

2.4. Fixed Points and Existence of Equilibrium

Brouwer's Fixed Point Theorem Given a non empty compact convex set X and a continuous function $\phi(X) \mapsto X$, then there exists a point $x^* \in X | \phi(x^*) = x^*$



Consider a function $\phi(x)$ for $x \in [0,1]$
The diagonal is $\phi(x) = x \; \forall x$
The curved function is $\phi(x) \in [0,1]$
Brouwer's Fixed point theorem says that
if $\phi(x) \mapsto x \; \exists \, x^*$ such that $\phi(x^*) = x^*$

Fig. 2.4.: Brouwer's Fixed Point Theorem

If we can prove that our system of equations for finding the equilibrium price vector can be transformed into one that satisfies the conditions of the Brouwer's Fixed point theorem then we can say that we have an equilibrium price vector.

Kakutani's Fixed Point Theorem Given a non empty compact convex set X and a upper hemi-continuous correspondence $\phi(X) \subseteq X$, then there exists a fixed point $x^* \in X | \phi(x^*) = x^*$.

The budget set is not a one to one mapping but a correspondence. So one income maps to many commodities. Kakutani's fixed point theorem is a generalisation of Brouwer's fixed point theorem for functions and extends to correspondences. So if we can transform our correspondence such that it satisfies the requirements of Kakutani's fixed point thereom we have found an equilibrium price vector.

2.4.1. Tatonnement Process

Consider the system of excess demands denoted by $\psi(\mathbf{p})$. In equilibrium $\psi(\mathbf{p}) = \mathbf{0}$ as excess demands will be $\mathbf{0}$ for all commodities. Convert to $\psi(\mathbf{p}) + \mathbf{p} = \mathbf{p} \; \forall \mathbf{p} > 0$ and write $\psi(\mathbf{p}) + \mathbf{p} = \phi(\mathbf{p})$. Thus by converting the set of non linear equations to a fixed point problem we can state that an equilibrium price vector exists by invoking the fixed point theorems.

The process that iterates on the price vector $\mathbf{p}^k$ till the excess demand $(\sum_{i=1}^n x_i^k = \sum_{i=1}^n e_i^k)$ for each of the k commodities equals $\mathbf{0}$. Given a space $\mathbb{R}^k$ of price vectors $\mathbf{p}^k$ such that $\sum_{k=1}^{K} p_k = 1$, consider a function $\phi(\mathbf{p}) \in \mathbb{R}^k \mapsto \mathbb{R}^k$

$$\phi_k(\mathbf{p}) = \frac{p_k + \max[0, X_k(\mathbf{p}) - e_k]}{\sum_{k'=1}^{K}(p_k^{'} + \max[0, X_k^{'}(\mathbf{p}) - e_k^{'}])} \tag{2.1}$$

Brouwer's fixed point theorem says that $\phi(\mathbf{p})$ has a fixed point $\mathbf{p}$. $\therefore$

$$p_k = \frac{p_k + \max[0, X_k(\mathbf{p}) - e_k]}{\sum_{k'=1}^{K}(p_k^{'} + \max[0, X_k^{'}(\mathbf{p}) - e_k^{'}])} \tag{2.2}$$

$\because \sum_{k=1}^{K} p_k^{'} = 1$, we have

$$p_k = \frac{p_k + \max[0, X_k(\mathbf{p}) - e_k]}{1 + \sum_{k'=1}^{K}(\max[0, X_k^{'}(\mathbf{p}) - e_k^{'}])} \tag{2.3}$$

$$p_k + p_k\left\{\sum_{k'=1}^{K}(\max[0, X_k^{'}(\mathbf{p}) - e_k^{'}])\right\} = p_k + \max[0, X_k(\mathbf{p}) - e_k] \tag{2.4}$$

$$p_k\left\{\sum_{k'=1}^{K}\max[0, X_k^{'}(\mathbf{p}) - e_k^{'}]\right\} = \max[0, X_k(\mathbf{p}) - e_k] \tag{2.5}$$

Multiply both sides by $X_k(\mathbf{p}) - e_k$ and summing over k we get

$$\left\{\sum_{k'=1}^{K} p_k(X_k(\mathbf{p}) - e_k)\right\}\left\{\sum_{k'=1}^{K}\max[0, X_k^{'}(\mathbf{p}) - e_k^{'}]\right\} = \sum_{k'=1}^{K}(X_k(\mathbf{p}) - e_k)\max[0, X_k(\mathbf{p}) - e_k] \tag{2.6}$$

By walras' law we have $\left\{\sum_{k'=1}^{K} p_k(X_k(\mathbf{p}) - e_k)\right\} = 0$, the aggregate value of excess demand $= \mathbf{0}$. $\because$ the LHS $= 0$, the RHS has to equal 0. The only possible way is if $X_k(\mathbf{p}) - e_k = 0$ for each k. So we have found an equilibrium price vector $\mathbf{p}^*$ such that demand equals supply for each k.

2.5. Illustrative Problem

Problem:
2 consumers A and B. 2 commodities f and c. The economy is characterised by utility functions of the 2 consumers and their endowments. There is no production. $\mathbb{U}_A = 0.4ln(f) + 0.6ln(c)$ and $\mathbb{U}_B = 0.5ln(f) + 0.5ln(c)$. $f_A = 10$, $c_A = 10$, $f_B = 10$, $c_B = 5$. Find the

(1) equilibrium price vector for f and c

(2) the final consumption vector for A and B

Solution:
By the method of marginal utilities

$$\frac{\mathrm{M}\mathbb{U}_{\mathrm{f}}}{\mathrm{M}\mathbb{U}_{\mathrm{c}}} = \frac{p_f}{p_c}$$

For Consumer A:

$$\begin{aligned}
\frac{\frac{0.4}{f_A}}{\frac{0.6}{c_A}} &= \frac{p_f}{p_c} \\
\frac{2}{3}\frac{c_A}{f_A} &= \frac{p_f}{p_c} \\
c_A &= \frac{3}{2}\frac{p_f}{p_c} f_A \\
p_f f_A + p_c \frac{3}{2}\frac{p_f}{p_c} f_A &= 10(p_f + p_c) \\
f_A &= 4\frac{p_f + p_c}{p_f} \\
c_A &= 6\frac{p_f + p_c}{p_c}
\end{aligned}$$

For Consumer B:

$$\begin{aligned}
\frac{\frac{0.5}{f_B}}{\frac{0.5}{c_B}} &= \frac{p_f}{p_c} \\
\frac{c_B}{f_B} &= \frac{p_f}{p_c} \\
c_B &= \frac{p_f}{p_c} f_B \\
p_f f_B + p_c \frac{p_f}{p_c} f_B &= 10p_f + 5p_c \\
f_B &= \frac{10p_f + 5p_c}{2p_f} \\
c_B &= \frac{10p_f + 5p_c}{2p_c}
\end{aligned}$$

$$\begin{aligned}
f_A + f_B &= 10 + 10 \\
c_A + c_B &= 10 + 5 \\
4\frac{p_f + p_c}{p_f} + \frac{10p_f + 5p_c}{2p_f} &= 20 \\
6\frac{p_f + p_c}{p_c} + \frac{10p_f + 5p_c}{2p_c} &= 15
\end{aligned}$$

Denote $\frac{p_c}{p_f} = k$

$$\begin{aligned}
4 + 4k + 5 + \frac{5}{2}k &= 20 \\
\frac{6}{k} + 6 + \frac{5}{k} + \frac{5}{2} &= 15
\end{aligned}$$

$\therefore k = \frac{22}{13}$. If one notices carefully one equation is redundant and therefore the solution consists of only RELATIVE prices and not ABSOLUTE prices.

(1) $f_A = \frac{140}{13}$ and $f_B = \frac{120}{13}$

(2) $c_A = \frac{210}{22}$ and $c_B = \frac{120}{22}$

2.6. Dynamic General Equilibrium Models

[a]The illustration of the elementary general equilibrium model can be extended into a more detailed model with multiple sectors, trade, single or multiple countries, incorporating environmental taxes to name a few. Production technologies like Leontief, or other production functions like $KLEM$ with a Constant Elasticity of Substitution *CES* are used to model the various response to exogenous shocks such as changes in tariffs, taxes, imposing new taxes for environment etc.

Essentially all these models approximate investment or savings behaviour in a naïve manner with a marginal propensity to save *mps* that is exogenously specified as a fraction of the income and the resulting savings are then added to the existing capital stock after accounting for depreciation and leads to a new capital stock. The capital stock is thus derived for as many years into the future as needed.

[a]this section follows [Heer and Maußner (2005)]

The need to model a more realistic approach to savings and investment creates the need for a dynamic and more forward looking approach to savings. Thus a new set of models that have their basis in optimal control where a terminal period, determines the end of horizon for decision making and the control or decision variables like level of consumption determining the discounted present value of the life time utility. To make the problem mathematically tractable, the utility is separable in time.

The various genre of models range from the initial Ramsey model, both finite and infinite horizon, stochastic Ramsey models, heterogeneous agents and overlapping generation models.

Stochastic Ramsey model deals with the path of capital stock that is no more deterministic but depends on a shock to the output and thus consumption.

Heterogeneous agent model account for heterogeneity amongst agents in terms of their endowments, education, employment, marital status etc. The most elementary models deal with two states of nature with regards to labour market, employment and unemployment. The driving force is a matrix of probabilities transition between the two states.

Overlapping generations models deal with a finite life individuals but with many such individuals living at the same time and thus can tackle a host of issues like demographics, pension reforms to name a few.

2.6.1. Deterministic Finite Horizon Ramsey Model

This model deals with agents having a finite life say $T = 80$ years and thus at the end of life the individual has zero consumption. So the consumption decisions of the individual are based on this terminal period T.

The individual has a time separable utility function $\mathbb{U}(C_0, C_1, .., C_T)$ which is a function of consumption $C_0, C_1, .., C_T$. The objective of the individual is to choose $C_0, C_1, .., C_T$, so that the discounted present value of the utility function is maximised. The link between any two time states t and $t + 1$, is the investment or savings, obtained by the Euler equation.

The model equations and the first order conditions are mentioned below.

$$\max_{(C_0, C_1, .., C_T)} \quad \mathbb{U}(C_0, C_1, .., C_T) \tag{2.7}$$

$$\left.\begin{aligned} K_{t+1} + C_t &= f(K_t) \\ C_t &\geq 0 \\ K_{t+1} &\geq 0 \end{aligned}\right\} t = 0, 1, .., T \tag{2.8}$$

The lagrangian is given by

$$\mathcal{L} = \mathbb{U}(C_0, C_1, .., C_T) + \lambda_t[f(K_t) - (K_{t+1} + C_t)] + \mu_t(C_t - 0) + \gamma_{t+1}(K_{t+1} - 0) \tag{2.9}$$

The first order conditions are

$$\begin{aligned}\frac{\partial \mathbb{U}(C_0, C_1, .., C_T)}{\partial C_t} - \lambda_t + \mu_t &= 0 \\ -\lambda_t + \lambda_{t+1} f^{'}(K_{t+1}) + \gamma_{t+1} &= 0 \\ \lambda_T - \mu_T &= 0 \\ \lambda_t[f(K_t) - (K_{t+1} + C_t)] &= 0 \\ \mu_t C_t &= 0 \\ \gamma_{t+1} K_{t+1} &= 0 \end{aligned} \tag{2.10}$$

We have $C_t > 0 \Rightarrow \mu_t = 0 \therefore$

$$\frac{\partial \mathbb{U}(C_0, C_1, .., C_T)/\partial C_t}{\partial \mathbb{U}(C_0, C_1, .., C_T)/\partial C_{t+1}} = f^{'}(K_{t+1}) \tag{2.11}$$

$$K_{t+1} = f(K_t) - C_t \tag{2.12}$$

2.6.2. Deterministic Infinite Horizon Ramsey Model

This model is similar to the finite horizon Ramsey model in all respects except the terminal time period is set to $T = \infty$. This does not force the capital stock to go down to zero after a period say $T = 80$ as the agent is infinitely lived.

We give the R programs later to solve and obtain the values of capital stock and consumption for both the finite and infinitely lived agents in the deterministic Ramsey models

2.6.3. Overlapping Generations Models

We give a a brief outline of the 2-period OLG model and then follow it with the general 60 period OLG model and give an outline of the solution algorithm.

2.6.3.1. *2-period OLG Model with Production*

Consider an economy with output denoted by Y_t at time t. Only one commodity is produced at time t using factors labour or level of employment L_t and capital K_t, that is inherited from the previous period. Time is denoted by t. Capital is fungible and can be used to produce new capital or for consumption. It also depreciates at a rate δ per unit time ($0 \le \delta \le 1$). The capital stock in time $t+1$ is composed of the investment (output-consumption) and the undepreciated capital $K_t - \delta K_t$. Thus next period capital stock is given by

$$K_{t+1} = (1 - \delta)K_t + Y_t - C_t \tag{2.13}$$

The 2-period overlapping generations model assumes that at time t there exist two generations of individuals. N_{t-1} the number of old and N_t the number of young.

In absence of population growth $N_t = N \; \forall t$ and $N_t = N_{t-1}(1+\eta_t)$, where η_t is the population growth rate at time t. For simplicity we assume zero population growth rate initially.

The young are born without any capital and also do not receive any bequests and therefore we have an egalitarian society in each generation. The old have savings/capital and cannot work. So the young use the capital of the old in exchange for meeting their consumption in the final period of their life. The young save and in turn when they get old at time $t+1$, they do the same as their parents.

Consumer Problem Each consumer has a utility function $\mathbb{U}(c_y, c_o)$, where c_y is consumption when young and c_o is consumption when old. The income is w_y when young and w_o when old. Assume that the young save a_y when young. The budget constraint is $c_y = w_y - a_y$ and $c_o = w_o + (1+r)a_y$, where r is the rate of interest for the period $t-1$ to t. Combining both the budget constraints we have

$$\begin{aligned} c_o &= w_o + (1+r)(w_y - c_y) \\ c_y + \frac{c_o}{(1+r)} &= w_y + \frac{w_o}{(1+r)} \end{aligned} \tag{2.14}$$

The consumer's problem is

$$\begin{aligned} &\max_{(c_y,c_o)} \;\; \mathbb{U}(c_y, c_o) \\ &s.t. \; c_y + \frac{c_o}{(1+r)} = w_y + \frac{w_o}{(1+r)} \end{aligned} \tag{2.15}$$

The lagrangian is given by

$$\mathcal{L} = \mathbb{U}(c_o, c_y) + \lambda\left[w_y + \frac{w_o}{(1+r)} - \left(c_y + \frac{c_o}{(1+r)}\right)\right] \tag{2.16}$$

The first order conditions are

$$\begin{aligned} \frac{\partial \mathbb{U}(c_o, c_y)}{\partial c_y} - \lambda &= 0 \\ \frac{\partial \mathbb{U}(c_o, c_y)}{\partial c_o} - \frac{\lambda}{(1+r)} &= 0 \\ \frac{\partial \mathbb{U}(c_o, c_y)/\partial c_y}{\partial \mathbb{U}(c_o, c_y)/\partial c_o} &= (1+r) \end{aligned} \tag{2.17}$$

Assuming a Cobb-Douglas utility function for simplicity we have $\mathbb{U}(c_y, c_o) = c_y^{\alpha} c_o^{1-\alpha}$. From equation 2.17 we have

$$\begin{aligned}
\frac{\alpha c_y^{\alpha-1} c_o^{1-\alpha}}{(1-\alpha) c_y^{\alpha} c_o^{-\alpha}} &= (1+r) \\
\frac{\alpha}{(1-\alpha)} \frac{c_o}{c_y} &= (1+r) \\
\frac{\alpha c_o}{(1+r)} &= (1-\alpha) c_y \\
c_y &= \alpha \left[\frac{c_o}{(1+r)} + c_y \right] = \alpha \left[\frac{w_o}{(1+r)} + w_y \right] \\
c_o &= (1-\alpha) \big[w_o + (1+r) w_y \big] \\
a_y &= (1-\alpha) w_y - \alpha \frac{w_o}{(1+r)}
\end{aligned} \tag{2.18}$$

Producer Problem The output is produced using factors labour L_t and capital K_t with a Constant Returns to Scale (CRS) technology. This assumption leads to indeterminacy of the scale of output and the output level is determined by the amount of factor inputs. Also the number of firms producing the output do not matter as they can all be merged into one aggregate firm, due to the assumption of perfect competition. The production function is denoted by

$$Y_t = F(K_t, L_t)$$

The CRS assumption implies

$$F(\lambda K_t, \lambda L_t) = \lambda F(K_t, L_t) \ \ \forall \lambda > 0$$

Or in more mathematical terms, the production function is **homogeneous of degree 1** [b]. The producers' problem is to

$$\begin{aligned}
\min_{(L_t, K_t)} \ & \rho_t K_t + w_t L_t \\
s.t. \ & F(L_t, K_t) = Y_t
\end{aligned} \tag{2.19}$$

where ρ_t is the rental rate of capital, which must be paid by those who borrow (young) capital to those who own it (old). The lagrangian is given by

$$\mathcal{L} = \rho_t K_t + w_t L_t + \lambda \big[Y_t - F(L_t, K_t) \big] \tag{2.20}$$

The first order conditions are

$$\begin{aligned}
-\lambda \frac{\partial F(L_t, K_t)}{\partial L_t} + w_t &= 0 \\
-\lambda \frac{\partial F(L_t, K_t)}{\partial K_t} + \rho_t &= 0
\end{aligned} \tag{2.21}$$

[b] A function $F(\mathbf{x})$ is homogeneous of degree k if $F(\lambda \mathbf{x}) = \lambda^k F(\mathbf{x})$

By Euler's theorem we have

$$Y_t = F(L_t, K_t) = \frac{\partial F(L_t, K_t)}{\partial K_t} K_t + \frac{\partial F(L_t, K_t)}{\partial L_t} L_t = \rho_t K_t + w_t L_t \tag{2.22}$$

Output per-Capita The CRS assumption enables us to work with only one variable, the capital to labour ratio $\kappa_t = \frac{K_t}{L_t}$. We convert all variables to per-capita terms by dividing by the amount of labour available L_t

$$\begin{aligned} \kappa_t &= \frac{K_t}{L_t} \\ y_t &= \frac{Y_t}{L_t} = F(\frac{K_t}{L_t}, \frac{L_t}{L_t}) = F(\kappa_t, 1) = f(\kappa_t) \end{aligned}$$

Given an exogenous level of employment or labour L at time t, the total output is

$$Y_t = Lf(\kappa_t)$$

and the profit Π is

$$\Pi = L\big[f(\kappa) - \rho\kappa - w\big]$$

The first order conditions $\Rightarrow$

$$\frac{\partial \Pi}{\partial \kappa} = L\left[\frac{\partial f(\kappa)}{\partial \kappa} - \rho\right] = 0 \Rightarrow \frac{\partial f(\kappa)}{\partial \kappa} = \rho$$

Assuming a Cobb-Douglas production function

$$\begin{aligned} F(K_t, L_t) &= AK^\beta L^{(1-\beta)} \\ F(\frac{K_t}{L_t}, 1) &= A\frac{K^\beta L^{(1-\beta)}}{L} \\ y = f(\kappa) &= A\kappa^\beta \end{aligned} \tag{2.23}$$

Maximising profit or minimising costs gives the first order condition

$$\begin{aligned} \frac{\partial f(\kappa)}{\partial \kappa} &= \beta A \kappa^{\beta-1} \\ \rho &= \beta A \kappa^{\beta-1} \\ \kappa &= \left[\frac{\beta A}{\rho}\right]^{\frac{1}{(1-\beta)}} \end{aligned} \tag{2.24}$$

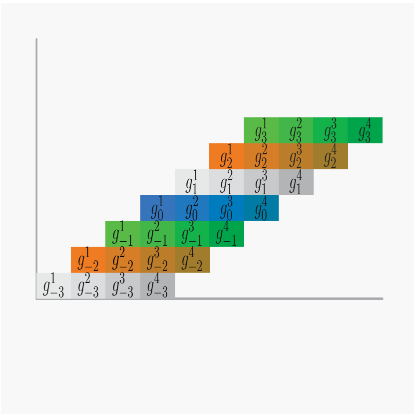
The zero profit condition $\Rightarrow$

$$\begin{aligned} wL + \rho K &= F(K, L) \\ w + \rho\kappa &= f(\kappa) \\ w &= A\kappa^\beta - \beta A \kappa^{\beta-1}\kappa \\ w &= (1-\beta)A\kappa^\beta \end{aligned} \tag{2.25}$$

The other way to obtain w is through the marginal product of labour

$$
\begin{aligned}
w &= \frac{\partial F(K,L)}{\partial L} = L\frac{\partial F(\frac{K}{L},1)}{\partial L} = L\frac{\partial f(\kappa)}{\partial L} = \frac{\partial (LA\kappa^{\beta})}{\partial L} \\
&= A\kappa^{\beta} + LA\beta\kappa^{\beta-1}\frac{\partial \kappa}{\partial L} = A\kappa^{\beta} + LA\beta\kappa^{\beta-1}\frac{-K}{L^2} \\
&= A\kappa^{\beta} - A\beta\kappa^{\beta-1}\frac{K}{L} = A\kappa^{\beta} - A\beta\kappa^{\beta-1}\kappa \quad \because \frac{K}{L} = \kappa \\
w &= A\kappa^{\beta}(1-\beta)
\end{aligned} \tag{2.26}
$$

2.6.3.2. *Multi-period OLG Model*



7 generations, each lives for 4 periods
When generation g_{-3} is in the last period 4, generation g_0 is in period 1
When generation g_0 is in the last period 4, generation g_3 is in period 1
Generation g_3 is connected to g_{-3} through g_0

Fig. 2.5.: 4 period OLG model

2.6.3.3. *Households*

Every year a generation of the same size is born. One can normalise the sum of all generations to one, so each generation is a fraction of the total. Households live for 60 years[c]. They work for $T = 40$ years and then lead a retired life for $TR = 20$ years. Since all generations are of the same size the measure of each generation is 1/60. The following notation is used. The superscript s denotes the age of the generation and the subscript t denotes the time. Thus c_t^s denotes the consumption of generation of age s at time t. During the first 40 years households supply labour n_t^s an enjoy leisure $l_t^s = 1 - n_t^s$. during their retirement the labour supply is zero and all households retire after 40 years. Thus $n_t^s = 0$ for $s > T$. The household

[c]they live for 80 years with the first 20 years being supported by their parents and enter workforce at 21

maximisation problem is as follows

$$\sum_{s=1}^{T+TR} \beta^{s-1} \mathbb{U}(c^s_{s+t-1}, l^s_{s+t-1}) \tag{2.27}$$

where β is the discount factor. The instantaneous utility is a function of both consumption and leisure given by

$$\mathbb{U}(c,l) = \frac{[(c+\psi)l^{\gamma}]^{1-\eta} - 1}{1-\eta} \tag{2.28}$$

A small constant $\psi = 0.001$ is added to ensure positive finite utility in case of zero consumption arising out of no income.

Agents are born without wealth $k^1_t = 0$ and leave no bequests $k^{61}_t = 0$. Agents receive income from capital and labour. The agents contribute an amount toward their pensions by paying taxes at the rate τ_t. At any instant of time the pensions of the retired agents is paid by the tax collection of the working agents. Since all generations are of the same size the tax collection and pension b_t is steady across all generations. The budget constraint is given by

$$\begin{aligned} k^{s+1}_{t+1} &= (1+r)k^s_t + (1-\tau_t)wn^s_t - c^s_t, \; s = 1,...,T \\ k^{s+1}_{t+1} &= (1+r)k^s_t + b - c^s_t, \qquad\qquad s = T+1,...,T+TR \end{aligned} \tag{2.29}$$

The price of leisure is $(1-\tau_t)w_t$, while the price of consumption is 1. The ratio of marginal utilities of consumption to leisure equals their price ratio. $\therefore$ the first order conditions are

$$\frac{\mathbb{U}_l}{\mathbb{U}_c} = \gamma \frac{c^s_t + \psi}{l^s_t} = (1-\tau_t)w_t \tag{2.30}$$

$$\begin{aligned} \frac{1}{\beta} &= \frac{\mathbb{U}_c(c^{s+1}_{t+1}, l^{s+1}_{t+1})}{\mathbb{U}_c(c^s_t, l^s_t)}(1+r_{t+1}) \\ &= \left[\frac{(c^{s+1}_{t+1}+\psi)}{(c^s_t+\psi)}\right]^{-\eta} \left[\frac{l^{s+1}_{t+1}}{l^s_t}\right]^{\gamma(1-\eta)} (1+r_{t+1}) \end{aligned} \tag{2.31}$$

2.6.3.4. *Production*

There is a single firm that produces output Y_t in period t using labour N_t and capital K_t. Labour is paid w_t and capital is paid r_t and depreciates at a rate δ. The production function is Cobb-Douglas but can also be any other function like CES.

$$Y_t = K^{\alpha}_t N^{1-\alpha}_t \tag{2.32}$$

The factor prices equal marginal products

$$w_t = (1-\alpha)\left[\frac{K_t}{N_t}\right]^{\alpha}$$

$$r_t = \alpha\left[\frac{N_t}{K_t}\right]^{1-\alpha} - \delta \tag{2.33}$$

2.6.3.5. *Government*

The government uses tax revenue from labour tax to finance expenditure on social security. The total pension payments in each period t equal the total tax collected from the working cohorts.

$$\tau_t w_t N_t = \sum_{s=1}^{TR} b^s$$

Since all cohorts get the same pension $b^s = b$ and is given only to the retired agents, the government's budget works out to

$$\tau_t w_t N_t = \frac{TR}{T+TR} b \tag{2.34}$$

2.6.3.6. *Equilibrium*

$$V^s(k_t^s, K_t, N_t) = \begin{cases} \max\limits_{k_{t+1}^{s+1}, c_t^s, k_t^s} \left[\mathbb{U}(c_t^s, l_t^s) + \beta V^{s+1}(k_{t+1}^{s+1}, K_{t+1}, N_{t+1})\right] & s = 1, .., T \\ \max\limits_{k_{t+1}^{s+1}, c_t^s, k_t^s} \left[\mathbb{U}(c_t^s, l_t^s) + \beta V^{s+1}(k_{t+1}^{s+1}, K_{t+1}, N_{t+1})\right] & s = T+1, .., T+TR-1 \end{cases} \tag{2.35}$$

subject to equation 2.29 and

$$V^{T+TR}(k_t^{T+TR}, K_t^{T+TR}, N_t^{T+TR}) = \mathbb{U}(c_t^{T+TR}, 1) \tag{2.36}$$

Given initial distribution of capital $\{k_0^s\}_{s=1}^{s=T+TR}$ and government policy b for pensions, the agents maximise the present discounted value of the value function $V^s(k_t^s, K_t, N_t)$ subject to their choice of consumption $c^s(k_t^s, K_t, N_t)$, labour $n^s(k_t^s, K_t, N_t)$ and capital $k^s(k_t^s, K_t, N_t)$. The factor prices, $\{w_t, r_t\}$ are endogenously determined such that

(1)

$$N_t = \sum_{s=1}^{T} \frac{n_t^s}{T+TR} \tag{2.37}$$

$$K_t = \sum_{s=1}^{T+TR} \frac{k_t^s}{T+TR} \tag{2.38}$$

The total labour supply at time t equals the labour supply of all working cohorts and total capital supply at time t equals the capital supply of each cohort. Since the total capital (K_t) and labour (N_t) is normalised to 1, the labour and capital of each cohort is a fraction $\frac{1}{T+TR}$ of the total as all generations are equal.

(2) Relative factor prices, $\{w_t, r_t\}$, solves the firm's optimisation problem satisfying equation 2.33

(3) Given the factor prices, $\{w_t, r_t\}$, government policy b, the individual policy choices for consumption $c^s(.)$, labour $n^s(.)$ and capital $k^s(.)$, the agents solve their dynamic problem 2.35- 2.36

(4) The total demand equals total supply, the capital stock in period $t+1$ is the production minus the consumption plus the non depreciated capital stock from the current period t.

$$K_t^\alpha N_t^{1-\alpha} = \sum_{s=1}^{T+TR} \frac{c^s}{T+TR} + K_{t+1} - (1-\delta)K_t \tag{2.39}$$

(5) the government budget 2.34 is balanced

2.6.3.7. *Computation of Steady State*

$$\frac{u_l(c_t^s, l_t^s)}{u_c(c_t^s, l_t^s)} = \gamma \frac{c_t^s + \psi}{l_t^s} = (1-\tau_t)w_t \tag{2.40}$$

$$\begin{aligned} \frac{1}{\beta} &= \frac{u_c(c_{t+1}^{s+1}, l_{t+1}^{s+1})}{u_c(c_t^s, l_t^s)}[1+r_{t+1}] \\ &= \left[\frac{(c_{t+1}^{s+1}+\psi)}{(c_t^s+\psi)}\right]^{-\eta} \left[\frac{(l_{t+1}^{s+1})}{(l_t^s)}\right]^{\gamma(1-\eta)} [1+r_{t+1}] \end{aligned} \tag{2.41}$$

For the working agents $s = 1, ..., T$

$$(1-\tau)w = \gamma \frac{(1+r)k^s + (1-\tau)wn^s - k^{s+1} + \psi}{1-n^s} \tag{2.42}$$

The equation for capital is

$$\frac{1}{\beta} = \left\{\frac{(1+r)k^{s+1} + (1-\tau)wn^{s+1} - k^{s+2} + \psi}{(1+r)k^s + (1-\tau)wn^s - k^{s+1} + \psi}\right\}^{-\eta} \left\{\frac{1-n^{s+1}}{1-n^s}\right\}^{\gamma(1-\eta)} [1+r] \tag{2.43}$$

For the agent who has reached the last year of work $s = T$

$$\frac{1}{\beta} = \left\{\frac{(1+r)k^{T+1} + b - k^{T+2} + \psi}{(1+r)k^T + (1-\tau)wn^T - k^{T+1} + \psi}\right\}^{-\eta} \left\{\frac{1}{1-n^T}\right\}^{\gamma(1-\eta)} [1+r] \tag{2.44}$$

For the retired agent, $s = T+1, .., T+TR$

$$\frac{1}{\beta} = \left\{\frac{(1+r)k^{s+1} + b - k^{s+2} + \psi}{(1+r)k^s + b - k^{s+1} + \psi}\right\}^{-\eta} [1+r] \tag{2.45}$$

2.6.3.8. *Algorithm for Computation of Steady State*

The optimal capital stock after death $k^{61} \equiv 0$ and in the first year of work $k^0 = 0$. One needs to compute 59 values for k^s, $s = 2, .., T + TR$ and 40 values for n^s, $s = 1, .., T$. To obtain these values one iterates recursively backwards from k^{59} to k^1. The algorithm will converge when $k^1 = 0$.

(1) begin with a steady state capital $\overline{K}$, labour $\overline{L}$ and tax rate τ

(2) obtain the wage rate w and return to capital r from 2.33

(3) obtain the pension b from 2.34

(4) iterate backwards from $s = T + TR, .., T + 1$ using 2.45 $\because n^s = 0$ for $s = T + 1, .., T + TR$

(5) for the last year of work obtain k^T and n^T from 2.44 and 2.42.

(6) for the working agent during $s = 1, .., T - 1$, use 2.42 and 2.43 to obtain k^s and n^s **simultaneously**.

$$\begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} (1-\tau)w - \gamma \frac{(1+r)k^s + (1-\tau)wn^s - k^{s+1} + \psi}{1-n^s} = 0 \\ \frac{1}{\beta} - \left\{ \frac{(1+r)k^{s+1} + (1-\tau)wn^{s+1} - k^{s+2} + \psi}{(1+r)k^s + (1-\tau)wn^s - k^{s+1} + \psi} \right\}^{-\eta} \left\{ \frac{1-n^{s+1}}{1-n^s} \right\}^{\gamma(1-\eta)} [1+r] = 0 \end{bmatrix} \tag{2.46}$$

(7) if $k^1 \neq 0$ modify k^{60}
start with iteration @i=1 $k_1^{60} = 0.4$ and @i=2 $k_2^{60} = 0.5$
$k_{i+2}^{60} = k_{i+1}^{60} - \frac{k_{i+1}^{60} - k_i^{60}}{k_{i+1}^1 - k_i^1} k_{i+1}^1$
goto step 1
else;
stop

2.7. Demand Functions, Identities and Welfare

[d]The cognitive abilities of humans and their responses to external stimuli are basically ordinal in nature. Individuals are at the best capable of ranking alternatives in a consistent manner from the least to the most preferred. The approach to assign numerical values to these alternatives transforms the choice into cardinal values. The logic of choice to preference to utility to make consistent decisions is based on certain axioms. In order to take decisions and rationalise them either ex-ante or ex-post one needs to prove without reasonable doubt that they are or were the best possible. Utility as represented by mathematical functions with specific properties is the best possible way to tackle this problem. In this section we give a very succinct overview that is necessary for further analysis.

Without delving into the aspects of axioms of choice and the links to utility and properties of indifference curves, we assume that we have indifference curves that show nice mathematical properties amenable to optimization and a unique one at that. Given a utility function that spans n commodities in the $\mathbb{R}_+^n$ space, and a known income, we first derive the various demand functions. We have a set of observables namely, the prices and incomes but not the utilities. To make a meaningful comparison, only monetary units will be useful. Changes will be

[d]this section follows [Mas-Colell *et al.* (1995)] and [Varian (1992)]

measured in terms of the impact on welfare and the different methods to compute welfare will be outlined.

We start by deriving the Marshallian or uncompensated demand function and then derive the Hicksian or compensated demand function. Later we show the relationship between these two demand functions and the need for such a link. A little digression with an example of a Cobb-Douglas function follows as an 'empirical' proof of the above. Finally we conclude with the implications for welfare using these two demand functions and a simple example of computing different welfare measures.

2.7.1. Demand Functions

2.7.1.1. *Marshallian Demand Function*

The consumer's optimisation problem, given a vector of n prices $\mathbf{p}\ |p_i > 0\ \forall\{i = 1, 2, ..., n\}$[e]. The size of the vector will be specified or clear from the context and income $\mathbb{I}$, is to maximise the utility function. The objective is to obtain the vector of consumption of commodities $\mathbf{x}|x_i > 0 \forall i = 1, 2, ..., n$

$$\begin{aligned} \max_{\mathbf{x}} \quad & \mathbb{U}(\mathbf{x}) \\ s.t.\ \ & \mathbf{p}.\mathbf{x} = \mathbb{I} \end{aligned} \tag{2.47}$$

The lagrangian is given by

$$\begin{aligned} \mathcal{L} &= \mathbb{U}(\mathbf{x}) + \lambda(\mathbb{I} - \mathbf{p}.\mathbf{x}) \\ \frac{\partial \mathcal{L}}{\partial \mathbf{x}} &= 0 \Rightarrow \frac{\partial \mathbb{U}(\mathbf{x})}{\partial \mathbf{x}} - \lambda \mathbf{p} = 0 \\ \frac{\partial \mathcal{L}}{\partial \lambda} &= 0 \Rightarrow \mathbb{I} - \mathbf{p}.\mathbf{x} = 0 \\ \frac{\partial \mathcal{L}}{\partial \mathbb{I}} &= \lambda \end{aligned} \tag{2.48}$$

gives rise to demand functions which are called in literature as Marshallian demand functions, denoted by $x(p, \mathbb{I})$.

2.7.1.2. *Indirect Utility Function*

Utility is not observable, however prices and incomes are. To obtain utility one must first obtain demand and then substitute in the utility function and repeat the whole procedure, each time the prices change. If one can obtain utility as a function of prices and income then one is saved the effort of computing $x(p, \mathbb{I})$. Substituting the marshallian demand $x(p, \mathbb{I})$ in the utility function gives rise to what is known as the indirect utility function denoted by $v(p, \mathbb{I})$

[e]a bold symbol denotes a vector. $\therefore \mathbf{p} \equiv \{p_1, p_2, ..., p_n\}$

2.7.1.3. *Hicksian Demand Function*

The dual to the utility maximisation problem is expenditure minimisation. It is the answer to the question What is the level of expenditure needed to achieve a pre-specified level of utility $\overline{\mathbb{U}}$?. Since the utility is known one moves in the south-west direction successively decreasing the expenditure till it is tangent to the indifference curve corresponding to the utility.

$$\min_{\mathbf{x}} \quad \mathrm{e}(\mathrm{p}, \mathbf{x}) \\ s.t. \ \mathbb{U} = \overline{\mathbb{U}} \tag{2.49}$$

The lagrangian is given by

$$\begin{aligned} \mathcal{L} &= \mathrm{e}(\mathrm{p}, \mathbf{x}) + \lambda(\overline{\mathbb{U}} - \mathbb{U}(\mathbf{x})) \\ \frac{\partial \mathcal{L}}{\partial \mathbf{x}} &= 0 \Rightarrow \frac{\partial \mathrm{e}(\mathrm{p}, \mathbf{x})}{\partial \mathbf{x}} - \lambda \frac{\partial \mathbb{U}}{\partial \mathbf{x}} = 0 \\ \frac{\partial \mathcal{L}}{\partial \lambda} &= 0 \Rightarrow \overline{\mathbb{U}} - \mathbb{U} = 0 \\ \frac{\partial \mathcal{L}}{\partial \overline{\mathbb{U}}} &= \lambda \end{aligned} \tag{2.50}$$

The solution to this dual problem is another set of demand functions termed in literature as Hicksian demand functions, denoted by $h(p, \mathbb{U})$.

2.7.1.4. *Expenditure Function*

$\because$ expenditure $\mathrm{e} = \boldsymbol{p}.\boldsymbol{h}(\mathrm{p}, \mathbb{U})$, we obtain the expenditure in terms of prices (p) and utility ($\mathbb{U}$). The expenditure function is denoted by $\mathrm{e}(\mathrm{p}, \mathbb{U})$

2.7.2. Relationships and Identities

2.7.2.1. *Relationship between Indirect Utility & Marshallian Demand*

Roy's Identity The benefit of the indirect utility function is that it permits evaluation of the utility in terms of the prices p and income $\mathbb{I}$. However if we are to recover the demand $x(p, \mathbb{I})$, then we need a relationship between the indirect utility v and the prices and income. Roy's identity does precisely that, enable us to obtain demand from the indirect utility function.

$$\mathrm{v}(\mathrm{p}, \mathbb{I}) = \mathbb{U}(\mathrm{x}(\mathrm{p}, \mathbb{I})) \tag{2.51}$$

Differentiating w.r.t. p_i, we have

$$\frac{\partial v(\mathbf{p},\mathbb{I})}{\partial p_i} = \sum_{i=1}^{n} \frac{\mathbb{U}(x(\mathbf{p},\mathbb{I}))}{\partial x_i}\frac{\partial x_i}{\partial p_i}$$

$$\frac{\partial \mathbb{U}}{\partial x_i} = \lambda p_i$$

$$\frac{\partial v}{\partial \mathbb{I}} = \lambda$$

$$\therefore \frac{\partial v(\mathbf{p},\mathbb{I})}{\partial p_i} = \frac{\partial v}{\partial \mathbb{I}}\sum_{i=1}^{n} p_i \frac{\partial x_i}{\partial p_i}$$

$$\because \sum_{i=1}^{n} p_i x_i = \mathbb{I}$$

$$\frac{\sum_{i=1}^{n} p_i x_i}{\partial p_j} = x_j + \sum_{j=1}^{n} p_j \frac{\partial x_i}{\partial p_j} = 0 \because \frac{\partial \mathbb{I}}{\partial p_j} = 0$$

$$\frac{\partial v(\mathbf{p},\mathbb{I})}{\partial p_i} = -x_j \frac{\partial v(\mathbf{p},\mathbb{I})}{\partial \mathbb{I}}$$

$$x_j = -\frac{\frac{\partial v(\mathbf{p},\mathbb{I})}{\partial p_i}}{\frac{\partial v(\mathbf{p},\mathbb{I})}{\partial \mathbb{I}}} \tag{2.52}$$

2.7.2.2. *Relationship between Expenditure Function & Hicksian Demand*

As mentioned before, the prices and expenditure are observable while the demand is not. Deriving a relationship between the expenditure function and Hicksian demand enables us to obtain the unobservable from the observable.

$$\mathrm{e}(\mathrm{p},\mathbb{U}) = \mathrm{p}.\mathrm{h}(\mathrm{p},\mathbb{U}) \tag{2.53}$$

$$\frac{\partial \mathrm{e}(\mathrm{p},\mathbb{U})}{\partial p_i} = \mathrm{h_i}(\mathrm{p},\mathbb{U}) + \sum_{\mathrm{j}=1}^{\mathrm{n}} \mathrm{p_j}\frac{\partial \mathrm{h}(\mathrm{p},\mathbb{U})}{\partial \mathrm{p_j}}$$

$$\frac{\partial \mathrm{e}(\mathrm{p},\mathbb{U})}{\partial x_i} = 0 \Rightarrow p_i = \lambda\frac{\partial \mathbb{U}}{\partial x_i} \quad [\min_{x_i}\sum_{i=1}^{n} p_i x_i + \lambda(\overline{\mathbb{U}} - \mathbb{U}(x_i))]$$

$$\frac{\partial \mathrm{e}(\mathrm{p},\mathbb{U})}{\partial p_i} = \mathrm{h_i}(\mathrm{p},\mathbb{U}) + \lambda\sum_{\mathrm{i}=1}^{\mathrm{n}} \frac{\partial \mathbb{U}}{\partial \mathrm{x_i}}\frac{\partial \mathrm{h}(\mathrm{p},\mathbb{U})}{\partial \mathrm{p_i}}$$

$$\mathbb{U}(\mathrm{x}) = \mathbb{U}(\mathrm{h}(\mathrm{p},\mathbb{U})) = \overline{\mathbb{U}}$$

$$\frac{\partial \mathbb{U}}{\partial p_j} = \sum_{j=1}^{n} \frac{\partial \mathbb{U}}{\partial x_j}\frac{\partial \mathrm{h}}{\partial p_j} = 0 \quad \because \frac{\partial \overline{\mathbb{U}}}{\partial p_j} = 0$$

$$\frac{\partial \mathrm{e}(\mathrm{p},\mathbb{U})}{\partial p_i} = \mathrm{h_i}(\mathrm{p},\mathbb{U}) \tag{2.54}$$

2.7.2.3. *Slutsky Equation: Linking Marshallian & Hicksian Demand*

$$
\begin{aligned}
\mathrm{h}(\mathrm{p}, \mathbb{U}) &= \mathrm{x}(\mathrm{p}, \mathrm{e}(\mathrm{p}, \mathbb{U})) \\
\frac{\partial h_i(p, \mathbb{U})}{\partial p_j} &= \frac{\partial x_i(p, \mathrm{e}(\mathrm{p}, \mathbb{U}))}{\partial p_j} + \frac{\partial x_i(p, \mathrm{e}(\mathrm{p}, \mathbb{U}))}{\partial \mathrm{e}(\mathrm{p}, \mathbb{U})} \frac{\partial \mathrm{e}(\mathrm{p}, \mathbb{U})}{\partial p_j} \\
&= \frac{\partial x_i(p, \mathbb{I})}{\partial p_j} + \frac{\partial x_i(p, \mathbb{I})}{\partial \mathbb{I}} h_j(p, \mathbb{U}) \\
&= \frac{\partial x_i(p, \mathbb{I})}{\partial p_j} + \frac{\partial x_i(p, \mathbb{I})}{\partial \mathbb{I}} x_j(p, \mathrm{e}(\mathrm{p}, \mathbb{U})) \\
&= \frac{\partial x_i(p, \mathbb{I})}{\partial p_j} + \frac{\partial x_i(p, \mathbb{I})}{\partial \mathbb{I}} x_j(p, \mathbb{I}) \qquad (2.55)
\end{aligned}
$$

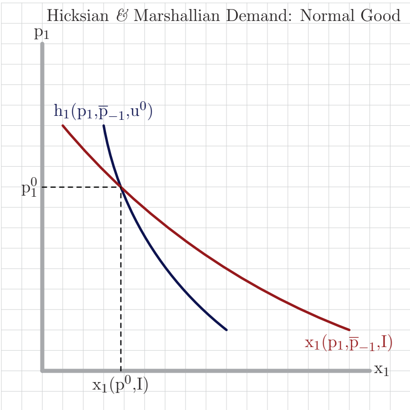
The hicksian demand $\mathrm{h}(\mathrm{p}, \mathbb{U})$ accounts for the change in demand after a change in price is compensated by an income change to maintain constant utility. In other words the utility is kept constant and only the relative prices are changed. This is the pure substitution effect. However in real life, consumers are never compensated for a change in prices and therefore the compensated component of the hicksian demand has to be reduced to obtain the uncompensated change or marshallian demand for a change in prices. Formally

$$
\frac{\partial x_i(p, \mathbb{I})}{\partial p_j} = \frac{\partial h_i(p, \mathbb{U})}{\partial p_j} - \frac{\partial x_i(p, \mathbb{I})}{\partial \mathbb{I}} x_j(p, \mathbb{I}) = \frac{\partial^2 e(p, \mathbb{U})}{\partial p_i \partial p_j} - \frac{\partial x_i(p, \mathbb{I})}{\partial \mathbb{I}} x_j(p, \mathbb{I}) \qquad (2.56)
$$

2.7.2.4. *Marshallian & Hicksian Demand: Graphical Exposition*

The change in marshallian demand of good i ∂x_i due to a change in price of good j $\left(\frac{\partial x_i(p,\mathbb{I})}{\partial p_j}\right)$, is the total of the substitution effect or hicksian demand $\frac{\partial h_i(p,\mathbb{U})}{\partial p_j}$ and the income effect $\frac{\partial x_i(p,\mathbb{I})}{\partial \mathbb{I}} x_j(p, \mathbb{I})$. The slope of the marshallian demand will depend on the income effect.

(1) For a normal good, $\frac{\partial x_i(p,\mathbb{I})}{\partial \mathbb{I}} > 0$, hence slope of marshallian demand $\frac{\partial x_i(p,\mathbb{I})}{\partial p_j}$ is more negative than slope of hicksian demand $\frac{\partial h_i(p,\mathbb{U})}{\partial p_j}$. $\therefore$ in the p x space the marshallian demand will be flatter.
(2) For an inferior good, $\frac{\partial x_i(p,\mathbb{I})}{\partial \mathbb{I}} < 0$, hence slope of marshallian demand $\frac{\partial x_i(p,\mathbb{I})}{\partial p_j}$ is less negative than slope of hicksian demand $\frac{\partial h_i(p,\mathbb{U})}{\partial p_j}$. $\therefore$ in the p x space the marshallian demand will be steeper.



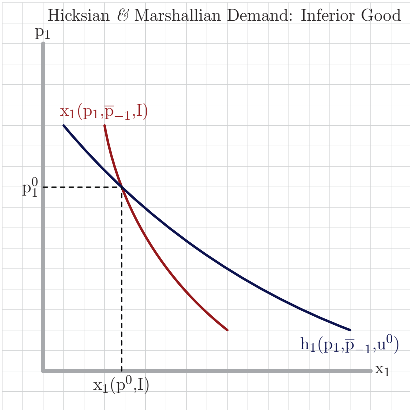


fig a: Normal Good

$$\frac{\partial x_1(p,\mathbb{I})}{\partial p_1} = \frac{\partial h_1(p,\mathbb{U})}{\partial p_1} - \frac{\partial x_1(p,\mathbb{I})}{\partial \mathbb{I}} x_1(p,\mathbb{I})$$
$$-ve = -ve - (+ve)(+ve)$$

fig b: Inferior Good

$$\frac{\partial x_1(p,\mathbb{I})}{\partial p_1} = \frac{\partial h_1(p,\mathbb{U})}{\partial p_1} - \frac{\partial x_1(p,\mathbb{I})}{\partial \mathbb{I}} x_1(p,\mathbb{I})$$
$$? = -ve - (-ve)(+ve)$$

Fig. 2.6.: Marshallian & Hicksian Demand

2.7.3. Functional Forms and Identities

2.7.3.1. *Cobb-Douglas*

Utility Maximisation Problem

$$\max_{x_i} \ \mathbb{U}(x_i) = \prod_{i=1}^{n} x_i^{\alpha_i}$$
$$s.t. \sum_{i=1}^{n} p_i x_i = \mathbb{I} \tag{2.57}$$

Form the Lagrangian $\mathcal{L}$

$$\mathcal{L} = \prod_{i=1}^{n} x_i^{\alpha_i} + \lambda[\mathbb{I} - \sum_{i=1}^{n} p_i x_i] \tag{2.58}$$

$$\frac{\partial \mathcal{L}}{\partial x_i} = \frac{\alpha_i \prod_{i=1}^{n} x_i^{\alpha_i}}{x_i} - p_i = 0 \tag{2.59}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \mathbb{I} - \sum_{i=1}^{n} p_i x_i = 0 \tag{2.60}$$

Taking ratio of conditions i and j

$$\frac{\frac{\alpha_i \prod_{i=1}^{n} x_i^{\alpha_i}}{x_i}}{\frac{\alpha_j \prod_{i=1}^{n} x_i^{\alpha_i}}{x_j}} = \frac{p_i}{p_j} = \frac{\alpha_i x_j}{\alpha_j x_i} \tag{2.61}$$

Solving for x_j we get

$$x_j = \frac{\alpha_j p_i x_i}{\alpha_i p_j} \tag{2.62}$$

substituting in the lagrangian we get

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \mathbb{I} - \sum_{j=1}^{n} p_j \frac{\alpha_j p_i x_i}{\alpha_i p_j} \Rightarrow \frac{p_i x_i}{\alpha_i} \sum_{j=1}^{n} \alpha_j = \mathbb{I} \Rightarrow x_i = \frac{\alpha_i}{\sum_{j=1}^{n} \alpha_j} \frac{\mathbb{I}}{p_i} \tag{2.63}$$

Marshallian demand $x(p, \mathbb{I})$ is homogeneous of degree **0** in income and prices. i.e. $x(p, \mathbb{I}) = x(\lambda p, \lambda \mathbb{I})$ The expenditure elasticity is

$$\frac{\partial x_i}{\partial \mathbb{I}} \frac{\mathbb{I}}{x_i} = \left[\frac{\alpha_i}{\sum_{j=1}^{n} \alpha_j} \frac{1}{p_i} \right] \left[\frac{\mathbb{I}}{\frac{\alpha_i}{\sum_{j=1}^{n} \alpha_j} \frac{\mathbb{I}}{p_i}} \right] = 1 \tag{2.64}$$

To obtain the indirect utility function $v(p, \mathbb{I})$

$$\begin{aligned}
\psi = v(x(p, \mathbb{I})) &= \prod_{i=1}^{n} x_i^{\alpha_i} \\
&= \prod_{i=1}^{n} \left[\frac{\alpha_i}{\sum_{j=1}^{n} \alpha_j} \frac{\mathbb{I}}{p_i} \right]^{\alpha_i} \\
&= \prod_{i=1}^{n} \left[\frac{\mathbb{I}}{\sum_{j=1}^{n} \alpha_j} \right]^{\alpha_i} \prod_{i=1}^{n} \left[\frac{\alpha_i}{p_i} \right]^{\alpha_i} \\
&= \left[\frac{\mathbb{I}}{\sum_{j=1}^{n} \alpha_j} \right]^{\sum_{i=1}^{n} \alpha_i} \prod_{i=1}^{n} \left[\frac{\alpha_i}{p_i} \right]^{\alpha_i}
\end{aligned} \tag{2.65}$$

Roy's identity states

$$\frac{\partial \psi}{\partial p_i} / \frac{\partial \psi}{\partial \mathbb{I}} = -x(p, \mathbb{I}) \tag{2.66}$$

To obtain Roy's identity for cobb-douglas utility function differentiate $\psi(p, \mathbb{I})$ w.r.t. $\mathbb{I}$ and p. Differentiating w.r.t. $\mathbb{I}$ we get

$$\begin{aligned}
\frac{\partial \psi}{\partial \mathbb{I}} &= (\sum_{i=1}^{n} \alpha_i) \mathbb{I}^{(\sum_{i=1}^{n} \alpha_i - 1)} \left[\frac{1}{\sum_{j=1}^{n} \alpha_j} \right]^{\sum_{i=1}^{n} \alpha_i} \prod_{i=1}^{n} \left[\frac{\alpha_i}{p_i} \right]^{\alpha_i} \\
&= \mathbb{I}^{(\sum_{i=1}^{n} \alpha_i - 1)} (\sum_{j=1}^{n} \alpha_j)^{(1 - \sum_{i=1}^{n} \alpha_i)} \left[\frac{1}{\sum_{j=1}^{n} \alpha_j} \right]^{\sum_{i=1}^{n} \alpha_i} \prod_{i=1}^{n} \left[\frac{\alpha_i}{p_i} \right]^{\alpha_i}
\end{aligned} \tag{2.67}$$

Differentiating w.r.t. p_k we get

$$\frac{\partial \psi}{\partial p_k} = \left[\frac{\mathbb{I}}{\sum_{j=1}^n \alpha_j}\right]^{\sum_{i=1}^n \alpha_i} \prod_{i=1}^n \left[\frac{\alpha_i}{p_i}\right]^{\alpha_i} \frac{-\alpha_k}{p_k} \tag{2.68}$$

Taking ratios of the above two equations we get

$$\frac{\frac{\partial \psi}{\partial p_k}}{\frac{\partial \psi}{\partial \mathbb{I}}} = \frac{\left[\frac{\mathbb{I}}{\sum_{j=1}^n \alpha_j}\right]^{\sum_{i=1}^n \alpha_i} \prod_{i=1}^n \left[\frac{\alpha_i}{p_i}\right]^{\alpha_i} \frac{-\alpha_k}{p_k}}{\mathbb{I}^{(\sum_{i=1}^n \alpha_i - 1)}(\sum_{j=1}^n \alpha_j)^{(1-\sum_{i=1}^n \alpha_i)} \left[\frac{1}{\sum_{j=1}^n \alpha_j}\right]^{\sum_{i=1}^n \alpha_i} \prod_{i=1}^n \left[\frac{\alpha_i}{p_i}\right]^{\alpha_i}}$$

$$= \frac{-\alpha_k}{p_k} \frac{\mathbb{I}}{\sum_{i=1}^n \alpha_i} = -x(p, \mathbb{I}) \tag{2.69}$$

2.7.3.2. *Expenditure Minimisation Problem*

$$\min_{x_i} \sum_{i=1}^n p_i x_i$$

$$s.t. \mathbb{U}(x_i) = \prod_{i=1}^n x_i^{\alpha_i} \tag{2.70}$$

Form the Lagrangian $\mathcal{L}$

$$\mathcal{L} = \sum_{i=1}^n p_i x_i + \lambda[\mathbb{U} - \prod_{i=1}^n x_i^{\alpha_i}] \tag{2.71}$$

$$\frac{\partial \mathcal{L}}{\partial x_i} = p_i - \frac{\alpha_i \prod_{i=1}^n x_i^{\alpha_i}}{x_i} = 0 \tag{2.72}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \mathbb{U} - \prod_{i=1}^n x_i^{\alpha_i} = 0 \tag{2.73}$$

Taking ratio of conditions i and j

$$\frac{p_i}{p_j} = \frac{\alpha_i x_j}{\alpha_j x_i} \tag{2.74}$$

Solving for x_j we get

$$x_j = \frac{\alpha_j p_i x_i}{\alpha_i p_j} \tag{2.75}$$

substituting in the lagrangian we get

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \Rightarrow \mathbb{U} - \prod_{j=1}^n \left[\frac{\alpha_j p_i x_i}{\alpha_i p_j}\right]^{\alpha_i} = 0 \Rightarrow \mathbb{U} = \prod_{j=1}^n \left[\frac{p_i}{\alpha_i}\right]^{\alpha_i} \prod_{j=1}^n \left[\frac{\alpha_j}{p_j}\right]^{\alpha_i} \prod_{j=1}^n x_i^{\alpha_i} \tag{2.76}$$

$$\mathbb{U} = \prod_{j=1}^{n}\left[\frac{\alpha_j}{p_j}\right]^{\alpha_j}\left[\frac{p_i}{\alpha_i}\right]^{\sum_{j=1}^{n}\alpha_j} x_i^{\sum_{j=1}^{n}\alpha_j}$$

$$x_i^{\sum_{j=1}^{n}\alpha_j} = \mathbb{U}\prod_{j=1}^{n}\left[\frac{p_j}{\alpha_j}\right]^{\alpha_j}\left[\frac{\alpha_i}{p_i}\right]^{\sum_{j=1}^{n}\alpha_j}$$

$$x_i = \left\{\mathbb{U}\prod_{j=1}^{n}\left[\frac{p_j}{\alpha_j}\right]^{\alpha_j}\right\}^{\frac{1}{\sum_{j=1}^{n}\alpha_j}}\left[\frac{\alpha_i}{p_i}\right]$$

$$h_i(p,\mathbb{U}) = \left\{\mathbb{U}\prod_{j=1}^{n}\left[\frac{p_j}{\alpha_j}\right]^{\alpha_j}\right\}^{\frac{1}{\sum_{j=1}^{n}\alpha_j}}\left[\frac{\alpha_i}{p_i}\right] \tag{2.77}$$

$$\mathrm{e}(\mathrm{p},\mathbb{U}) = \sum_{i=1}^{n} p_i x_i = (\sum_{i=1}^{n}\alpha_i)\left\{\mathbb{U}\prod_{j=1}^{n}\left[\frac{p_j}{\alpha_j}\right]^{\alpha_j}\right\}^{\frac{1}{\sum_{j=1}^{n}\alpha_j}} \tag{2.78}$$

Identities

(1) To Prove That: $\mathrm{e}(\mathrm{p},\mathbb{U}) = \mathrm{e}(\mathrm{p},\mathrm{v}(\mathrm{p},\mathbb{I})) = \mathbb{I}$ Using equations 2.43 and 2.30 we have

$$\begin{aligned}\mathrm{e}(\mathrm{p},\mathbb{U}) &= \sum_{i=1}^{n} p_i x_i = (\sum_{i=1}^{n}\alpha_i)\left\{\left[\frac{\mathbb{I}}{\sum_{j=1}^{n}\alpha_j}\right]^{\sum_{i=1}^{n}\alpha_i}\prod_{i=1}^{n}\left[\frac{\alpha_i}{p_i}\right]^{\alpha_i}\prod_{j=1}^{n}\left[\frac{p_j}{\alpha_j}\right]^{\alpha_j}\right\}^{\frac{1}{\sum_{j=1}^{n}\alpha_j}}\\ &= (\sum_{i=1}^{n}\alpha_i)\left\{\left[\frac{\mathbb{I}}{\sum_{j=1}^{n}\alpha_j}\right]^{\sum_{i=1}^{n}\alpha_i}\right\}^{\frac{1}{\sum_{j=1}^{n}\alpha_j}}\\ &= (\sum_{i=1}^{n}\alpha_i)\left[\frac{\mathbb{I}}{\sum_{j=1}^{n}\alpha_j}\right] = \mathbb{I}\end{aligned} \tag{2.79}$$

(2) To Prove That: $\mathrm{v}(\mathrm{p},\mathbb{I}) = \mathrm{v}(\mathrm{p},\mathrm{e}(\mathrm{p},\mathbb{U})) = \mathbb{U}$ Using equations 2.43 and 2.30 we have

$$\begin{aligned}v(x(p,\mathbb{I})) &= \left[\frac{(\sum_{i=1}^{n}\alpha_i)\left\{\mathbb{U}\prod_{j=1}^{n}\left[\frac{p_j}{\alpha_j}\right]^{\alpha_j}\right\}^{\frac{1}{\sum_{j=1}^{n}\alpha_j}}}{\sum_{j=1}^{n}\alpha_j}\right]^{\sum_{i=1}^{n}\alpha_i}\prod_{i=1}^{n}\left[\frac{\alpha_i}{p_i}\right]^{\alpha_i}\\ &= \mathbb{U}\prod_{j=1}^{n}\left[\frac{p_j}{\alpha_j}\right]^{\alpha_j}\prod_{i=1}^{n}\left[\frac{\alpha_i}{p_i}\right]^{\alpha_i} = \mathbb{U}\end{aligned} \tag{2.80}$$

(3) To Prove That: $\mathrm{x}(\mathrm{p}, \mathrm{e}(\mathrm{p}, \mathbb{U})) = \mathrm{h}(\mathrm{p}, \mathbb{U})$. Using equations 2.43 and 2.28 we have

$$
\begin{aligned}
x_i &= \frac{\alpha_i}{\sum_{j=1}^{n} \alpha_j} \frac{\mathbb{I}}{p_i} \\
&= \frac{\alpha_i}{p_i} \frac{(\sum_{i=1}^{n} \alpha_i)\left\{\mathbb{U}\prod_{j=1}^{n} \left[\frac{p_j}{\alpha_j}\right]^{\alpha_j}\right\}^{\frac{1}{\sum_{j=1}^{n} \alpha_j}}}{\sum_{j=1}^{n} \alpha_j} \\
&= \frac{\alpha_i}{p_i} \left\{\mathbb{U}\prod_{j=1}^{n} \left[\frac{p_j}{\alpha_j}\right]^{\alpha_j}\right\}^{\frac{1}{\sum_{j=1}^{n} \alpha_j}} = h(p, \mathbb{U})
\end{aligned} \tag{2.81}
$$

(4) To Prove That: $\mathrm{h}(\mathrm{p}, \mathrm{v}(\mathrm{p}, \mathbb{I})) = \mathrm{x}(\mathrm{p}, \mathbb{I})$. Using equations 2.43 and 2.28 we have

$$
\begin{aligned}
h_i(p, \mathbb{U}) &= \left\{\mathbb{U}\prod_{j=1}^{n} \left[\frac{p_j}{\alpha_j}\right]^{\alpha_j}\right\}^{\frac{1}{\sum_{j=1}^{n} \alpha_j}} \left[\frac{\alpha_i}{p_i}\right] \\
&= \left\{\left[\frac{\mathbb{I}}{\sum_{j=1}^{n} \alpha_j}\right]^{\sum_{i=1}^{n} \alpha_i} \prod_{i=1}^{n} \left[\frac{\alpha_i}{p_i}\right]^{\alpha_i} \prod_{j=1}^{n} \left[\frac{p_j}{\alpha_j}\right]^{\alpha_j}\right\}^{\frac{1}{\sum_{j=1}^{n} \alpha_j}} \left[\frac{\alpha_i}{p_i}\right] \\
&= \left[\frac{\alpha_i}{\sum_{j=1}^{n} \alpha_j}\right] \left[\frac{\mathbb{I}}{p_i}\right] = x(p, \mathbb{I})
\end{aligned} \tag{2.82}
$$

The demand obtained from the expenditure minimisation problem is termed as the HICKSIAN DEMAND ($\mathrm{h}(\mathrm{p}, \mathbb{U})$). Shephard's lemma states that

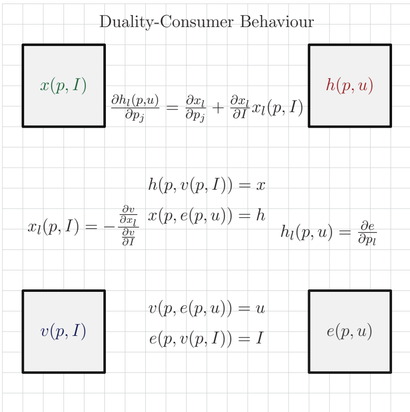
$$
\frac{\partial \mathrm{e}}{\partial p_k} = h(p, \mathbb{U}) \tag{2.83}
$$

To prove Shephard's lemma differentiate $\mathrm{e}(p, \mathbb{U})$ w.r.t. p_k to obtain

$$
\begin{aligned}
\frac{\partial \mathrm{e}(\mathrm{p}, \mathbb{U})}{\partial p_k} &= (\sum_{i=1}^{n} \alpha_i) \frac{1}{\sum_{j=1}^{n} \alpha_j} \left\{\mathbb{U}\prod_{j=1}^{n} \left[\frac{p_j}{\alpha_j}\right]^{\alpha_j}\right\}^{\left[\frac{1}{\sum_{j=1}^{n} \alpha_j} - 1\right]} \left[\frac{1}{\alpha_j}\right]^{\alpha_j} \alpha_k p_k^{(\alpha_j - 1)} \\
&= \left\{\mathbb{U}\prod_{j=1}^{n} \left[\frac{p_j}{\alpha_j}\right]^{\alpha_j}\right\}^{\left[\frac{1}{\sum_{j=1}^{n} \alpha_j} - 1\right]} \left[\frac{p_j}{\alpha_j}\right]^{\alpha_j} \frac{\alpha_k}{p_k} \\
&= \left\{\mathbb{U}\prod_{j=1}^{n} \left[\frac{p_j}{\alpha_j}\right]^{\alpha_j}\right\}^{\frac{1}{\sum_{j=1}^{n} \alpha_j}} \frac{\alpha_k}{p_k} = x_k
\end{aligned} \tag{2.84}
$$

2.7.3.3. *Duality in Consumption*

The figure 2.7 sumarises graphically and analytically the dual relationships between the various demand functions, utility and expenditure functions.



(1) $\mathrm{e(p, \mathbb{U}) = e(p, v(p, \mathbb{I})) = \mathbb{I}}$

(2) $\mathrm{v(p, \mathbb{I}) = v(p, e(p, \mathbb{U})) = \mathbb{U}(x(p, \mathbb{I}))}$

(3) $\mathrm{x(p, e(p, \mathbb{U})) = h(p, \mathbb{U})}$

(4) $\mathrm{h(p, v(p, \mathbb{I})) = x(p, \mathbb{I})}$

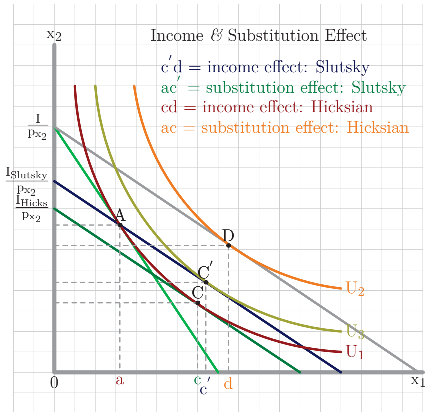
(5) $\mathrm{x_l(p, \mathbb{I})} = -\frac{\frac{\partial \mathrm{v}}{\partial \mathrm{x_l}}}{\frac{\partial \mathrm{v}}{\partial \mathbb{I}}}$

(6) $\frac{\partial \mathrm{h_l(p, \mathbb{U})}}{\partial p_j} = \frac{\partial \mathrm{x_l}}{\partial p_j} + \frac{\partial \mathrm{x_l}}{\partial \mathbb{I}} \mathrm{x_l(p, \mathbb{I})}$

Fig. 2.7.: Duality in Consumption

2.7.4. Income and Substitution Effects

We decompose a change in price on demand graphically and numerically by computing the income and substitution effects by two methods, the Hicksian decomposition and Slutsky decomposition. This is also important from the perspective of Hicksian or compensated demand and Marshallian or uncompensated demand. The figure 2.8 shows the income and substitution effect. The Marshallian demand curve passed through **AD** while the Hicksian demand curve passes through **AC**. The change in uncompensated demand is more than the compensated demand (utility maintained at $\mathbb{U}_1$), thus showing for a normal good the Marshallian demand is flatter than the Hicksian demand when price is on the y-axis and quantity on the x-axis.



Consumer with income **I** faces relative prices $p^1_{x_1}$ & $p^1_{x_2}$. Price of good 1 falls from $p^1_{x_1}$ to $p^2_{x_1}$. The consumer shifts from **A** on U_1 to **D** on U_2

The shift from A to D is decomposed into 2 parts :- Income Effect & Substitution effect. Maintaining original utility U_1 @ **new** relative prices $\frac{p^2_{x_1}}{p^1_{x_2}}$, the consumer reaches **C**. Thus there is a substitution from A to C. Now keeping the relative prices constant, to reach utility U_2 additional income is necessary. This additional income is the income effect. $\therefore \overline{AD} = \overline{AC}$ (substitution) + $\overline{CD}$ (income) effect.

Fig. 2.8.: Income & Substitution Effects

2.7.4.1. *Hicksian Decomposition*

I=100	$p^o_1 = 1$	$p^o_2 = 1$	$\alpha = 0.5$	U= $x_1^{\alpha} x_2^{(1-\alpha)}$
	$p^n_1 = 2$	$p^n_2 = 1$		
$x_1[I^o, p^o] = x^o_1 = \frac{\alpha I}{p^o_1} = 50$				
$x_2[I^o, p^o] = x^o_2 = \frac{(1-\alpha)I}{p^o_2} = 50$				
Maintain Utility at new prices				
$50^{\frac{1}{2}} 50^{\frac{1}{2}} = \left[\frac{\alpha I^n}{p^n_1}\right]^{\alpha} \left[\frac{(1-\alpha)I^n}{p^n_2}\right]^{(1-\alpha)} = \left[\frac{I^n}{2.2}\right]^{\frac{1}{2}} \left[\frac{I^n}{2.1}\right]^{\frac{1}{2}} \Rightarrow I^n = 100\sqrt{2} \approx 140$				
$x_1[I^n, p^n] = x^n_1 = \frac{\alpha I^n}{p^n_1} = \frac{0.5 \times 140}{2} = 35$				
$x_2[I^n, p^n] = x^n_2 = \frac{(1-\alpha)I^n}{p^n_2} = \frac{0.5 \times 140}{1} = 70$				
$x_1[I^o, p^n] = \frac{\alpha I^o}{p^n_1} = \frac{0.5 \times 100}{2} = 25$				
$x_2[I^o, p^n] = \frac{(1-\alpha)I^o}{p^n_2} = \frac{0.5 \times 100}{1} = 50$				
Substitution Effect $= x_1[I^o, p^o] - x_1[I^n, p^n]@U = U^o = 50 - 35 = 15$				
Income Effect $= x_1[I^n, p^n] - x_1[I^o, p^n]@U = U^n = 35 - 25 = 10$				

2.7.4.2. *Slutsky Decomposition*

I=100	$p_1^o = 1$	$p_2^o = 1$	$\alpha = 0.5$	$U = x_1^{\alpha} x_2^{(1-\alpha)}$
	$p_1^n = 2$	$p_2^n = 1$		

$x_1[I^o, p^o] = x_1^o = \frac{\alpha I}{p_1^o} = 50$

$x_2[I^o, p^o] = x_2^o = \frac{(1-\alpha)I}{p_2^o} = 50$

Maintain Income at new prices

$\triangle I = \triangle p \times q = (2-1) \times 50 = 50 \therefore I^n = I^o + \triangle I = 150$

$x_1[I^n, p^n] = x_1^n = \frac{\alpha I^n}{p_1^n} = \frac{0.5 \times 150}{2} = 37.5$

$x_2[I^n, p^n] = x_2^n = \frac{(1-\alpha)I^n}{p_2^n} = \frac{0.5 \times 150}{1} = 75$

$x_1[I^o, p^n] = \frac{\alpha I^o}{p_1^n} = \frac{0.5 \times 100}{2} = 25$

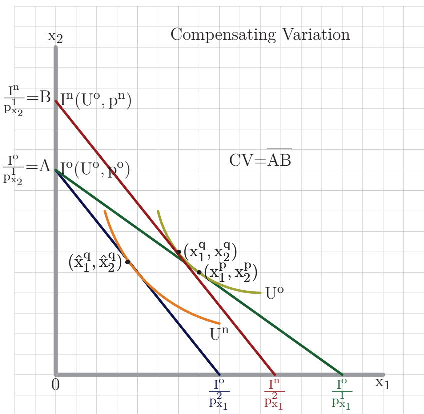
$x_2[I^o, p^n] = \frac{(1-\alpha)I^o}{p_2^n} = \frac{0.5 \times 100}{1} = 50$

Substitution Effect $= x_1[I^o, p^o] - x_1[I^n, p^n]@U = U^o = 50 - 37.5 = 12.5$

Income Effect $= x_1[I^n, p^n] - x_1[I^o, p^n]@U = U^n = 37.5 - 25 = 12.5$

2.7.5. Welfare

2.7.5.1. *Compensating Variation*

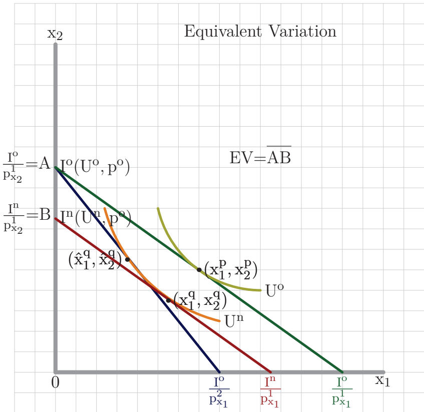


With an income $\mathbf{I}^o$ and price vector $\mathbf{p}^o$, an individual achieves utility $\mathbf{U}^o$ consuming $\{x_1^p, x_2^p\}$. Price of good 1 changes from $p_{x_1}^1$ to $p_{x_1}^2$. Price of good 2 remains unchanged. The consumer is now **worse off** due to this price rise and will now achieve utility $\mathbf{U}^n$ consuming $\{\hat{x}_1^q, \hat{x}_2^q\}$. At new relative prices, to achieve utility $\mathbf{U}^o$, the consumer requires income $\mathbf{I}^n > \mathbf{I}^o$ ($\because \mathbf{U}^o > \mathbf{U}^n$) The rise in income $\triangle M = \mathbf{I}^o$ - $\mathbf{I}^n$ is the money that the consumer is paid to COMPENSATE for a price rise. This is called COMPENSATING VARIATION

$$\therefore CV = \mathbb{I}^o(\mathbb{U}^n, p^n) - \mathbb{I}^n(\mathbb{U}^o, p^n)$$
$$= \mathbb{I}^o(\mathbb{U}^o, p^o) - \mathbb{I}^n(\mathbb{U}^o, p^n)$$

Fig. 2.9.: Compensating Variation

2.7.5.2. *Equivalent Variation*



With an income $\mathbf{I}^o$ and price vector $\mathbf{p}^o$, an individual achieves utility $\mathbf{U}^o$ consuming $\{x_1^p, x_2^p\}$. Price of good 1 changes from $p^1_{x_1}$ to $p^2_{x_1}$. Price of good 2 remains unchanged. The consumer is now **worse off** due to this price rise and will now achieve utility $\mathbf{U}^n$ consuming $\{\hat{x}_1^q, \hat{x}_2^q\}$. At original relative prices, to achieve utility $\mathbf{U}^n$, the consumer requires income $\mathbf{I}^n < \mathbf{I}^o$ ($\because \mathbf{U}^n < \mathbf{U}^o$) The fall in income $\triangle M = \mathbf{I}^o$ - $\mathbf{I}^n$ is the money that the consumer is willing to pay to AVOID a price change. This is called EQUIVALENT VARIATION

$$\begin{aligned}\therefore EV &= \mathbb{I}^n(\mathbb{U}^n, p^o) - \mathbb{I}^o(\mathbb{U}^o, p^o)\\ &= \mathbb{I}^n(\mathbb{U}^n, p^o) - \mathbb{I}^o(\mathbb{U}^n, p^n)\end{aligned}$$

Fig. 2.10.: Equivalent Variation

2.7.5.3. *Computing Compensating and Equivalent Variation*

(1) Let Utility be $U = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$.
(2) Income is spent on x_1 & x_2. $\therefore I = p_1 x_1 + p_2 x_2$.
(3) $MU_{x_1} = \frac{1}{2}[\frac{x_2}{x_1}]^{\frac{1}{2}}$
(4) $MU_{x_2} = \frac{1}{2}[\frac{x_1}{x_2}]^{\frac{1}{2}}$
(5) $\because \frac{MU_{x_1}}{MU_{x_2}} = \frac{p_1}{p_2} \Rightarrow \frac{x_2}{x_1} = \frac{p_1}{p_2} \Rightarrow p_1 x_1 = p_2 x_2$
(6) $\therefore x_1 = \frac{I}{2p_1}$ & $x_2 = \frac{I}{2p_2}$.
(7) Let I = 100; $[p_1, p_2] = [1,1] \Rightarrow [x_1, x_2] = [50,50]$.
(8) Change prices to [2,1] $\Rightarrow$ new bundle [25,50].
(9) Income @ new prices [2,1] required to achieve old utility @ [50,50] is $[\frac{I}{2\times 2}]^{\frac{1}{2}}$ $[\frac{I}{2\times 1}]^{\frac{1}{2}} = 50^{\frac{1}{2}} 50^{\frac{1}{2}} \Rightarrow I = 100\sqrt{2}$ = compensating variation = 100 - $100\sqrt{2}$ $\approx$ -41
(10) To obtain NEW bundle [25,50] @ prices [1,1] the consumer would require an income $[\frac{I}{2\times 1}]^{\frac{1}{2}}$ $[\frac{I}{2\times 1}]^{\frac{1}{2}} = 25^{\frac{1}{2}} 50^{\frac{1}{2}} \Rightarrow I = 50\sqrt{2}$ equivalent variation = 50 $\sqrt{2}$ - 100 $\approx$ -30

2.7.5.4. *Hicksian Demand and Welfare*

Figure 2.11: a shows the hicksian demand $h_1(p_1, \overline{p}_{-1}, \mathbb{U}^1)$ where price of commodity 1 is p_1, maintaining all other $\overline{p}_{-1}$ prices constant @ utility $\mathbb{U}^1$. Now

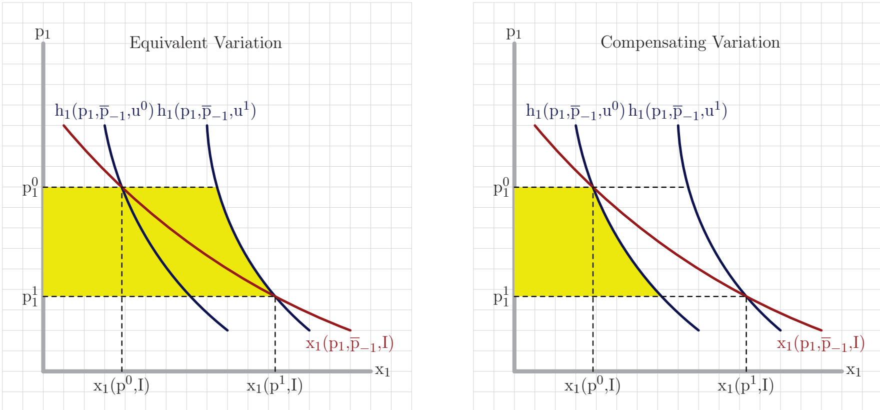
$h_1(p, \mathbb{U}) = \frac{\partial \mathrm{e}(\mathrm{p},\mathbb{U})}{\partial p_1}$ and $\mathbb{I} = \mathrm{e}(\mathrm{p}^1, \mathbb{U}^1) = \mathrm{e}(\mathrm{p}^0, \mathbb{U}^0)$. The equivalent variation is given by $\mathbb{I}(\mathbb{U}^n, p^o) - \mathbb{I}(\mathbb{U}^o, p^o)$, while the compensating variation is given by $\mathbb{I}(\mathbb{U}^n, p^n) - \mathbb{I}(\mathbb{U}^o, p^n)$. If one changes only price of commodity **1** out of a bundle of n commodities, then one can draw the conventional diagram of change in consumer surplus with hicksian demand curves depicting the compensating and equivalent variation on it.

The area to the left of the hicksian demand curve $h_1(p_1, \overline{p}_{-1}, \mathbb{U}^1)$ between prices p^0 and p^1 show the consumer surplus. $\therefore$ CS $= \int_{p^0}^{p^1} h_1(p_1, \overline{p}_{-1}, \mathbb{U}^1)$. If the price changes from p^0 and p^1 the consumer is better off or worse off depending on whether $p^0 < / > p^1$. Thus the equivalent variation will equal the change in consumer surplus at the new utility level $\mathbb{U}^1$ and the compensating variation will equal the change in consumer surplus at old utility level $\mathbb{U}^0$

$$\begin{aligned} \mathrm{EV}(\mathrm{p}^0, \mathrm{p}^1, \mathbb{I}) &= \mathbb{I}(\mathbb{U}^1, p^0) - \mathbb{I}(\mathbb{U}^0, p^0) \\ &= \mathbb{I}(\mathbb{U}^1, p^0) - \mathbb{I}(\mathbb{U}^1, p^1) \\ &= \int_{p_1^1}^{p_1^0} h_1(p_1, \overline{p}_{-1}, \mathbb{U}^1) \end{aligned} \tag{2.85}$$

Figure 2.11: b shows the hicksian demand $h_0(p_1, \overline{p}_{-1}, \mathbb{U}^0)$ where price of commodity 1 is p_1, maintaining all other $\overline{p}_{-1}$ prices constant @ utility $\mathbb{U}^0$.

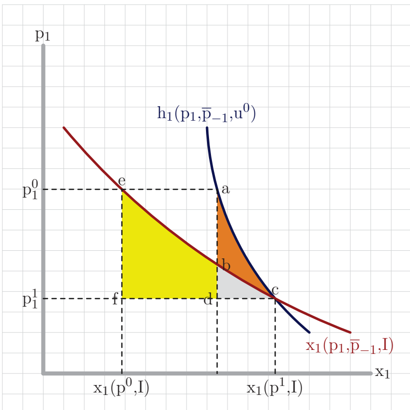
$$\begin{aligned} \mathrm{CV}(\mathrm{p}^0, \mathrm{p}^1, \mathbb{I}) &= \mathbb{I}(\mathbb{U}^1, p^1) - \mathbb{I}(\mathbb{U}^0, p^1) \\ &= \mathbb{I}(\mathbb{U}^0, p^0) - \mathbb{I}(\mathbb{U}^0, p^1) \\ &= \int_{p_1^1}^{p_1^0} h_1(p_1, \overline{p}_{-1}, \mathbb{U}^0) \end{aligned} \tag{2.86}$$



a: Equivalent Variation b: Compensating Variation

Fig. 2.11.: Welfare Measures: Marshallian & Hicksian Demand

Figure 2.12 shows the difference in welfare based on the Hicksin and Marshallian demand functions.



$\text{CS}_M = \int_{p^0}^{p^1} x_1(p_1, \overline{p}_{-1}, \mathbb{I})$

$\text{CS}_H = \int_{p^0}^{p^1} h_1(p_1, \overline{p}_{-1}, \mathbb{U}^1)$

The difference in welfare is CS_M-CS_H.

The dead weight loss in the marshallian case is $bcd + bdfe$ while the loss in the hicksian case is $abc + bcd$

Fig. 2.12.: Error in Welfare Measures: Marshallian & Hicksian Demand

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3

Programs for a Stylised CGE Model and Ramsey Model

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Contents

3.1. 4x4x3 Stylised GAMS Model

In this chapter we outline the basic structure of a CGE model in its most elementary form. This structure can be easily modified to build more complex models suited to various policy requirements.

*CEEAplA

The illustration consists of a **4** sectors x **4** commodities x **3** households closed country model with no external trade, replete with different taxes such as excise, sales, labour and tax on capital used as a proxy for corporate income taxes.

The chapter outline is as follows. The basic building block of an CGE model, the SAM is explained in brief. The detailed explanation is reserved for Chapter 4. The data set with the Input-Output (IO) table follows with the various details about employment and capital use by sectors, tax rates on commodities, demand system of households, transfers and taxes on households, along with their endowments. This is followed by the calibration procedure and the calibrated values with the GAMS code concluding.

3.2. Structure of a SAM

Table 3.1 shows the major players in the model. As one reads across a row, it shows the incomes for that player. As one moves down a column, it shows the expenditure incurred by that player.

Player	Source of Income	Items of Expenditure
A: Activities	Selling commodities to other sectors B, Exports H and Export subsidies if any from government F	Paying intermediate goods B, wages C, rental of capital D and indirect taxes to Government F
B: Commodities	Selling commodities for production A, consumption to households E, Government F and investment G	Other commodities in production A and Imports H
C: Labour	wages C	spends on Labour income of Households E
D: Capital	rentals D	spend on capital payments to Households E and capital taxes to government F
E: Households	earn from wages C and rental income D	spend on commodities B, pay taxes (direct) to government F and savings G
F: Government	Indirect taxes from activities A, capital taxes from Capital D, direct taxes from Households E, and sales taxes from Government F, investment G and Rest of the World H	Export subsidy to Activity A, own consumption F, transfers to households E, sales taxes on its consumption F, Savings G and foreign reserves H
G: Investment	Private E and government Savings E	investment goods B, capital inflow to households E and sales taxes to government F
H: Rest of the World	imports of commodities B and reserve accumulation by government F	exports A, sales taxes to government F

Source: author

3.3. 4 x 4 x 3 Model Data Set

3.3.1. IO Table

The IO table shows the input of commodity i in sector j, denoted by a_{ij}. As we move along a row, it shows the consumption of that commodity by different agents like sectors 1-4, which comprise of intermediate consumption IUse and final demand FD. In more complicated models, we will have Government consumption, Change in Stocks, Exports and Imports as a part of final demand. Some data might also differentiate the inputs a_{ij} by source of origin namely domestic d_{ij} and imported m_{ij} such that $a_{ij} = d_{ij} + m_{ij}$. One can use this to have imperfect substitutes assumption often called the Armington function for intermediate inputs. This is especially true of big countries with own sources of oil that import and export oil simultaneously. Also the quality of crude is important for the final composition that results after cracking and there is a limit to which one can alter the composition of the output. This is another reason to have 'homogeneous' commodities from different sources. The consumption is in physical units but normally scaled up after converting the price to **1€**. So a cow worth €1000 producing 10 liters of milk a day at 0.3€ for 365 days gives an output of €1095 is converted to 1000 cows at 1€ each produce 1095 litres of milk at €1. Ideally horizontally the units are all different like metric tons, barrels of oil, mega watts of electricity, tons of cement which are homogenised to value using price. As we move down the column the elements are accounting values in monetary terms. Here the taxes and subsidies, trade and transport margins come into play. A cow costing €1000 can get a farmer a 50% subsidy thus costing only €500, but still it takes one cow to produce an average of 10 litres of milk.

In table 3.2 commodity **1** is consumed in sectors 1, 2, 3, 4 respectively as 50, 10, 10, 40 implying an intermediate use of 110 and final demand of 93.5 leading to a total consumption of 203.5 units. Financially sector **1** incurs an expense on commodities 1, 2, 3, 4 respectively of 50, 10, 10, 30. Pays taxes worth 9.5 (excise:0.1x10+0.2x10+0.1x30=6 and sales: 0.1x10+0.1x10+0.05x30=3.5, no tax on internal consumption of its own produce). It pays 60 to labour and 6 as payroll taxes at 10%, 20 as rental and 8 as capital taxes at 40%. The value of output is value of intermediate inputs which equals 100 (50+10+10+30) and value added which equals 103.5 equalling 203.5€. Similar procedure is adopted for all sectors.

Table 3.1.: Structure of a SAM

	A	B	C	D	E	F	G	H	I
A		Domestic Commodity Supplies				Export Subsidy		Exports	
B	Intermediate Inputs				Private Consumption	Government Consumption	Investment		
C	Wages								
D	Rentals								
E			Labour Income	Capital Income		Transfers	Capital Inflow		
F	Indirect Taxes			Capital Taxes	Direct Taxes	Sales Taxes	Sales Taxes	Sales Taxes	
G					Private Saving	Government Saving			
H		Imports				Reserve Accumulation			
I	Total Costs	Total Absorption	Factor Income	Factor Income	Household Income	Government Expenditure	Investment	Foreign Exchange Inflow	

Source: Author and [Dervis *et al.* (1982)]

Key

A: Activities
B: Commodities
C: Labour
D: Capital
E: Households
F: Government
G: Investment
H: Rest of the World
I: Total

Table 3.2.: Stylised Input-Output Table

	1	2	3	4	IUse	FD	Q
1	50	10	10	40	110	93,5	203,5
2	10	60	40	20	130	211,5	341,5
3	10	50	80	20	160	140,5	300,5
4	30	40	40	80	190	188,0	378,0
NIT	9,5	21,5	14,5	12,0	57,5	633,5	
L	60	80	60	100	300		
K	20	60	40	80	200		
tL	6	8	6	10	30		
tk	8	12	10	16	46		
VA	103,5	181,5	130,5	218,0	633,5		
VQ	203,5	341,5	300,5	378,0			

Source: Author

3.3.2. Tax Rates

The hypothetical tax rates on different commodities are mentioned in table 3.3. tl is tax on labour or payroll tax to employ an individual, tk is tax on rental or corporate tax on accounting profits, te is excise tax paid by industries on consumption of intermediate goods and ts is sales tax rate on consumption of any good by any agent, industries or households.

Table 3.3.: Stylised Tax Rates

	1	2	3	4
tl	0,10	0,10	0,10	0,10
tk	0,40	0,20	0,25	0,20
te	0,00	0,10	0,20	0,10
ts	0,05	0,10	0,10	0,05

Source: Author

3.3.3. Final Demand

The stylised consumption of 3 households is shown in table 3.4. The total physical consumption of households for each commodity matches that shown under final demand FD in table 3.2. The taxes paid on consumption tax_C equals 0.05x40+0.1x70+0.1x50+0.05x40=16.

Table 3.4.: Stylised Household Demand

	H1	H2	H3	total
1	40	30	23,5	93,5
2	70	70	71,5	211,5
3	50	50	40,5	140,5
4	40	40	98,0	188,0
tax_C	16	16	17,275	49,275
tax_I	17	16	17	50
TExp	216	216	250,775	682,775

Source: Author

3.3.4. Endowments and Transfers

The table 3.5 shows the stylised endowment of labour and capital for each household. H1 is the richest as it has the most capital while H3 is poorest as it has the least amount of capital. The income tax rates are tD 10% for all but the transfer rates TR are different for each household.

Table 3.5.: Stylised Household Endowments and Transfers

	H1	H2	H3	total
LE	50	100	150	300
KE	120	60	20	200
tD	0,1	0,1	0,1	
TR	0,27	0,31	0,42	

Source: Author

3.3.5. Taxes

The total tax collection from various sources, intermediate use or IO is 57.5, factor use (labour:30, capital:46) is 76, household consumption is 49.275 and income tax is 50, leading to a total of 232.775. For consistency the value added 633.5 equals the expenditure 633.5

Table 3.6.: Stylised Tax Collection

Head	IO	F	C	I
Value	57,5	76,0	49,275	50,0
Total	232,775			

Source: Author

3.4. Calibration

In the base case, we have modified the system to such that all prices are 1. So when one solves the system of non-linear equations from an arbitrary starting point, it must converge to an equilibrium value and replicate the original SAM. Since the parameters for the production function and demand system are not estimated through an econometric procedure, they need to have values that replicate the base values when prices equal 1. Even if the parameters are estimated using econometric methods there is no guarantee that using them will replicate the original values. The next best possible approach is used. One would use the parameters estimated from econometric studies and obtain the other parameters to replicate the original values. This process is called calibration.

3.4.1. Production Function

Here we illustrate the calibration procedure for a Cobb-Douglas production function. A similar approach can be applied for other production functions.

3.4.1.1. *Cobb-Douglas*

The endogenous variables are wage rate w and return to capital r, labour use L, capital use K and value added VA. tl, tk are exogenous parameters that are policy variables, the tax on labour and capital respectively. The parameters of the production function are δ and ϕ. So we calculate the values of δ and ϕ, given the base case values of other variables. Thus in the simulations, when these values are used, we will replicate the base case.

$$VA(L,K) = \phi[L^{\delta}K^{(1-\delta)}]$$

$$\delta = \frac{w(1+tl)L}{[w(1+tl)L + r(1+tk)K]}$$

$$\phi = \frac{[w(1+tl)L + r(1+tk)K]}{L^{\delta}K^{(1-\delta)}}$$

3.4.2. Calibrated Values

Using equation 3.1 the calibrated values are as shown in table 3.7.

Table 3.7.: Calibration Stylised Production Function

	1	2	3	4
δ	0,7021	0,5500	0,5690	0,5340
ϕ	2,3928	2,5823	2,5904	2,4189

Source: Author

3.4.3. Demand System

Similar to the production function, the demand system needs to be calibrated. We assume a Cobb-Douglas demand function as it is easy to illustrate. The endogenous variables are x_i, consumption of commodity i, prices p_i and income $\mathbb{I}$, while exogenous variables are ts_i, sales tax on commodity i. The parameter α needs to be calibrated to replicate the base case demands as shown in the SAM.

3.4.3.1. *Cobb-Douglas*

$$\mathbb{U} = \prod_{i=1}^{n} x_i^{\alpha_i} \quad | \sum_{i=1}^{n} \alpha_i = 1$$
$$\alpha_i = \frac{p_i(1 + ts_i)x_i}{\mathbb{I}}$$

3.4.4. Calibrated Values

The equation 3.1 shows formulae to obtain calibrated values of the Cobb-Douglas demand system for any values of base case variables. The calibrated values are as shown in table 3.8

Table 3.8.: Calibration Stylised Demand System

	H1	H2	H3
α_1	0,1944	0,1458	0,0984
α_2	0,3565	0,3565	0,3136
α_3	0,2546	0,2546	0,1776
α_4	0,1944	0,2431	0,4103

Source: Author

3.4.5. Leontief Fixed IO-Coefficients

The technology follows a Leontief IO structure with fixed coefficients to produce output. One needs to calibrate that too, and one obtains these coefficients by dividing the use of each commodity i in sector j by the value of output of sector j, as shown in table 3.9.

Table 3.9.: Calibration Stylised Leontief Technology

	1	2	3	4
1	0,2457	0,0293	0,0333	0,1058
2	0,0491	0,1757	0,1331	0,0529
3	0,0491	0,1464	0,2662	0,0529
4	0,1474	0,1171	0,1331	0,2116
VA	0,5086	0,5315	0,4343	0.5767

Source: Author

3.5. GAMS Code

For introduction to programming in GAMS, refer [Rosenthal (2006)]. We give an illustrative GAMS code for solving a simple general equilibrium model whose data was shown in the earlier part of this chapter. One can proceed in a similar manner to build other models with government, trade, multi-country or any other format that one chooses. The basic procedure of creating a SAM and calibration remains the same.

```
$offlisting offsymlist offsymxref offuellist offuelxref
*$onupper

Sets
i commodities /c1, c2, c3, c4/
h households  /h1, h2, h3/
f factors     /l, k/
t taxes       /e, s/
tt taxes transfers /ty, tr/;

alias (i,j);

Table a(i,j) use of commodity i in sector j
         c1      c2      c3      c4
c1       50      10      10      40
c2       10      60      40      20
c3       10      50      80      20
c4       30      40      40      80 ;

Table fu(f,j) use of factor f in sector j
         c1      c2      c3      c4
l        60      80      60      100
k        20      60      40      80 ;

Table fe(f,h) endowment of factor f in household h
         h1      h2      h3
l        50      100     150
k        120     60      20      ;

Table tf(f,j) tax on use of factor f in sector j
         c1      c2      c3      c4
```

```
l        0.1     0.1     0.1     0.1
k        0.4     0.2     0.25    0.2 ;

Table ti(t,j) tax on int use of commodity i in sector j
         c1      c2      c3      c4
e        0.0     0.1     0.2     0.1
s        0.05    0.1     0.1     0.05 ;

Table th(tt,h) income tax and transfers to household h
         h1      h2      h3
ty       0.1     0.1     0.1
tr       63      72      97.775     ;

Table fd(i,h) demand of commodity i in household h
         h1      h2      h3
c1       40      30      23.5
c2       70      70      71.5
c3       50      50      40.5
c4       40      50      98.0     ;


Parameter taxi(i,j) tax rate on use of cdty i in sector j;
taxi(i,j) = sum(t, ti(t,j));
taxi(i,i) = 0;

Parameter nit0(i) net indirect tax paid by sector j;
nit0(i) = sum(j, taxi(i,j)*a(j,i));

Parameter ft0(i) factor tax paid by sector j on use of factor i;
ft0(j) = sum(f, tf(f,j)*fu(f,j));

Parameter va0 value added in sector j;
va0(j) = nit0(j) + sum(f, fu(f,j)*(1+tf(f,j)));

Parameter vq0 value of output in sector j;
vq0(j) = va0(j) + sum(i, a(i,j));

Parameter fd0(i) final demand of good i;
fd0(i) = sum(h, fd(i,h));

Parameter q0 output in sector j;
q0(i) = fd0(i) + sum(j, a(i,j));

Parameter
delta(j) distribution parameter in va for sector j
phi(j)   scale parameter in va for sector j
eta(j)   elasticity of substitution in sector j;

delta(j) = ((1+tf('l',j))*fu('l',j))/sum(f,(1+tf(f,j))*fu(f,j));
phi(j) = va0(j)/(fu('l',j)**delta(j)*fu('k',j)**(1-delta(j)));

Parameter
y0(h) income of household h;
y0(h) = sum(i,(1+ti('s',i))*fd(i,h));
```

```
Parameter ct0(i) consumption tax paid by household h for using commodity i;
ct0(i) = sum(h, ti('s',i)*fd(i,h));

Parameter it0(h) income tax paid by household h;
it0(h) = th('ty',h)*sum(f, fe(f,h));

Parameter tt0 total tax collected from all sources;
tt0 = sum(h, it0(h)) + sum(i, ct0(i)+nit0(i)+ft0(i));

*calculating the transfer rate
th('tr',h) = th('tr',h)/tt0;

Parameter
alpha(i,h) demand coefficient for good i for consumer h;
alpha(i,h) = (1+ti('s',i))*fd(i,h)/y0(h);

Parameter
io(i,j) input output coefficient
vacoef  fixed coefficient for value added;
io(i,j) = a(i,j)/vq0(j);
vacoef(j) = va0(j)/vq0(j);

display y0,taxi,nit0,ft0,ct0,it0,tt0,va0,vq0,fd0,q0,delta,phi,alpha,io,vacoef;

equations
obj          'maximize fictitious obj z=0'
grinc(h)     'gross income of household h'
netinc(h)    'net income of household h'
cdemand(i,h) 'demand of commodity i by household h'
prva(i)      'price of value added in sector j'
numfd(i)     'numerator in factor demand equation'
denfd(i)     'denomiator in factor demand equation'
lfdd(j)      'labour demand by sector j'
kfdd(j)      'capital demand by sector j'
costl(j)     'labour cost in sector j'
costk(j)     'capital cost in sector j'
expisec(j)   'total expenditure on good i by sector j'
indtax(i)    'indirect tax per unit output paid by sector j for using i'
totnit(i)    'total indirect tax paid by sector j'
factax(i)    'factor tax paid by sector j for using f'
constax(i)   'consumption tax paid by house h for using i'
incmtax(h)   'income tax paid by household h'
totaltax     'total tax collection'
iuse(i)      'intermediate use of commodity i'
fuse(i)      'final use of commodity i'
demand(i)    'demand for commodity i'
voutput(i)   'value of output of commodity i'
ssddbal(i)   'supply demand balance for commodity i'
lbal         'balance for labour demand'
kbal         'balance for capital demand'
transfers    'total transfers'
deficit      transfers equals taxes

Positive variables p(i), q(i), pf(f), nit(i), nitp(i), ft(i), ct(i), it(h);
Positive variables gi(h), ni(h), cd(i,h), iu(i), finu(i), va(i), facd(f,i);
```

```
Positive variables dd(i), ttax, ttr, pva(j);
Positive variables fdnum(j), fdden(j), labd(j), kapd(j);
Positive variables secui(j), luse(j), kuse(j);
variables z;

obj..               z =e= 0;
grinc(h)..       gi(h) =e= sum(f,pf(f)*fe(f,h));
netinc(h)..      ni(h) =e= (1-th('ty',h))*gi(h) + th('tr',h)*ttr;
cdemand(i,h).. cd(i,h) =e= alpha(i,h)*ni(h)/(p(i)*(1+ti('s',i)));
fuse(i)..      finu(i) =e= sum(h,cd(i,h));
iuse(i)..        iu(i) =e= sum(j,io(i,j)*q(j));
demand(i)..      dd(i) =e= finu(i)+iu(i);
prva(j)..       pva(j) =e= vacoef(j);
numfd(j)..    fdnum(j) =e= delta(j)*pf('k')*(1+tf('k',j));
denfd(j)..    fdden(j) =e= (1-delta(j))*pf('l')*(1+tf('l',j));
lfdd(j)..      labd(j) =e= pva(j)/phi(j)*(fdnum(j)/fdden(j))**(1-delta(j));
kfdd(j)..      kapd(j) =e= pva(j)/phi(j)*(fdden(j)/fdnum(j))**delta(j);
expisec(j)..  secui(j) =e= sum(i,io(i,j)*p(i));
indtax(j)..    nitp(j) =e= sum(i,io(i,j)*p(i)*taxi(j,i));
costl(j)..     luse(j) =e= pf('l')*labd(j)*(1+tf('l',j));
costk(j)..     kuse(j) =e= pf('k')*kapd(j)*(1+tf('k',j));
voutput(j)..      p(j) =e= secui(j)+nitp(j)+luse(j)+kuse(j);
totnit(i)..     nit(i) =e= nitp(i)*q(i);
factax(j)..      ft(j) =e= (pf('l')*labd(j)*tf('l',j)+pf('k')*kapd(j)*tf('k',j))*q(j);
constax(j)..     ct(j) =e= p(j)*finu(j)*ti('s',j);
incmtax(h)..     it(h) =e= th('ty',h)*gi(h);
totaltax..        ttax =e= sum(j,nit(j)+ft(j)+ct(j))+sum(h,it(h));
ssddbal(i)..     dd(i) =e= q(i);
lbal..      sum(h,fe('l',h)) =e= sum(j,labd(j)*q(j));
kbal..      sum(h,fe('k',h)) =e= sum(j,kapd(j)*q(j));
transfers..    ttax =e= sum(h,th('tr',h))*ttr;
deficit..       ttr =e= ttax;

p.l(i)     = 1.0;
q.l(i)     = q0(i);
pf.l(f)    = 1.0;
ttax.l     = tt0;
gi.l(h)    = sum(f,pf.l(f)*fe(f,h));
ni.l(h)    = (1-th('ty',h))*gi.l(h) + th('tr',h)*ttax.l;
cd.l(i,h)  = alpha(i,h)*ni.l(h)/(p.l(i)*(1+ti('s',i)));

finu.l(i)  = fd0(i);
iu.l(i)    = sum(j,io(i,j)*q.l(j));
dd.l(i)    = finu.l(i)+iu.l(i);

pva.l(j)   = vacoef(j);
fdnum.l(j) = delta(j)*pf.l('k')*(1+tf('k',j));
fdden.l(j) = (1-delta(j))*pf.l('l')*(1+tf('l',j));
labd.l(j)  = pva.l(j)/phi(j)*(fdnum.l(j)/fdden.l(j))**(1-delta(j));
kapd.l(j)  = pva.l(j)/phi(j)*(fdden.l(j)/fdnum.l(j))**delta(j);

secui.l(j) = sum(i,io(i,j)*p.l(i));

luse.l(j)  = pf.l('l')*labd.l(j)*(1+tf('l',j));
kuse.l(j)  = pf.l('k')*kapd.l(j)*(1+tf('k',j));
```

```
nitp.l(j)  = sum(i,io(i,j)*p.l(i)*taxi(j,i));
ft.l(j)    = (pf.l('l')*labd.l(j)*tf('l',j)+pf.l('k')*kapd.l(j)*tf('k',j))*q.l(j);
ct.l(j)    = p.l(j)*finu.l(j)*ti('s',j);
it.l(h)    = th('ty',h)*gi.l(h);
nit.l(i)   = nitp.l(i)*q.l(i);
ttax.l     = sum(j,nit.l(j)+ft.l(j)+ct.l(j))+sum(h,it.l(h));
ttr.l      = ttax.l;

* setting numeraire
pf.fx('l') = 1.0;

* setting lower bounds
p.lo(i)     = 0.001*p.l(i)    ;
q.lo(i)     = 0.001*q.l(i)    ;
pf.lo(f)    = 0.001*pf.l(f)   ;
gi.lo(h)    = 0.001*gi.l(h)   ;
ni.lo(h)    = 0.001*ni.l(h)   ;
cd.lo(i,h)  = 0.001*cd.l(i,h) ;
finu.lo(i)  = 0.001*finu.l(i) ;
iu.lo(i)    = 0.001*iu.l(i)   ;
dd.lo(i)    = 0.001*dd.l(i)   ;
pva.lo(j)   = 0.001*pva.l(j)  ;
fdnum.lo(j) = 0.001*fdnum.l(j);
fdden.lo(j) = 0.001*fdden.l(j);
labd.lo(j)  = 0.001*labd.l(j) ;
kapd.lo(j)  = 0.001*kapd.l(j) ;
secui.lo(j) = 0.001*secui.l(j);
luse.lo(j)  = 0.001*luse.l(j) ;
kuse.lo(j)  = 0.001*kuse.l(j) ;
nit.lo(j)   = 0.001*nit.l(j)  ;
ft.lo(j)    = 0.001*ft.l(j)   ;
ct.lo(j)    = 0.001*ct.l(j)   ;
it.lo(h)    = 0.001*it.l(h)   ;
ttax.lo     = 0.001*ttax.l    ;
ttr.lo      = 0.001*ttr.l     ;

*Model cge443 /all/;
Model cge443
/
obj
grinc
netinc
cdemand
prva
numfd
denfd
lfdd
kfdd
costl
costk
expisec
indtax
factax
constax
```

```
incmtax
totaltax
iuse
fuse
demand
voutput
totnit
ssddbal
*lbal
kbal
transfers
deficit

/;

*option iterlim = 0;
*option nlp = pathnlp;

Solve cge443 using nlp minimizing z;

scalar walras;
walras = sum(h,fe('l',h)) - sum(j,labd.l(j)*q.l(j));

display walras;

display p.l,pf.l,q.l,ttax.l,ttr.l,nit.l,ft.l,ct.l,it.l,cd.l, gi.l, ni.l;
```

3.6. Tools for Dynamic Models

In this section [a] we give a brief outline of the tools used to solve some of the dynamic models like finite and infinite horizon Ramsey models using the method of Value Function Iterations. The R code [R Development Core Team (2009)] is given with the objective of linking the theory to practice and to enable the reader to experiment with various parameters.

3.6.1. Chebyshev Regression

The problem is to find an approximation $\tilde{f}(x)$ of a function $f(x)$, where $x \in [lb, ub]$, where lb is the lower bound and ub is the upper bound, for the domain of the function $f(x)$. Since we are approximating $f(x)$ with $\tilde{f}(x)$, we will choose a set of points called collocation points at which $\tilde{f}(x) = f(x)$.

Chebyshev polynomials are one approach to approximate the required function and exhibit an important property of orthogonality. Unfortunately Chebyshev polynomials have a range and domain of [-1,1]. So to approximate any function in an arbitrary range $[lb, ub]$, we first need to convert it to [-1,1]. Before proceeding further we first define the Chebyshev polynomials and state its important property

[a]for details [Heer and Maußner (2005)]

of orthogonality. Then we proceed to outline the method of approximation of the desired function.

A Chebyshev polynomial of order i defined over the domain $x \in [-1, 1]$ is given by

$$T_i(x) = \cos(i \cos^{-1} x) \tag{3.1}$$

An easier expression for recursively computing all orders of the Chebyshev polynomial for a given x exists and is given by

$$\begin{aligned} T_{i+1}(x) &= 2xT_i(x) - T_{i-1}(x) \\ T_0(x) &= 1 \\ T_1(x) &= x \end{aligned}$$

As mentioned before Chebyshev polynomials belong to a class of polynomial defined as orthogonal polynomials. The classical definition of an orthogonal polynomial is as follows. Given an interval $x \in [x_1, x_2]$ where $x_1 = -\infty$ and $x_2 = \infty$ and the existence of a weighting function $w(x) \mapsto \mathbb{R}$ such that $\int_{x_1}^{x_2} f(x)w(x)dx$ is finite, we have for any two polynomials $f(x)$ and $g(x)$, define $\langle f, g\rangle = \int_{x_1}^{x_2} f(x)g(x)w(x)dx$ such that $\langle f, g\rangle = 0$

For the Chebyshev polynomials the weighting function is $w(x) = \frac{1}{\sqrt{(1-x^2)}}$ and for any two polynomials $T_i(x)$ and $T_j(x)$ $i \neq j$, we have

$$\langle T_i(x), T_j(x)\rangle = \begin{cases} \int_{x_1}^{x_2} T_i(x)T_j(x)w(x)dx = 0 & i \neq j \\ \int_{x_1}^{x_2} T_i(x)T_j(x)w(x)dx = \pi & i = j = 0 \\ \int_{x_1}^{x_2} T_i(x)T_j(x)w(x)dx = \frac{\pi}{2} & i = j \neq 0 \end{cases} \tag{3.2}$$

For the discrete case we have

$$\langle T_i(x), T_j(x)\rangle = \begin{cases} \sum_{k=1}^{n} T_i(x_k)T_j(x_k) = 0 & i \neq j \\ \sum_{k=1}^{n} T_i(x_k)T_j(x_k) = n & i = j = 0 \\ \sum_{k=1}^{n} T_i(x_k)T_j(x_k) = \frac{n}{2} & i = j \neq 0 \end{cases} \tag{3.3}$$

The Chebyshev polynomial $T_n(x)$ has n distinct roots in $[-1, 1]$. These are the points $\{x_i\}_{i=1}^{n}$, where $T_n(x) = 0$, and are given by

$$x_i = -\cos\left[\frac{(2i-1)\pi}{2n}\right] \tag{3.4}$$

So reverting back to the original problem of approximating $f(x)$ with $\tilde{f}(x) = \sum_{i=0}^{n} \theta_i T_i(x)$, where we have to compute the values of the $n+1$ unknown θ_is we do the following

1: fix the order of Chebyshev polynomials to $n + 1 \because$ we need $n + 1$ θ_is
2: we need to fix the collocation points where the approximation $\tilde{f}(x)$ equals the original function $f(x)$. Collocation points will be the roots of the Chebyshev polynomial of the highest order $n + 1$.

3: obtain the values of the roots for the Chebyshev polynomial of order $m = n + 1$ using $z_i = -\cos\left(\frac{(2i-1)\pi}{2m}\right)$, where $z \in [-1, 1]$
4: convert the range of $z \in [-1, 1]$ to $f(x)$ $x \in [lb, ub]$, using the formula

$$x_i = lb + (ub - lb)\frac{z+1}{2} \tag{3.5}$$

5: We have the following

$$\tilde{f}(z_i) = f(z_i)$$

Substituting for $\tilde{f}(z_i)$, we have

$$\sum_{k=0}^{n} \theta_k T_k(z_i) = f(z_i)$$

Multiply both sides by $T_j(z_i)$ for some $0 \leq j \leq n$

$$\sum_{k=0}^{n} \theta_k T_k(z_i) T_j(z_i) = T_j(z_i) f(z_i)$$

Summing across all m roots, z_i

$$\sum_{i=1}^{m} \sum_{k=0}^{n} \theta_k T_k(z_i) T_j(z_i) = \sum_{i=1}^{m} T_j(z_i) f(z_i)$$

For $j \neq k$, we have the LHS $= 0$ due to orthogonality and what remains is

$$\sum_{i=1}^{m} \theta_j T_j(z_i) T_j(z_i) = \sum_{i=1}^{m} T_j(z_i) f(z_i)$$

for $j = 0$ we have $T_j(z_i) = 1 \forall i$

$$\sum_{i=1}^{m} \theta_0 = \theta_0 m = \sum_{i=1}^{m} T_j(z_i) f(z_i) = \sum_{i=1}^{m} f(z_i)$$

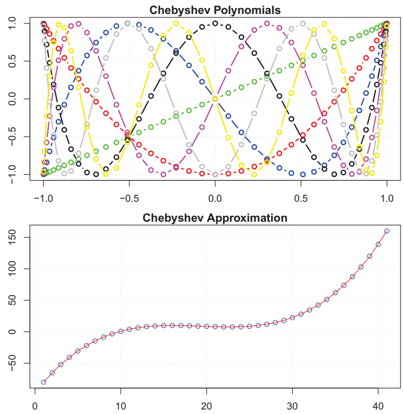
$$\theta_0 = \frac{1}{m} \sum_{i=1}^{m} f(z_i)$$

for $0 < j \leq n$, we have

$$\theta_j \frac{m}{2} = \sum_{i=1}^{m} T_j(z_i) f(z_i)$$

therefore

$$\theta_j = \frac{2}{m} \sum_{i=1}^{m} T_j(z_i) f(z_i)$$



Chebyshev polynomial of order i is defined as $T_i(x) = \cos(i \cos^{-1}(x))$ $x \in [-1, 1]$; is equivalent to $T_{i+1}(x) = 2xT_i(x) - T_{i-1}(x)$ with $T_0(x) = 1$ and $T_1(x) = x$
The top part of the figure shows Chebyshev polynomials upto order 7. A chebyshev polynomial of order k will touch the extremities -1 and 1 at $k+1$ points

Fig. 3.1.: Chebyshev Polynomials and Approximation of $f(x) = -(x^2 - \frac{((x+2)^3)}{4} + 4x - 6)$

3.6.1.1. R *code for Chebyshev Regression*

For an introduction to R, please refer [Verzani (2004)].

```
#ChebyApprx.R but with functions
# steps are as follows
# decide m: number of points
# decide a: number of polynomials to be used for approximation
# given fn : the objective function to approximate like log(x), exp(x)
# given LB: lower bound and UB: upper bound of function
# 1. get z in (-1,1) it has m points
# 2. get T[i,j] 'm' rows and 'a' cols for each polynomial
# 3. get x mapping from (-1,1) of z to (LB,UB) of the function
# 4. evaluate fn @ x[i] i=1,..,m
# 5. obtain cfa[j], j=1,..a for each polynomial using the chebyshev formula
# 6. check if approx function is close to original function

# calculating m points of chebyshev collocation
m <- 40 # no of points
a <- 30 # no of polynomials used for approximation
z <- matrix(nr=m+1,nc=1)
T <- matrix(nr=m+1,nc=a) #T[m x a] m rows, a columns

for (i in 1:(m+1))
  z[i] <- -cos( (pi*(2*i-1))/(2*(m+1)) )

#getT function to obtain the matrix T[m x a] of chebyshev polynomial
getT <- function(z)
{
  k = length(z)
  for (j in 1:k)
  {
    T[j,1] <- 1
```

```
    T[j,2] <- z[j]
  }

  for (i in 3:a)
  for (j in 1:k)
    T[j,i]<-2*z[j]*T[j,i-1]-T[j,i-2]

return(T)
}
# calculating T
T <- getT(z)

LB <- -8 # defining lower bound
UB <- 8 # defining upper bound
x <- matrix(nr=m,nc=1)

#getX function converting from (-1,1) of z in chebyshev
# to (LB,UB) of the original function
getX <- function(z,LB,UB)
{
  k = length(z)
  for (i in 1:k)
    x[i] <- LB + (UB-LB)*(z[i]+1)/2

  return(x)
}

x <-getX(z,LB,UB)

getXA <- function(m,LB,UB)
{
  dx = (UB-LB)/m
  for (i in 1:(m+1))
    xa[i] <- LB + (i-1)*dx

  return(xa)
}

xa <- matrix(nr=(m+1),nc=1)
xa <- getXA(m,LB,UB)

# function to evaluate objective
feval <- function(x)
{
# tricky objective function and gets caught in local minima or maxima
    obj = -(x^2 -((x+2)^3)/4 + 4*x - 6)
#    obj = exp(x)
#    obj = cos(x)
#    obj = max(0,(x-1))
    return(obj)
}

fnx  <- matrix(nr=(m+1),nc=1)
fnxa <- matrix(nr=(m+1),nc=1)
```

```
for (i in 1:(m+1))
 fnx[i] <- feval(x[i])

for (i in 1:(m+1))
 fnxa[i] <- feval(xa[i])

# coefficient vector cfa [a x 1]
cfa <- matrix(nr=a,nc=1)

# checking approximate function chkf [m x 1]
chkf <- matrix(nr=(m+1),nc=1)

# gcoef(fnx,T) computes the coefficients in chebyshev collocation
# given fnx[i]: function values at i=1:m and T[m x a]
# it returns cfa[a], coefficients for each of "a" polynomials
gcoef <- function(fnx,T)
{
  k = length(fnx)
  a = length(T[1,])
  sum = 0.0
  for (i in 1:k)
    sum = sum + fnx[i]
     cfa[1] <- sum/k

  for (j in 2:a)
  {
    sum = 0.0
    for (i in 1:k)
      sum = sum + T[i,j]*fnx[i]

    cfa[j] <- 2*sum/k
  }
  return(cfa)
}

# calculating the chebysheff coefficients
cfa <- gcoef(fnxa,T)

Aprx <- function(T,cfa)
{
  m = length(T[,1])
  a = length(cfa)
  for (i in 1:m)
  {
     sum = 0.0;
     for (j in 1:a)
        sum = sum + T[i,j]*cfa[j]

     chkf[i]<-sum
  }
  return(chkf)
}

chkf <- Aprx(T,cfa)
```

```
par(mfrow=c(2,1))
par(mai=c(0.5,0.5,0.25,0.25))

plot(z,T[,2],type="b",col='green',lwd=2,panel.first=grid())
 lines(z,T[,3],type="b",col='red',lwd=2)
 lines(z,T[,4],type="b",col='blue',lwd=2)
 lines(z,T[,5],type="b",col='black',lwd=2)
 lines(z,T[,6],type="b",col='magenta',lwd=2)
 lines(z,T[,7],type="b",col='grey',lwd=2)
 lines(z,T[,8],type="b",col='yellow',lwd=2)
title(main="Chebyshev Polynomials", sub="", xlab="", ylab="")

plot(fnxa,col='blue',panel.first=grid())
lines(chkf,type="l",col='red')
title(main="Chebyshev Approximation", sub="", xlab="", ylab="")
```

3.6.2. Optimization

In order to find the optimum we first define the general class of problems and then focus only on the subset that is of immediate use in solution of the Ramsey problem.

[Nocedal and Wright (2006)] define optimization as either minimization or maximization of a real valued function subject to constraints on its variables. The notation they use is as follows

- $\mathbf{x}$ is a vector of variables or unknowns that we need to find
- $f(\mathbf{x})$ is the objective scalar function of $\mathbf{x}$ that we need to optimize
- $c_i(\mathbf{x})$ are the equality and inequality constraints on $\mathbf{x}$

In mathematical notation

$$\min_{\mathbf{x}\in\mathbb{R}^n} f(\mathbf{x}) \text{ subject to } \begin{matrix} c_i(\mathbf{x})\geq 0 & i\in\mathcal{I} \\ c_i(\mathbf{x})=0 & i\in\mathcal{E} \end{matrix} \tag{3.6}$$

where $\mathcal{E}$ and $\mathcal{I}$ are sets of indices for equality and inequality constraints respectively.

The problem is to find a suitable starting point and then proceed to update the initial guess till one finds the optimum. The problems that one can encounter is that functions may have multiple optima and the method needs to find the global optimum.

For the Ramsey problem we will restrict the exposition to finding the optimum for a function $f(x)$ of a single variable $x \in [x_{min}, x_{max}]$ without derivatives. This method is known as the golden section search. For the use of derivative methods the reader is referred to [Nocedal and Wright (2006)]

3.6.2.1. *Golden Section Search*

1: set the parameter ϵ as the convergence parameter
2: set the limits $[a, b]$ which contain the optimum
3: if $|a - b| < \epsilon$ STOP. Solution is $x_{opt} = \frac{a+b}{2}$
4: compute $f(a)$ and $f(b)$

5: obtain two more points c, d such that $a < c < d < b$

$$c = a + \frac{3-\sqrt{5}}{2}(b-a)$$
$$d = a + \frac{\sqrt{5}-1}{2}(b-a)$$

6: compute compute $f(c)$ and $f(d)$
7: if $f(c) \geq f(d)$, the minimum is in $[c, b]$, replace $a = c$ and go to step **3**
8: if $f(c) < f(d)$, the minimum is in $[a, d]$, replace $b = d$ and go to step **3**

As mentioned earlier this method is incapable of finding the global optimum

3.6.2.2. R *code for Golden Section Search*

```
# this function for linear interpolation will be used later in the
# Value Function iteration with the Finite Element Method
# it is bunched here with the Golden Section Search for Convenience
# linear interpolation LI
LI <- function(y,x,xi)
{
   emax <- max(x)
   emin <- min(x)

   if (xi > emax || xi < emin)
      print("element out of range")

   s <- length(y)
   for (i in 1:(s-1))
   {
     if (xi >= x[i] && xi <= x[i+1])
        {
            yi = y[i] + (y[i+1]-y[i])/(x[i+1]-x[i])*(xi-x[i])
        }
   }
   return(yi)
}

# function to evaluate objective
feval <- function(x)
{
# tricky objective function and gets caught in local minima or maxima if
# one uses the usual GOLDEN SECTION SEARCH
     obj = -(x^2 -((x+2)^3)/4 + 4*x - 6)
#     obj = exp(x)
     obj = cos(x)
    return(obj)
}

#function golden section search
# given initial points low:a and high:b and iterations n
# calculate the minimum of the function defined by feval
gdnsrch <- function(a,b,iter)
{
    tol = 0.000001
    ts5 <- 3 - sqrt(5)
```

```
    s51 <- sqrt(5)-1

  for (i in 1:iter)
  {
    while ((abs(a-b) > tol) || i > iter)
    {
        fa = feval(a);
        fb = feval(b);
        if (abs(a-b) < tol)
        {
            solution = (a+b)/2;
            value = feval(solution);
        }
        else
        {
                if (a > b)
                {
                    temp = a;
                    a = b;
                    b = temp;
                }

                c = a + ts5*(b-a)/2;
                d = a + s51*(b-a)/2;
                fc = feval(c);
                fd = feval(d);
                if (fc >= fd)
                {
                   a = c
                }
                else if (fc < fd)
                {
                   b = d
                }
        }
    }
  }
    solution = (a+b)/2
    minima = feval(solution)
    res <- list(X=solution,Y=minima)
}

a <- -2.0
b <- 4.0
iter <- 100
gsmin <- gdnsrch(a,b,iter)

# testing functions from here
x <- rnorm(100)
y <- rnorm(100)
xi = (max(x)-min(x))/2

np <- 100
dp <- (b-a)/np
fy <- matrix(nr=np,nc=1)
```

```
fx <- matrix(nr=np,nc=1)

for (i in 1:np)
{
  fx[i] = a + (i-1)*dp
  fy[i] = feval(a + (i-1)*dp)
}

plot(fx,fy,type="l",col = "red",panel.first=grid())
```

3.7. Solution of Infinite Horizon Ramsey Model- Value Function Iteration

This section deals with the solution of the infinite horizon Ramsey model using the Value Function iteration. We first outline the methodology of the solution and follow it with a code in R.

To recapitulate in brief, the recursive formulation for the infinite horizon Ramsey model is as follows

$$F(K_t, 1) = F(\kappa_t) = z\kappa_t^{\alpha} \tag{3.7a}$$
$$K_{t+1} = F(K_t, 1) + (1-\delta)K_t - C_t \tag{3.7b}$$
$$C_t \geq 0 \tag{3.7c}$$
$$K_t \geq 0 \tag{3.7d}$$
$$\mathbb{U}(C_t) = \frac{C_t^{(1-\sigma)}}{(1-\sigma)} \quad \text{or} \tag{3.7e}$$
$$\mathbb{U}(C_t) = \log[C_t] \tag{3.7f}$$
$$V(K_t) = \max_{C_t, K_t}\{\mathbb{U}(C_t) + \beta V(K_{t+1})\} \tag{3.7g}$$

where $F(K_t, 1)$ is the output at time t using capital stock K_t with total factor productivity z and parameter α, δ is the depreciation, C_t is the consumption in time t and K_{t+1} is the saving or investment in the next period $t+1$, $\mathbb{U}(C_t)$ is the utility associated with C_t [b], β is the discount factor and $V(K_t)$ is the value function that needs to be maximised with K_{t+1} playing the role of the control variable as the amount of savings will determine the consumption and hence the utility.

What we need is at each time t a value of K_t that will give the maximum of equation 3.7g.

The approach is as follows. We arbitrarily select a range of capital stock $k \in [K_{min}, K_{max}]$ and hope that the steady state capital stock K^* lies in that range. Since it is impossible to have a continuous change in the capital stock, we discretize the interval into say 100 or 250 or 1000 grid points [c] nk. We can use the linear scale wherein all grid points are equidistant or use the log scale where in they are bunched

[b] $\mathbb{U}(C_t)$ is unique but two different forms have been shown as used in the program
[c] the larger the grid points nk, slower is the rate of convergence

closer to the lower end K_{min}. Then we use Bellman principle of optimisation and at each period calculate the value of the capital stock that gives the maximum of the value function. We repeat until convergence. The different approaches to convergence gives us 3 methods that can be used to obtain the steady state capital stock in the infinite horizon Ramsey model. They are

(1) Value Function Iteration with Discretization
(2) Value Function Iteration with Finite Element Method
(3) Value Function Iteration with Chebyshev Collocation / Regression

3.7.1. Value Function Iteration with Discretization

1: Fix K_{min} and K_{max}. For the Ramsey problem without depreciation we set $K_{min} = 0.01$ and $K_{max} = 1$
2: select a parameter ϵ for convergence like 0.0001
3: fix the number of grid points nk and obtain the values of $K_i \quad \forall i = 1, .., nk+1$, using $d = \frac{K_{max}-K_{min}}{nk}$ and $K_i = K_{min} + (i-1) * d$
4: Initialise the Value function $V^0 = \{V_i^0\}_1^{nk+1}$ to 0
5: Using equation 3.7b we obtain the consumption C_t and then the corresponding utility $\mathbb{U}(C_t)$ and then using equation 3.7g we obtain the new value function V^1. Thus

$$V_{i,j}^1 = \mathbb{U}(zF(K_i, 1) + (1-\delta)K_i - K_j) + \beta V_j^0$$

In simple words for each K_i, we iterate over the grid of capital stocks using K_j to find the value of K_j that gives the maximum for each i. In case $zF(K_i, 1) + (1-\delta)K_i - K_j < 0$ assign a large negative value like -1000 so that this K_j is never selected. Since we have $nk+1$ K_is we will fill the vector V^1 with i maximum values for a corresponding K_j.
6: We also store the value of each j or indirectly K_j that gives the maximum for each K_i in a vector g_i
7: Now we have two vectors V^0 and V^1 and we compare if they are close using the maximum norm given by

$$normd = \max_{i=1,2..,nk+1} |V_i^0 - V_i^1|$$

8: if $normd > \epsilon$ goto step **5** updating $V_i^0 = V_i^1 \ \forall i$
9: if $normd < \epsilon$ we have found the capital stock and the vector g_i is the optimal one
10: check for bounds that is if $g_1 = 1$ or $g_{nk+1} = nk+1$, then the range $[K_{min}, K_{max}]$ is too small and one needs to increase it
11: repeat until the above condition is not true that is $g_i \in \{2, 3, .., nk\}$

3.7.2. Value Function Iteration with Finite Element Method

Given an interval between $[x_1, y_1]$ and $[x_2, y_2]$ and in the absence of any function that generates values between this interval, if we need to find a value of y corresponding to a value of x, such that $x_1 \leq x \leq x_2$, we resort to linear interpolation. Therefore y is given by

$$y = y_1 + \frac{(y_2 - y_1)}{(x_2 - x1)}(x - x_1)$$

The steps for the Value Function iteration with the finite element method are same as with discretization upto step **5**, except that we use

$$V_i^0 = \frac{\mathbb{U}(zF(K_i, 1) + (1-\delta)K_i - K_i)}{1-\beta}$$

for V^0

(1) The objective is to find K, $K_{j-1} < K < K_j$ using linear interpolation with V_{j-1} and V_j playing the part of y_1 and y_2 and K_{j-1} and K_j playing the part of x_1 and x_2. We use the Golden Section Search to find the optimal K of

$$g_i = \underset{K \in [K_{min}, K_{max}]}{\arg\max} \{\mathbb{U}(zF(K_i, 1) + (1-\delta)K_i - K) + \beta V^0(K)\}$$
$$V_i^1 = \mathbb{U}(zF(K_i, 1) + (1-\delta)K_i - g_i) + \beta V^0(g_i)$$

(2) the next steps are again similar to the discretization method with computing the maximum norm between V^0 and V^1 till they converge

3.7.3. Value Function Iteration with Chebyshev Collocation / Regression

We have seen the use of Chebyshev collocation / regression for approximating any function $f(x)$. To solve the Ramsey problem, we substitute the Value function $V(K)$ in place of $f(x)$ and approximate it using the Chebyshev collocation / regression.

The approach is as follows. We need to find out the value of the set of a coefficients $\theta_i \quad \forall i \in 1, .., a$, where a is the number of Chebyshev polynomials of order a. We also use $m = n + 1$ collocation points which are the roots of the Chebyshev polynomial. If we use $m = a$, then it is called collocation and if $m > a$ it is called regression. In the earlier code we had used $m = 40$ and $a = 3$ and hence it is regression.

Obtaining the roots of the Chebyshev polynomial is straight forward as explained earlier.

The steps in brief are as follows

1: Choose a, say $a = 3$
2: Choose m, say $m = 40$
3: Fix ϵ, the convergence parameter, say $\epsilon = 0.0001$
4: Fix the upper and lower bounds on capital stock K_{max} and K_{min}
5: Choose the functional form and fix parameters for Utility, production function
6: Compute collocation points $\{z_i\}_{i=1}^m \quad \forall z_i \in [-1, 1]$
7: Convert z_i to ks_i for $\forall ks_i \in [K_{min}, K_{max}]$
8: Compute $T_{m \times a}$, the $m \times a$ matrix of Chebyshev polynomials
9: Initialise the value function $V^0(K)$ for $V_i^0(K) \quad \forall i \in [1, m]$
10: Value function is approximated by $V(K) = \sum_{j=1}^a \theta_j T_j(K)$
11: Obtain the initial value of θ^0
12: with this valueof θ^0 obtain the approximate value of the Value function $V^0(K)$
13: **for all** i such that $1 \le i \le m$ **do**
14: $\quad g_i = \arg\max_{K_j \in [K_{min}, K_{max}]} \{\mathbb{U}(zF(K_i, 1) + (1-\delta)K_i - K_j) + \beta V^0(K)\}$
15: **end for**
16: update the value function $V^1(K) = \{\mathbb{U}(zF(K_i, 1) + (1-\delta)K_i - g_i) + \beta V^0(g_i)\}$
17: obtain the new Chebyshev coefficients $\theta_j^1 \quad \forall j = 1, .., a$
18: Check if $\max_j |\theta_j^0 - \theta_j^1| < \epsilon$
19: If yes, STOP. $V^1(K)$ is the optimal value function and g_i are the optimal capital stock
20: If not, $\theta_j^0 = \theta_j^1 \quad j = 1, .., a$ and go back to step 12

3.7.4. R code for Infinite Horizon Ramsey Model

```
# Ramsey Model with NO depreciation
# Steady State Capital Stock:K* lies between 0 & 1
# with depreciation K* is greater than 1
NEG <- -10000

sigma <- 0.5 # CRRA sigma
alpha <- 0.5 # F(L,K) alpha
delta <- 1.00 # depreciation
tfp   <- 1 # z: total factor productivity
beta  <- 0.90 # beta: discount factor
Kl    <- 0.01 # Kmin
Ku    <- 1.00 # Kmax
nk    <- 250 # nk: number of grid points
tol   <- 0.001 # tol
niter <- 20 # niter

PF <- function(K,alpha)
{
   OP <- K^(alpha)
   return(OP)
}

UF1 <- function(C,sigma)
{
   CRRA <- (C^(1-sigma))/(1-sigma)
   return(CRRA)
}

UF2 <- function(C)
{
   CRRA <- log(C)
   return(CRRA)
}

kd <- (Ku-Kl)/nk
Kgrid <- matrix(nr=nk,nc=1)
for (i in 1:nk)
  Kgrid[i] <- Kl + i*kd

oo <- matrix(nr=nk,nc=1)
for (i in 1:nk)
  oo[i] <- 0 + i*kd

V1 <- matrix(nr=nk,nc=1)
for (i in 1:nk)
  V1[i] <- 0

V0 <- matrix(nr=nk,nc=1)

for (i in 1:nk)
  V0[i] <- UF2(PF(Kgrid[i],alpha))

norm <- matrix(nr=nk,nc=1)
```

```
for (i in 1:nk)
  norm[i] <- abs(V0[i]-V1[i])

diff <- max(norm)
ni = 0

Vj <- matrix(nr=nk,nc=1)

g <- matrix(nr=nk,nc=1) # optimal capital stock
gn <- matrix(nr=nk,nc=1) # used tocheck if capital stock is at extremes

for (i in 1:nk)
{
  g[i] <- Kgrid[i]
  gn[i] <- i
}

result <- matrix(nr=nk,nc=3)

while (diff > tol || ni < niter)
{
  ni = ni + 1
  for (i in 1:nk)
    V1[i] <- 0

  for (i in 1:nk)
  {
    for (j in 1:nk)
      {
        Cij = tfp*PF(Kgrid[i],alpha) + (1-delta)*Kgrid[i]-Kgrid[j];
          if (Cij < 0)
            Vj[j] <- NEG
          else
            Vj[j] <- UF2(Cij)+beta*V0[j]
      }

# finding the element of V1 which is max and selecting K according to it
    elmax  <- max(Vj)
    n_cmax <- which.max(Vj)
    V1[i]  <- elmax
    g[i]   <- Kgrid[n_cmax]
    gn[i]  <- n_cmax
  }

  for (i in 1:nk)
    norm[i] <- abs(V0[i]-V1[i])

  diff <- max(norm)

  {
    if (diff > tol)
    {
      for (i in 1:nk)
        V0[i] = V1[i];
    }
```

```
    else
    {
      for (i in 1:nk)
      {
        result[i,1] = g[i];
        result[i,2] = V1[i];
        result[i,3] = gn[i];
      }
    }
  }

}

plot(oo,type = "l", col = "red", lwd=2,panel.first=grid())
lines(g,,type = "l", col = "blue", lwd=2)
```

3.7.5. R code for Finite Horizon Ramsey Model

```
# This is modified from Ramsey.R with a finite time interval of 60 years
# Ramsey Model with NO depreciation
# Steady State Capital Stock:K* lies between 0 & 1
# with depreciation (delta < 1) K* is greater than 1
NEG <- -100

sigma <- 0.5 # CRRA sigma
alpha <- 0.5 # F(L,K) alpha
delta <- 1.0 # depreciation
tfp   <- 1 # z: total factor productivity
beta  <- 0.90 # beta: discount factor
Kl    <- 0.01 # Kmin
Ku    <- 1.00 # Kmax
nk    <- 100   # nk: number of grid points
tol   <- 0.001 # tol
niter <- 20 # niter

T <- 60
K0 <- 0.05

PF <- function(K,alpha)
{
   OP <- K^(alpha)
   return(OP)
}

UF1 <- function(C,sigma)
{
   CRRA <- (C^(1-sigma))/(1-sigma)
   return(CRRA)
}

UF2 <- function(C)
{
   CRRA <- log(C)
   return(CRRA)
```

```
}

kd <- (Ku-Kl)/nk
Kgrid <- matrix(nr=nk,nc=1)
for (i in 1:nk)
  Kgrid[i] <- Kl + i*kd

# for a log grid so that it is more dense at lower K
kdl <- (log(Ku)-log(Kl))/nk
KgL <- matrix(nr=nk,nc=1)
for (i in 1:nk)
  KgL[i] <- exp(log(Kl) + i*kdl)

Kgrid <- KgL

oo <- matrix(nr=nk,nc=1)
for (i in 1:nk)
  oo[i] <- 0 + i*kd

VF <- matrix(nr=T,nc=1)

# grid for Value function at different grid points
# initialised to ZERO
VGrid <- matrix(nr=nk,nc=(T+1))
for (i in 1:nk)
  for (j in 1:T)
    VGrid[i,j] <- 0

Vi <- matrix(nr=nk,nc=1)
Vj <- matrix(nr=nk,nc=1)

g <- matrix(nr=(T+1),nc=1) # index of optimal capital stock
gn <- matrix(nr=nk,nc=1) # used tocheck if capital stock is at extremes
ks <- matrix(nr=T,nc=1) # optimal capital stock

# vector for storing the index of max K for each period
WN_max <- matrix(nr=T,nc=1)

for (i in 1:T)
{
  VF[i] <- NEG
  g[i] <- 0
}
  g[T+1] <- 0
# we know that in period T+1 K_{T+1} = 0 and V_{T+1} = 0
# so we calculate V_{T} at each grid point so as to work out the
#optimal K for earlier periods

for (i in 1:nk)
{
  VGrid[i,T] <- UF1((tfp*Kgrid[i]^alpha),sigma)
  VGrid[i,(T+1)] <- 0
}
  VF[T] <- max(VGrid[,T])
  n_cmax <- which.max(VGrid[,T])
```

```
  g[T]  <- Kgrid[n_cmax]
  WN_max[T] <- n_cmax

for (k in T:3)
{
  for (i in 1:nk)
  {
    for (j in 1:nk)
    {
        Cij = tfp*PF(Kgrid[i],alpha) + (1-delta)*Kgrid[i] - Kgrid[j];
        if (Cij < 0)
          Vj[j] <- NEG
        else
          Vj[j] <- UF1(Cij,sigma)+beta*VGrid[j,(k+1)]
    }
    Vi[i] <- max(Vj)
    n_cmax <- which.max(Vj)
    g[k]  <- Kgrid[n_cmax]
    VGrid[i,k] <- Vi[i]
    VF[k] <- max(VGrid[,k])
    WN_max[k] <- n_cmax
  }
}

# for time period 1 where capital stock is already known
    for (j in 1:nk)
    {
        Cij = tfp*PF(K0,alpha) + (1-delta)*K0 - Kgrid[j]
        if (Cij < 0)
          Vj[j] <- NEG
        else
          Vj[j] <- UF1(Cij,sigma)+beta*VGrid[j,3]
    }
    VGrid[,2]<- Vj
    n_cmax <- which.max(Vj)
    g[2]  <- Kgrid[n_cmax]
    VF[2] <- max(VGrid[,2])
    g[1] <- K0
    VF[1] <- tfp*PF(K0,alpha)
    WN_max[2] <- n_cmax

    for (j in 1:nk)
      VGrid[j,1]<- tfp*PF(K0,alpha)

Ck <- matrix(nr=T+1,nc=1)
for (i in 1:T)
  Ck[i] <- tfp*PF(g[i],alpha) + (1-delta)*g[i] - g[i+1]
Ck[T+1] <- 0

plot(g,type = "b", col = "green", lwd=2, panel.first=grid() )
```

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PART 2

CGE Modelling of a Small Island Economy: the Azores

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4

The Azores: A Succinct Introduction

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Contents

4.1. Introduction

The examples of CGE models presented in the following chapters are all applied to the Açores. For this reason, this chapter provides a brief introduction to the geography, politics and economy of the region. The objective is to give an overview of the islands and the challenges faced on the economic front and the policy options available to the policy makers in the islands, mainland Portugal and the European Union as it receives funds from all these sources.

In the sections that follow we characterize this region on the basis of its geography, the political organization, its demographics and its economy. We finalize with a section on some relevant policy issues

4.2. Geography

The archipelago of the Azores, is an Autonomous Portuguese territory in the Atlantic Ocean comprising nine islands. It was first inhabited in the 15th century and has always been under Portuguese dominance. Some marking characteristics of the

*CEEAplA

region are its location in the middle of the North Atlantic Ocean, one third of the way between Lisbon and New York, the small size of each island and their great dispersion relative to each other. When dealing with the Azores we should almost always think of a micro dimension.

This location in the middle of the Atlantic Ocean has brought prosperity from time to time as transportation technology changed and as a land bases were considered indispensable. However, just as technology brought functions to the islands it also, invariably, has evolved to render them unnecessary

4.3. Politics

The region is called autonomous because it has its own regional parliament and government. National laws are followed with the capacity of the regional legislature to adapt them, in many circumstances, to its specificities, in many domains.

The regional budget is independent of the national budget except for the transfers that are decided nationally. Otherwise the regional assembly has the power to approve the budget, including deciding over tax rates, within some bounds, and all expenditures.

The main revenue sources are all the taxes collected locally or imputable to activities that are locally generated, transfers from the national budget to municipalities and to the regional budget as a final destination of the resources, transfers from the EU and debt. The main expenditures go to support the public administration, education, health and investment in various areas.

Each island is divided into municipalities which are further divided in parishes. Political representation in the legislative assembly includes all the major national parties with socialists and social democrats in the forefront.

Table 4.1 shows the number of voters by island and the seats in the legislative assembly for the year 2004. It also shows the number of municipalities and parishes along with the land area of each island.

The islands also have the right to fill five seats in the national parliament. Transfers to the municipalities are governed by a local finance law that determines the division of total transfers, determined as a function of tax revenues in previous years, on the basis of need, population size and territory variables.

Transfers to the regional government are made on the basis of a regional finance law that determines not only the total amount of resources to be transferred annually to the two autonomous regions, the Azores and Madeira, but also how this amount is split between the two regions. This law has been the subject of some profound discussions. The islands can also, by law, resort to debt financing within some limitations. In the recent past, however, due to the national budget constraints, debt has not been allowed except in some very specific situations associated to the recent crisis.

Table 4.1.: Area and Administrative Divisions by Island

No.	Island	Municipalities No.	Parishes No	Area km^2	Eligible Voters	Deputies
1	Corvo	1	1	17.1	350	2
2	Faial	1	13	173.1	11451	4
3	Flores	2	11	141.0	3211	3
4	Graciosa	1	4	60.7	3817	3
5	Pico	3	17	444.8	11820	4
6	Santa Maria	1	5	96.9	4508	3
7	São Jorge	2	11	243.6	7967	4
8	São Miguel	6	64	744.6	99854	19
9	Terceira	2	30	400.3	44787	10
	Total	19	165	2322.0	187765	52

Source: [SREA (Serviço Regional de Estatística dos Açores)]

Table 4.2.: Population in the Islands of the Azores

	Average Population (Estimate)	2001	2002	2003	2004	2005
1	Santa Maria	5451	5490	5496	5504	5518
2	São Miguel	129434	130154	130839	131183	131866
3	Terceira	54743	54996	55252	55349	55523
4	Graciosa	4665	4708	4748	4763	4795
5	São Jorge	9454	9522	9539	9549	9542
6	Pico	14454	14579	14666	14698	14741
7	Faial	14785	14934	15072	15148	15284
8	Flores	3907	4099	3967	3980	4008
9	Corvo	422	435	445	448	456
TOTAL	R.A.Azores	237315	238917	240024	240622	241733

Source: [SREA (Serviço Regional de Estatística dos Açores)]

4.4. Demographics

The estimated population of the islands stood, in 2005, at around 242.000. By island, the population is distributed according to table 4.2. Note that 55% of the population lives in the island of S. Miguel.

4.5. Economics

4.5.1. Evolution of Employment

Table 4.3 shows the active population by sex identifying the shares of males and females that are employed and unemployed. It also shows the employed population by sex for the major sectors of the economy. So in the year 2001, the primary sector accounts for 13.7% of the employed population in the labour force of which 92% are males and 8% are females. Similarly the secondary sector accounts for 28.3% of the total employment with 82% males and 18% females. The tertiary sector

Table 4.3.: Employed and Unemployed Population by Activity 2001-2005

No.		Sex	2001	2002	2003	2004	2005
1		*Active Population*					
1a		Persons	100645	103645	105099	108585	109773
1b		Males	64414	65892	66262	67922	68510
1c		Females	36231	37753	38837	40663	41263
2		*Employment and Unemployment*					
2a	Employed	Persons	98360	100974	102066	104892	105283
		Males	63602	64917	65255	66743	66662
		Females	34758	36057	36811	38149	38621
2b	Unemployed	Persons	2286	2671	3033	3694	4490
3	*Employed population according to sector of main activity %*						
3a	Primary	Persons	13.7%	13.4%	12.8%	12.5%	12.4%
		Males	92	93	92	92	92
		Females	8	7	8	8	8
3b	Secondary	Persons	28.3%	29.2%	28.2%	26.4%	25.4%
		Males	82	85	86	85	85
		Females	18	15	14	15	15
3c	Tertiary	Persons	58.0%	57,4%	59.0%	61.1%	62.2%
		Males	50	47	48	49	49
		Females	50	53	53	51	51

Source: [SREA (Serviço Regional de Estatística dos Açores)]

accounts for 58% of the employment with males and females having an equal share of 50% each. The Azores have one of the lowest participation rates in the country, particularly for women.

The active population as a fraction of the total population is hovering around the 45% mark while employed population as a fraction of total population is around 43%. One observes the share of employment in primary and secondary sectors in the economy falling steadily from 42% in 2001 to approximately 38% in 2005.

4.5.2. Evolution of Government Revenue and Expenditures

Table 4.4 shows the revenues by year in the Azores. The total revenues are made up of current revenues and capital revenues. The current revenues are mostly transfers and fiscal receipts accruing from various direct and indirect taxes. The capital revenues accrue from investments, sale of assets and transfers. Funds received or spent that are here marked and classified under a specific head of accounts termed as *Conta de Ordem*

Table 4.5 shows the sources of tax revenue collection in the islands. Income tax on individuals and Value Added Tax is the largest source of revenue for the government.

Table 4.6 shows the breakup of the expenditures by the government. By far compensation of employees and transfers make up the largest segment of current expenditures. On the capital expenditure side, planned and unspecified expenditure

Table 4.4.: Sources of Revenue in Azores million €

No	Account Head	2001	2002	2003	2004	2005
1	Revenues Summary					
1a	Current Revenues	399.33	513.74	504.84	542.74	589.70
1a1	Direct taxes	75.08	125.64	119.01	153.09	159.22
1a2	Indirect taxes	273.00	292.16	303.95	333.78	343.62
1a3	Contributions for social security			2.74	2.81	2.90
1a4	Taxes and others penalties	4.94	4.53	2.44	1.80	3.87
1a5	Property income	1.13	0.78	0.27	0.51	1.46
1a6	Transfers	43.89	89.89	72.50	50.00	77.80
1a7	Sales of current goods and services	0.54	0.47	0.66	0.37	0.39
1a8	Other current revenues	0.75	0.27	3.27	0.38	0.46
2a	Capital Revenues	302.89	214.73	203.00	211.51	223.04
2a1	Sales of investment assets	1.78	0.14	2.10	0.29	0.09
2a2	Transfers	198.96	153.93	188.35	207.05	181.65
2a3	Financial assets	1.40	1.64	10.04	1.69	38.78
2a4	Financial liabilities	90.73	56.59			
2a5	Other capital revenues	7.85	0.18	0.17	0.16	0.18
2a6	Recoveries	2.16	2.25	2.32	2.31	2.32
3	Conta de ordem	205.76	310.86	291.29	332.96	302.85
4	Total of charged revenues	908.00	1039.34	999.12	1087.21	1115.59
5	Balance of the previous years	19.13	14.94	15.65	21.60	45.78
	Total Revenues	927.11	1054.28	1014.77	1108.82	1161.37

Source: [SREA (Serviço Regional de Estatística dos Açores)]

Table 4.5.: Tax Revenue in Azores million €

No	Account Head	2001	2002	2003	2004	2005
1a1	Direct taxes	75.08	125.64	119.01	153.09	159.22
1a1a	Income Tax of Natural Persons (IRS)	63.03	93.25	90.10	109.43	116.58
1a1b	Corporate income tax (IRC)	11.26	31.87	27.99	43.31	42.39
1a1c	Others direct taxes	0.78	0.52	0.91	0.34	0.24
1a2	Indirect taxes	272.99	292.16	303.95	323.62	327.94
1a2a	Imposto de Selo (Stamp Duty)	12.08	13.60	19.81	31.05	18.90
1a2b	IVA (Value Added Tax)	226.11	244.72	253.15	260.73	271.95
1a2c	Tax on vehicles	14.18	14.22	11.27	11.62	14.39
1a2d	Tax on Tobacco	16.88	17.66	17.95	19.76	22.12
1a2e	Others indirect taxes	3.74	1.96	1.78	0.45	0.57
2	Taxes, fines and others penalties	4.94	4.53	5.18	1.80	3.87
	Total	353.02	422.33	428.14	478.50	491.03

Source: [SREA (Serviço Regional de Estatística dos Açores)]

is a major source of expenses.

Table 4.7 shows the gross value added by type of activity and the gross domestic product in the islands. As mentioned earlier, Public Administration and defence for a major part of the value added in the islands.

Table 4.6.: Expenditures in Azores million €

No	Account Head	2001	2002	2003	2004	2005
1	Current expenditure	435.56	482.90	493.82	503.86	515.93
1a	Compensation of employees	236.86	248.66	249.34	252.13	261.78
1b	Acquisition of goods and Services	13.36	14.80	15.35	16.32	17.07
1c	Interest on current liabilities	9.47	9.07	7.59	7.37	7.16
1d	Current transfers	167.09	200.63	212.18	218.18	219.47
1e	Subsidies	0	0	0	0	0
1f	Other current expenditures	8.78	9.73	9.40	9.85	10.44
2	Capital expenditure	62.24	28.79	1.73	2.28	3.25
2a	Acquisition of goods and services	1.28	1.44	1.37	1.37	1.32
2b	Capital Transfers	0.11	0.12	0.12	0.62	1.62
2c	Financial assets	0	0	0	0	0
2d	Financial liabilities	60.55	26.66	0	0	0
2e	Other capital expenditure	0.30	0.57	0.25	0.29	0.30
3	Plan expenditure	204.43	216.87	212.30	226.14	303.37
4	Conta de ordem	192.27	323.56	285.31	327.75	315.08
	Total	894.50	1052.11	993.17	1060.03	1137.64

Source: [SREA (Serviço Regional de Estatística dos Açores)]

4.6. Some Key Policy Issues

In approaching the development of the model presented in this book we had to address the policy issues that we would like to be taken up. Looking at the some of the main driving issues of economic policy the financing of the regional budget, regional tax policy, sector policies and the analysis of the impact of special programs were selected as key policy issues.

The economy of the Azores is driven by the production of its various production sectors but also depends, to a great extent on transfers from various sources: the national budget, EU funds, other transfers and debt financing.

Transfers from the national budget are determined by a specific law but have been the subject of considerable debate. Their impact on the regional economy should be considered of paramount importance.

Another source of financing of the regional budget is the transfers from the EU, here marked for different economic sectors among which agriculture and fisheries are very important. Construction and training should also be included as directly affected because of the resources that have been channelled by the regional development funds for the construction of infrastructure and through the social fund for training.

Together the EU transfers and the transfers from the national budget account for close to 50% of the regional budget. These are very sensitive policies because they provide resources that are fundamental in explaining the current development of the region. Since the Azores have some capacity to adapt the national tax rates (increase them by as much as 10% or decrease them by as much as 30%), these policies are important since they will significantly impact on the cost of final and

Table 4.7.: GVA and GDP in Azores million €

	Sector	2001	2002	2003	2004	2005
A	Agriculture, hunting and forestry	258	267	267	275	269
B	Fishing	37	44	46	48	52
C	Mining and quarrying	10	9	9	9	7
D	Manufacturing	128	146	154	156	166
E	Electricity, gas and water supply	55	61	69	83	92
F	Construction	166	172	161	173	162
G	Wholesale and retail trade; repair of motor vehicles, motorcycles and personal and household goods	217	253	271	283	289
H	Hotels and restaurants	75	76	82	95	105
I	Transport, storage and communication	192	188	191	191	200
J	Financial intermediation	89	83	95	90	95
K	Real estate, renting and business activities	236	249	276	283	311
L	Public administration and defence; compulsory social security	308	335	356	372	385
M	Education	184	194	193	196	199
N	Health and social work	158	174	178	187	193
O	Other community, social and personal service activities	34	40	43	40	40
P	Private households with employed persons	23	26	28	29	32
	GVA (RAAzores)	2171	2318	2421	2510	2597
	GDP (RAAzores)	2488	2666	2785	2887	3018

Source: [SREA (Serviço Regional de Estatística dos Açores)]

of intermediate products. The Azores have, in fact adapted their tax rates in 1998, implementing considerable tax reduction policies.

Other policy issues that are of importance in the Azores would be those that pertain to specific sectors such as fisheries, agriculture, tourism, etc. Some of the main issues in each sector have to do with the EU policy itself as would be the case of the funding of the fisheries or agriculture activities.

Equally important one might consider the impact of specific projects that are of some dimension. Such is the case of the American Base of Lajes, in Terceira, or any major project that might lend itself to interesting analysis.

In fact two of the projects analysed and reported in the following chapters pertain to the hypothetical closure of the base and to a, de facto, project developed in the island of S. Miguel.

Whichever the policy, beside its impact on GDP or on employment, distributional impacts are also of interest. In the end who benefits the most and who pays the most is a relevant policy question to be addressed.

These policy issues are some we have considered important for the purpose of this project and have, as such influenced the development of the model we use. These are, obviously not the only interesting questions to address and might, in some perspectives not even be the most important. They are, in our perspective key questions and, most of all, interesting questions to address when looking, specifically, at the Azores.

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5

Dynamic General Equilibrium Model of the Açorean Economy

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Contents

5.1. Introduction

The main objective of this project is to develop a multi-sectoral, multi-regional dynamic modelling platform of the Azores economy integrated within the European

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and global context. The platform will have the highest capabilities of analysis and forecasting in Azores for problems related to structural sectoral and regional issues, agriculture, labour markets, public finance, trade, EU funds, regional development, environment, and energy. The modelling platform is intended to act as an analytical and quantitative support for policy-making.

5.2. Model Description

The current version of the modelling platform of the Azores economy is represented by a dynamic multi-sectoral computable general equilibrium model (CGE), which incorporates the economic behaviour of six economic agents: firms, households, regional government, Mainland government, European Commission and the external sector.

The goods-producing sectors, consisting of both public and private enterprises, are disaggregated into 45 branches of activity. Households are divided into six income groups, to analyze the distributional effects of various policy measures. Special attention is paid to the economic links between the regional government, the Mainland government and the European Commission. With regard to the rest of the world the economy is treated as a small open economy with no influence on (given) world market prices. Trade relations are differentiated according to four main trade partners: Mainland, EU, US and the rest of the world. The behaviour of each agent in the model is described in detail below.

The model has been solved by using the general algebraic modelling system [Rosenthal (2006)].

The following conventions are adopted for the presentation of the model. Variable names are given in capital letters, small letters denote parameters calibrated from the database (SAM) and elasticity parameters. The subscript s stands for one of the production activities (45 branches of activity). The subscript c stands for one of the commodities (45 types of commodities). The subscript qu stands for one of the households' income groups (6 households' income groups). The subscript ctm stands for one of the trade and transport services (7 types of trade and transport services), while nctm stands for all the other commodities except trade and transport services (38 types of commodities).

5.2.1. Households

Household Income Six households groups $[qu]$ receive as income: capital (net operating surplus), labour, unemployment benefits from the mainland and other net transfers from the regional and mainland governments.

$$\begin{aligned} YH_{qu} = {} & shYKH_{qu}\sum_{s}[PK_sKSK_s] + shYLH_{qu}\sum_{s}[PL(1+premLSK_s)]LSK_s \\ & + TRHML_{qu}\,.\,ERML + shUNEMPB_{qu}\,.\,trep\,.\,PL\,.\,UNEMP \\ & + TRHG_{qu}\,.\,PCINDEX \end{aligned} \tag{5.1}$$

	Description	Status
YH_{qu}	Income of household qu	variable
$shYKH_{qu}$	share of household qu in capital income	parameter
PK_s	price of capital in sector s	variable
KSK_s	capital demand in sector s	variable
$shYLH_{qu}$	share of household qu in labour income	parameter
PL	price of labour	variable
$premLSK_s$	wage premium over average wage to labour in sector s	parameter
LSK_s	labour demand in sector s	variable
$TRHML_{qu}$	transfers to households from mainland	variable
$ERML$	exchange rate mainland	variable
$shUNEMPB_{qu}$	share of unemployment benefits of household qu	parameter
$trep$	replacement rate out of national average wage	parameter
$UNEMP$	number of unemployed	variable
$TRHG_{qu}$	transfers to households from local government	variable
$PCINDEX$	consumer price index	variable

Each household pays income taxes and saves a share of the income. The savings by income group is given by

$$SH_{qu} = MPS_{qu}[1 - ty_{qu}]YH_{qu} \tag{5.2}$$

	Description	Status
SH_{qu}	savings of household qu	variable
MPS_{qu}	marginal propensity to save of household qu	parameter
ty_{qu}	rate of income tax on household qu	parameter

Household propensity to save reacts to changes in the after-tax return to capital according to

$$MPS_{qu} = MPSZ_{qu}\left\{\frac{[1 - ty_{qu}]PKavr}{[1 - tyz_{qu}]PKavrZ}\right\}^{elasS_{qu}} \tag{5.3}$$

	Description	Status
$PKavr_{qu}$	real average return to capital qu	variable
$PKavrZ_{qu}$	benchmark level of $PKavr_{qu}$ qu	parameter
$MPSZ_{qu}$	benchmark marginal propensity to save of household qu	parameter
tyz_{qu}	benchmark rate of income tax on household qu	parameter
$elasS_{qu}$	elasticity of savings with respect to after-tax rate of return of household qu	parameter

Household Disposable Income The disposable income of households for consumption is

$$CBUD_{qu} = (1 - ty_{qu})YH_{qu} - SH_{qu} \tag{5.4}$$

	Description	Status
$CBUD_{qu}$	Disposable income of household qu	variable

Household Utility The disposable income is allocated between different goods and services based on a Stone-Geary utility function. The households maximise the utility

$$U(C_{c,qu}) = \prod_c [C_{c,qu} - \mu H_{c,qu}]^{\alpha H_{c,qu}} \tag{5.5}$$

	Description	Status
$C_{c,qu}$	consumption of commodity c by household qu @ purchasers price	variable
$\mu H_{c,qu}$	minimum consumption of commodity c by household qu	parameter
$\alpha H_{c,qu}$	marginal budget shares in Stone-Geary utility function: commodity c household qu	parameter

Household Budget Constraint The households maximise utility subject to the following budget constraint

$$CBUD_{qu} = \sum_c \left\{ [P_c + \sum_{ctm} tchtm_{ctm,c,qu} P_{ctm} (1 + texc_{c,qu})(1 + tc_{c,qu} + vatc_{c,qu})] C_{c,qu} \right\}$$

$$\sum_c \alpha H_{c,cu} = 1 \tag{5.6}$$

	Description	Status
P_c	Price of commodity c NET of taxes	variable
$tchtm_{ctm,c,qu}$	quantity of commodity ctm as trade-transport service per unit of commodity c	parameter
P_{ctm}	price of commodities receiving trade-transport margins $ctm \subset c$	variable
$texc_{c,qu}$	excise duty on consumption of commodity c by household qu	parameter
$tc_{c,qu}$	other tax rate on consumption of commodity c by household qu	parameter
$vatc_{c,qu}$	value added tax on consumption of commodity c by household qu	parameter

Household Demand for Commodities Maximising utility subject to budget constraint leads to demand for commodities valued at purchasers price that includes trade-transport margins $\sum_{ctm} tchtm_{ctm,c,qu} P_{ctm}$, excise duties, other taxes and value added tax. The consumer has a pre-determined minimum expenditure per commodity $\mu H_{c,qu}$ and the remaining is allocated to all commodities as per marginal budget shares $\alpha H_{c,qu}$

$$\left\{ P_c + \sum_{ctm} tchtm_{ctm,c,qu} P_{ctm} (1 + texc_{c,qu})(1 + tc_{c,qu} + vatc_{c,qu}) \right\} C_{c,qu} =$$

$$\mu H_{c,qu} \left\{ [P_c + \sum_{ctm} tchtm_{ctm,c,qu} P_{ctm}](1 + texc_{c,qu})(1 + tc_{c,qu} + vatc_{c,qu}) \right\} +$$

$$\alpha H_{c,qu} \left\{ CBUD_{qu} - \mu H_{c,qu} [P_c + \sum_{ctm} tchtm_{ctm,c,qu} P_{ctm}](1 + texc_{c,qu})(1 + tc_{c,qu} + vatc_{c,qu}) \right\} \tag{5.7}$$

Total Private consumption by commodity This is the total consumption by all households for each commodity

$$CCT_c = \sum_{qu} C_{c,qu} \tag{5.8}$$

	Description	Status
CCT_c	Total private consumption of commodity c	variable

Average Income tax Rate on Households This is the total income tax of all households as a share of the total income of all households

$$tyavr = \frac{\sum_{qu} ty_{qu} YH_{qu}}{\sum_{qu} YH_{qu}} \tag{5.9}$$

	Description	Status
$tyavr$	Average income tax rate for the economy	variable

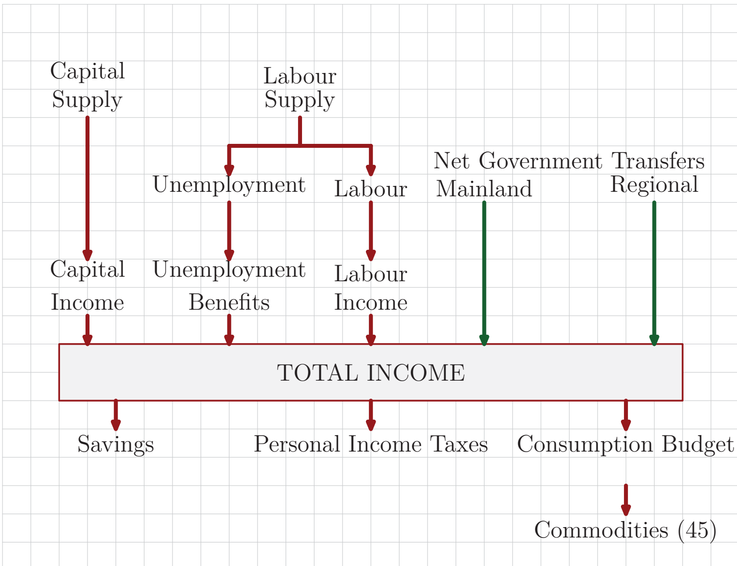

Fig. 5.1.: Decision Structure of Representative Household by Income Group

Welfare Measures Welfare gains and losses are measured using the Equivalent Variation, which is based on the indirect utility function. The Equivalent Variation is given by

$$EV_{qu} = \prod_c \left\{ \frac{PZ_c + \sum_{ctm} tchtmz_{ctm,c,qu} PZ_{ctm}(1+texcz_{c,qu})(1+tcz_{c,qu}+vatcz_{c,qu})}{\alpha H_{c,qu}} \right\}^{\alpha H_{c,qu}} \times \left[VU_{qu} - VUI_{qu} \right] \quad (5.10)$$

$$VU_{qu} = \left\{ CBUD_{qu} - \sum_c \left[P_c + \sum_{ctm} tchtm_{ctm,c,qu} P_{ctm} \right] (1+texc_{c,qu})(1+tc_{c,qu}+vatc_{c,qu}) \mu H_{c,cq} \right\} \quad (5.11)$$

$$\times \left\{ \frac{\alpha H_{c,qu}}{P_c + \sum_{ctm} tchtm_{ctm,c,qu} P_{ctm}(1+texc_{c,qu})(1+tc_{c,qu}+vatc_{c,qu})} \right\}^{\alpha H_{c,qu}}$$

$$VUI_{qu} = \left\{ CBUDZ_{qu} - \sum_c \left[PZ_c + \sum_{ctm} tchtmz_{ctm,c,qu} PZ_{ctm} \right] (1+texcz_{c,qu})(1+tcz_{c,qu}+vatcz_{c,qu}) \mu H_{c,cq} \right\}$$

$$\times \left\{ \frac{\alpha H_{c,qu}}{PZ_c + \sum_{ctm} tchtmz_{ctm,c,qu} PZ_{ctm}(1+texcz_{c,qu})(1+tcz_{c,qu}+vatcz_{c,qu})} \right\}^{\alpha H_{c,qu}} \quad (5.12)$$

5.2.2. Firms

The firms operate in a an environment of perfect competition. This implies price equals marginal costs with all firms taking price as given. With increasing or decreasing returns to scale the price will equal average costs which are the sum of marginal and average fixed costs.

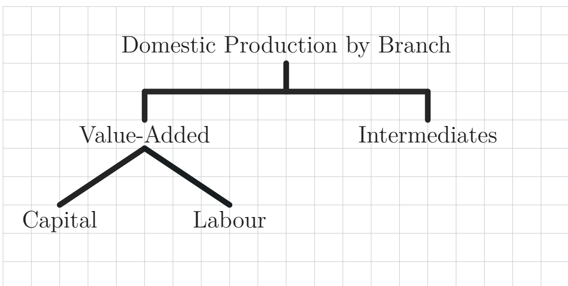

Fig. 5.2.: Nested CES-Leontief production Technology for Domestic Production

Value Added The production structure is modelled as a fixed-coefficient Leontief technology between intermediate inputs and value added. The value added is a constant elasticity of substitution (CES) function between labour and industry specific capital.

$$\begin{aligned} KL_s &= aKL_s XD_s \\ KL_S &= aF_s\left[\gamma FK_s KSK_s^{-\rho F_s} + \gamma FL_s LSK_s^{-\rho F_s}\right]^{-\frac{1}{\rho F_s}} \\ \sigma F_s &= \frac{1}{1+\rho F_s} \end{aligned} \tag{5.13}$$

	Description	Status
γFK_s	CES distribution parameter for capital in production	parameter
γFL_s	CES distribution parameter for labour in production	parameter
ρF_s	elasticity of substitution parameter in production	parameter
σF_s	elasticity of substitution in production	parameter
aF_s	efficiency parameter in CES production function	parameter
aKL_s	fixed share of value added in production	parameter
KL_s	value added in production	variable
KSK_s	capital stock in sector s	variable
LSK_s	number of employees in branch s	variable
XD_s	domestic production in sector s	variable

Cost Function The factor cost is given by

$$Cost_s(KSK_s, LSK_s) = \left[PK_s(1+tk_s) + d_s PI\right]KSK_s + \left[PL(1+premLSK_s)(1+\frac{tl_s}{1+tl_s})\right]LSK_s$$

	Description	Status
PI	price index of composite investment good	variable
d_s	depreciation rate in sector s	parameter
tk_s	corporate tax rate in sector s	parameter
tl_s	social insurance contribution rate in sector s	parameter

Demand for Labour and Capital Minimising the cost function subject to the value added we obtain the demand for labour and capital

$$KSK_s = KL_s\left[\frac{PKL_s}{(PK_s(1+tk_s)+d_sPI)}\right]^{\sigma F_s} \gamma FK_s^{\sigma F_s} aF_s^{\sigma F_s - 1} \tag{5.14}$$

$$LSK_s = KL_s\left\{\frac{PKL_s}{PL(1+premLSK_s)(1+\frac{tl_s}{1+tl_s})}\right\}^{\sigma F_s} \gamma FL_s^{\sigma F_s} aF_s^{\sigma F_s - 1} \tag{5.15}$$

The domestic production is valued at basic prices net of taxes but includes direct subsidies from the regional government, European Agricultural Guidance and Guarantee Fund (EAGGF), Financial Instrument for Fisheries Guidance (FIFG), European Regional Development Fund (ERDF), European Social Fund (ESF) and from the United States of America (USA). It is the sum of value added @ basic prices and the intermediate inputs valued at commodity prices excluding subsidies but including trade-transport margins, value added taxes on intermediate consumption.

Savings of Firms Firms save a fixed fraction of their capital income, given by

$$SF = shYKF \sum_s PK_s KSK_s \tag{5.16}$$

	Description	Status
SF	Savings of firms	variable
$shYKF$	share of savings in capital income of firms	parameter

5.2.3. Foreign Trade

The model assumes that the country is "small" implying a price taker in both, import and export markets. There are four trading partners, mainland, EU, USA and Rest of the World.

Imports The domestic users consume a composite goods comprising domestic and imported goods according to an Armington function

$$X_c = aA_c\Big\{\gamma A1_c MML_c^{-\rho A_c} + \gamma A2_c MEU_c^{-\rho A_c} + \gamma A3_c MUS_c^{-\rho A_c} + \gamma A4_c MROW_c^{-\rho A_c} + \gamma A5_c XDD_c^{-\rho A_c}\Big\}^{-\frac{1}{\rho A_c}} \quad (5.17)$$

The cost to the domestic consumer is

$$Cost_c[.] = PMML_c MML_c + PMEU_c MEU_c + PMUS_c MUS_c + PMROW_c MROW_c + PDD_c XDD_c \quad (5.18)$$

Minimising cost subject to the Armington function, we obtain

$$MML_c = X_c\Big\{\frac{P_c}{PMML_c}\Big\}^{\sigma A_c} \gamma A1_c^{\sigma A_c} aA_c^{\sigma A_c - 1} \quad (5.19)$$

$$MEU_c = X_c\Big\{\frac{P_c}{PMEU_c}\Big\}^{\sigma A_c} \gamma A2_c^{\sigma A_c} aA_c^{\sigma A_c - 1} \quad (5.20)$$

$$MUS_c = X_c\Big\{\frac{P_c}{PMUS_c}\Big\}^{\sigma A_c} \gamma A3_c^{\sigma A_c} aA_c^{\sigma A_c - 1} \quad (5.21)$$

$$MROW_c = X_c\Big\{\frac{P_c}{PMROW_c}\Big\}^{\sigma A_c} \gamma A4_c^{\sigma A_c} aA_c^{\sigma A_c - 1} \quad (5.22)$$

$$XDD_c = X_c\Big\{\frac{P_c}{PDD_c}\Big\}^{\sigma A_c} \gamma A5_c^{\sigma A_c} aA_c^{\sigma A_c - 1} \quad (5.23)$$

The zero profit condition is

$$P_c X_c = PMML_c MML_c + PMEU_c MEU_c + PMUS_c MUS_c + PMROW_c MROW_c + PDD_c XDD_c \quad (5.24)$$

	Description	Status
PDD_c	Domestic price of good from domestic producers	variable
$PMEU_c$	Domestic price of imported good from EU	variable
$PMML_c$	Domestic price of imported good from mainland	variable
$PMROW_c$	Domestic price of imported good from ROW	variable
$PMUS_c$	Domestic price of imported good from USA	variable
MEU_c	Demand for imports from EU	variable
MML_c	Demand for imports from mainland	variable
$MROW_c$	Demand for imports from ROW	variable
MUS_c	Demand for imports from USA	variable
X_c	Demand for composite domestic and imported good	variable
XDD_c	Domestic demand for goods	variable
σA_c	elasticity of substitution between imported and domestic goods	parameter
$\gamma A1_c$	distribution parameter for imports from mainland	parameter
$\gamma A2_c$	distribution parameter for imports from EU	parameter
$\gamma A3_c$	distribution parameter for imports from USA	parameter
$\gamma A4_c$	distribution parameter for imports from ROW	parameter
$\gamma A5_c$	distribution parameter for demand from domestic production	parameter
aA_c	efficiency parameter in Armington function	parameter

Exports The domestic producers differentiate goods based on four export markets and the domestic market according to a Constant Elasticity of Substitution Function

$$XDDE_c = aT_c\Big\{\gamma T1_c EML_c^{-\rho T_c} + \gamma T2_c EEU_c^{-\rho T_c} + \gamma T3_c EUS_c^{-\rho T_c} + \gamma T4_c EROW_c^{-\rho T_c} + \gamma T5_c XDD_c^{-\rho T_c}\Big\}^{-\frac{1}{\rho T_c}} \quad (5.25)$$

The revenue to the domestic producer is

$$Cost_c[.] = PEML_c EML_c + PEEU_c EEU_c + PEUS_c EUS_c + PEROW_c EROW_c + PDD_c XDD_c \quad (5.26)$$

Minimising cost subject to the Armington function, we obtain

$$EML_c = XDDE_c\left\{\frac{PDDE_c}{PEML_c}\right\}^{\sigma T_c}\gamma T1_c^{\sigma A_c} aA_c^{\sigma T_c - 1} \quad (5.27)$$

$$EEU_c = XDDE_c\left\{\frac{PDDE_c}{PEEU_c}\right\}^{\sigma T_c}\gamma T2_c^{\sigma A_c} aA_c^{\sigma T_c - 1} \quad (5.28)$$

$$EUS_c = XDDE_c\left\{\frac{PDDE_c}{PEUS_c}\right\}^{\sigma T_c}\gamma T3_c^{\sigma A_c} aA_c^{\sigma T_c - 1} \quad (5.29)$$

$$EROW_c = XDDE_c\left\{\frac{PDDE_c}{PEROW_c}\right\}^{\sigma T_c}\gamma T4_c^{\sigma A_c} aA_c^{\sigma T_c - 1} \quad (5.30)$$

$$XDD_c = XDDE_c\left\{\frac{PDDE_c}{PDD_c}\right\}^{\sigma T_c}\gamma T5_c^{\sigma T_c} aT_c^{\sigma T_c - 1} \quad (5.31)$$

The zero profit condition is

$$PDDE_c XDDE_c = PEML_c EML_c + PEEU_c EEU_c + PEUS_c EUS_c + PEROW_c EROW_c + PDD_c XDD_c \quad (5.32)$$

	Description	Status
$PDDE_c$	Price index corresponding to domestic production $XDDE_c$	variable
$PEEU_c$	Domestic price of imported good from EU	variable
$PEML_c$	Domestic price of imported good from mainland	variable
$PEROW_c$	Domestic price of imported good from ROW	variable
$PEUS_c$	Domestic price of imported good from USA	variable
EEU_c	Demand for exports to EU	variable
EML_c	Demand for exports to mainland	variable
$EROW_c$	Demand for exports to ROW	variable
EUS_c	Demand for exports to USA	variable
$XDDE_c$	Demand for domestic production of commodity c	variable
σT_c	elasticity of substitution between imported and domestic goods	parameter
$\gamma T1_c$	distribution parameter for exports from mainland	parameter
$\gamma T2_c$	distribution parameter for exports from EU	parameter
$\gamma T3_c$	distribution parameter for exports from USA	parameter
$\gamma T4_c$	distribution parameter for exports from ROW	parameter
$\gamma T5_c$	distribution parameter for demand from domestic production	parameter
aT_c	efficiency parameter in CET function	parameter

Export Demand To clear the export market the demand for exports has to equal the supply of exports. The supply of exports to different trading partners depends on the the price elasticity of exports, benchmark level of export demand, the price of exports in domestic currency and the corresponding exchange rate

$$EDML_c = EDIML_c\{\frac{PWEML \times ERML}{PEML_c}\}^{elasE_c} \quad (5.33)$$

$$EDEU_c = EDIEU_c\{\frac{PWEEU \times EREU}{PEEU_c}\}^{elasE_c} \quad (5.34)$$

$$EDUS_c = EDIUS_c\{\frac{PWEUS \times ERUS}{PEUS_c}\}^{elasE_c} \quad (5.35)$$

$$EDROW_c = EDIROW_c\{\frac{PWEROW \times ERROW}{PEROW_c}\}^{elasE_c} \quad (5.36)$$

	Description	Status
$PWEEU_c$	Foreign currency price of exports to EU	variable
$PWEML_c$	Foreign currency price of exports to mainland	variable
$PWEROW_c$	Foreign currency price of exports to ROW	variable
$PWEUS_c$	Foreign currency price of exports to USA	variable
$EDEU_c$	Demand for exports to EU	variable
$EDML_c$	Demand for exports to mainland	variable
$EDROW_c$	Demand for exports to ROW	variable
$EDUS_c$	Demand for exports to USA	variable
$EDIEU_c$	Benchmark demand for exports to EU	variable
$EDIML_c$	Benchmark demand for exports to mainland	variable
$EDIROW_c$	Benchmark demand for exports to ROW	variable
$EDIUS_c$	Benchmark demand for exports to USA	variable
$elasE_c$	price elasticity of export demand	parameter

Export Prices They are determined by equating the export supply and demand

$$EDML_c = EML_c \quad (5.37)$$

$$EDEU_c = EEU_c \quad (5.38)$$

$$EDUS_c = EUS_c \quad (5.39)$$

$$EDROW_c = EROW_c \quad (5.40)$$

Balance of Payments This is expressed in units of foreign currency and accounts for all trade and capital flows and differentiated by trade partners

$$SML = \sum_c \{\frac{MML_c PWMML_c - EML_c PWEML_c}{ERML}\} + SGML \quad (5.41)$$

$$SEU = \sum_c \{\frac{MEU_c PWMEU_c - EEU_c PWEEU_c}{EREU}\} + SGEC \quad (5.42)$$

$$SUS = \sum_c \{\frac{MUS_c PWMUS_c - EUS_c PWEUS_c}{ERUS}\} - TRGUS \quad (5.43)$$

$$SROW = \sum_c \{\frac{MROW_c PWMROW_c - EROW_c PWEROW_c}{ERROW}\} - TRGW \quad (5.44)$$

	Description	Status
$ERML$	Exchange rate with respect to mainland	variable
$EREU$	Exchange rate with respect to EU	variable
$ERUS$	Exchange rate with respect to USA	variable
$ERROW$	Exchange rate with respect to ROW	variable
SEU	Current account balance with EU	variable
SML	Current account balance with mainland	variable
SUS	Current account balance with USA	variable
$SROW$	Current account balance with ROW	variable
$SGML$	Net transfers to Azores from mainland	variable
$SGEC$	Net transfers to Azores from European Commission	variable
$TRGUS$	Transfers to Regional Govt of Azores from USA	variable
$TRGW$	Transfers to Regional Govt of Azores from ROW	variable

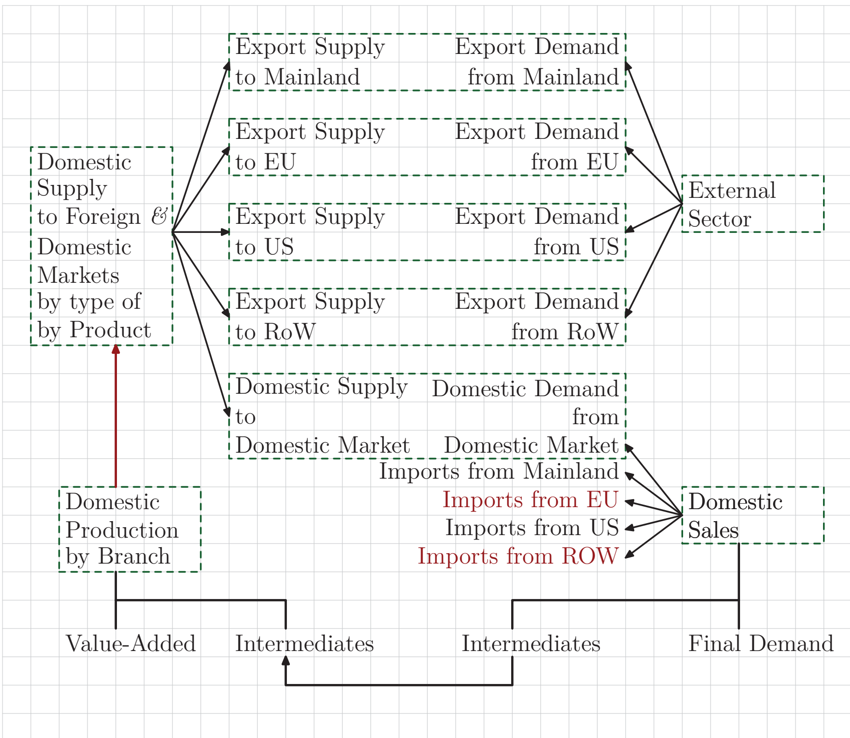

Fig. 5.3.: Foreign Trade Specification

5.2.4. Government

Revenue The regional government generates revenue through tax collections on income and wealth, products and production and receives transfers from the main-

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land government, EU and external sector

$$GREV = TRPROP + TRPROD + TRANSR \tag{5.45}$$

	Description	Status
$GREV$	Total Revenue of the Regional Government	variable
$TRPROP$	Revenue of Regional Government from taxes on income and wealth	variable
$TRPROD$	Revenue of Regional Government from taxes on products and production	variable
$TRANSR$	Total transfers received by the regional government	variable

Taxes on Income and Wealth These are given by

$$TRPROP = \sum_{qu}[ty_{qu}YH_{qu}] + \sum_{s}[tk_s KSK_s PK_s] \tag{5.46}$$

Taxes on Products and Production These are given by

$$\begin{aligned} TRPROD = & \sum_{s} tp_s XD_s PD_s \\ & + \sum_{c,qu} \{[P_c + \sum_{ctm} tchtm_{ctm,c,qu} P_{ctm}][texc_{c,qu} + (1 + texc_{c,qu})][tc_{c,qu} + vatc_{c,qu}]C_{c,qu}\} \\ & + \sum_{c}[P_c + \sum_{ctm} tcitm_{ctm,c} P_{ctm}] vatic_c \,.\, I_c \\ & + \sum_{c,s} \{(1 - tsic_{c,s})P_c + \sum_{ctm} tcictm_{ctm,c,s} P_{ctm}\} vatic_{c,s} \,.\, io_{c,s} XD_s \\ & + \sum_{c}[tmus_c PWMUS_c ERUS \,.\, MUS_c] \\ & + \sum_{c}[tmrw_c ERROW \,.\, PWMROW_c MROW_c] \end{aligned} \tag{5.47}$$

	Description	Status
I_c	Investment demand for commodity c	variable
$io_{c,s}$	input-output coefficient corresponding to intermediate consumption	
$tcictm_{ctm,c,s}$	quantity of commodity ctm used as trade-transport margin per unit of intermediate consumption	
$tcitm_{ctm,c}$	quantity of commodity ctm used as trade-transport margin on per unit investment good c	
$tmus_c$	Tariffs on commodity c imported from USA	
$tmrw_c$	Tariffs on commodity c imported from ROW	
$vatic_{c,s}$	value added tax on intermediate consumption of commodity c by sector s	
$vatic_c$	value added tax on investment of commodity c	

Transfers The total transfers received by the regional government are given by the sum total of transfers from the mainland, EU as direct subsidies and other transfers, UA and the Rest of the World, all in local currency

$$\begin{aligned} TRANSR = & TRGML \,.\, ERML + TRGEU \,.\, EREU + TRGEC \,.\, EREU \\ & + TRGUS \,.\, ERUS + TRGW \,.\, ERROW \end{aligned} \tag{5.48}$$

	Description	Status
$TRGEC$	Transfers from the EU as production subsidies	variable
$TRGEU$	Other transfers from the EU	variable

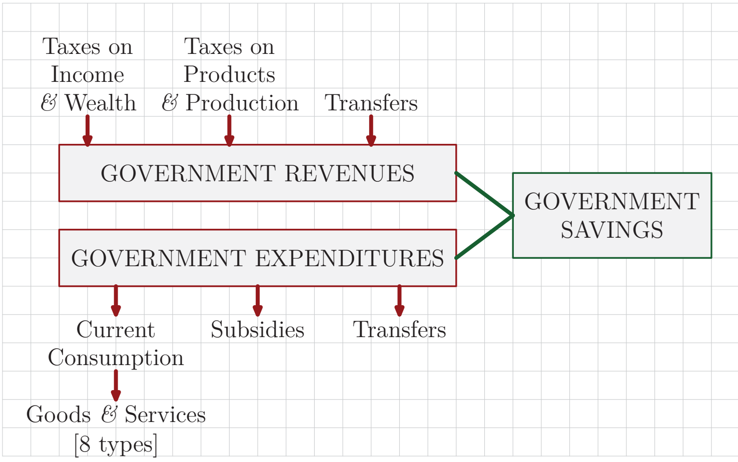


Fig. 5.4.: Structure of Regional Government Budget

Regional Government expenditures Comprise of government expenditures, public current consumption and total transfers by the government

$$GEXP = CGBUD + TRANS + SUBSID \tag{5.49}$$

	Description	Status
$GEXP$	Government Expenditure	variable
$CGBUD$	Current public consumption	variable
$TRANS$	Government transfers	variable
$SUBSID$	Government subsidies on products and production	variable

Public Consumption This is determined by maximising a Cobb-Douglas production function over commodities c

$$U(CG_c) = \prod_c CG_c^{\alpha CG_c} \tag{5.50}$$

subject to the budget constraint

$$CGBUD = \sum_c P_c \centerdot CG_c \quad \text{where} \quad \sum_c \alpha CG_c = 1 \tag{5.51}$$

This yields the public demand for commodities

$$CG_c = \frac{\alpha CG_c \centerdot CGBUD}{P_c} \tag{5.52}$$

	Description	Status
CG_c	Government Expenditure	variable
αCG_c	Current public consumption of commodity c	parameter

Transfers include transfers to households weighted by a Laspeyeres consumer price index

$$TRANS = \sum_{qu} TRHG_{qu} \cdot PCINDEX \tag{5.53}$$

	Description	Status
$PCINDEX$	consumer price index	variable

Subsidies One part of subsidies are from the EU that are transferred to the regional government, which allocates them across different sectors according to

$$TRGEC \cdot EREU = MUtspeu \sum_{s} [tspeuea_s + tspeufi_s + tspeuer_s + tspeues_s] XD_s \cdot PD_s \tag{5.54}$$

The total subsidies on production are further divided as

$$\begin{aligned} SUBSID = & \sum_{c,s} tsic_{c,s} \cdot P_c \cdot io_{c,s} \cdot XD_s \\ & + \sum_{s} \big(tsp_s + [tspeuea_s + tspeufi_s + tspeuer_s + tspeues_s] \cdot MUtspeu + tspusa_s\big) XD_s \cdot PD_s \end{aligned} \tag{5.55}$$

	Description	Status
$MUtspeu$	scaling parameter to ensure consistency between EU funds and subsidy by sectors	parameter
tsp_s	subsidy rate on production in sector s	parameter
$tspeuea_s$	subsidy rate on production in sector s from the European Agricultural Guidance and Guarantee Fund (EAGGF)	parameter
$tspeufi_s$	subsidy rate on production in sector s from Financial Instrument for Fisheries Guidance (FIFG)	parameter
$tspeuer_s$	subsidy rate on production in sector s from the European Regional Development Fund (ERDF)	parameter
$tspeues_s$	subsidy rate on production in sector s from the European Social Fund (ESF)	parameter
$tspusa_s$	subsidy rate on production in sector s	parameter

Government Savings It is the difference between government revenue and government expenditures

$$SG = GREV - GEXP \tag{5.56}$$

	Description	Status
SG	government saving	variable

Accounting Ratios The various ratios for accounting government revenue and expenditure as a fraction of the GDP at market prices are

$$rTRPROPGDP = \frac{TRPROP}{GDPC}100 \tag{5.57}$$

$$rTRPRODGDP = \frac{TRPROD}{GDPC}100 \tag{5.58}$$

$$rTRANSRGDP = \frac{TRANSR}{GDPC}100 \tag{5.59}$$

$$rCGBUDGDP = \frac{CGBUD}{GDPC}100 \tag{5.60}$$

$$rTRANSGDP = \frac{TRANS}{GDPC}100 \tag{5.61}$$

$$rSUBSIDGDP = \frac{SUBSID}{GDPC}100 \tag{5.62}$$

$$rSGGDP = \frac{SG * GDPDEF}{GDPC}100 \tag{5.63}$$

	Description	Status
$rCGBUDGDP$	ratio of regional government current expenditure to GDP	variable
$rSGGDP$	ratio of regional government saving to GDP	variable
$rSUBSIDGDP$	ratio of total subsidies of regional government to GDP	variable
$rTRANSGDP$	ratio of total transfers of regional government to GDP	variable
$rTRANSRGDP$	ratio of total transfers RECEIVED by regional government to GDP	variable
$rTRPRODGDP$	ratio of total revenues of regional government from taxes on products and production to GDP	variable
$rTRPROPGDP$	ratio of total revenues of regional government from taxes on income and wealth to GDP	variable

5.2.5. Mainland Government

It collects all social security contributions, provides unemployment benefits and makes transfers to households and regional government.

Social security contribution are derived by applying the social security contributions rate to gross wages. Unemployment benefits received by each household income group are determined by the combination of the replacement rate, the national wage rate, the total number of unemployed and the share of unemployed subject to unemployment benefits in each household income group.

The net transfers from the mainland government to the Azores are given by

$$\begin{aligned} SGML = & \sum_s \left\{ \frac{tl_s}{1 - tl_s} LSK_s PL \frac{(1 + premLSK_s)}{ERML} \right\} - \sum_{qu} TRHML_{qu} \\ & - \sum_{qu} \left\{ shUNEMPB_{qu} trep \,.\, PL \,.\, \frac{UNEMP}{ERML} \right\} - TRGML \end{aligned} \tag{5.64}$$

	Description	Status
$TRGML$	transfers to regional government from mainland	variable

5.2.6. European Commission

The commission provides EU funds as direct subsidies to the production sectors and other EU funds to the regional government. The net transfers to the Azores by the commission are given by

$$SGEC = -TRGEC - TRGEU \tag{5.65}$$

5.2.7. Investment

The total savings used to buy investment goods is given by

$$S = \sum_{qu} SH_{qu} + SF + SG \, . \, GDPDEF + SML \, . \, ERML + SEU \, . \, EREU + SUS \, . \, ERUS \\ + SROW \, . \, ERROW + \sum_{s} DEP_s \, . \, PI \tag{5.66}$$

	Description	Status
DEP_s	depreciation related to public and private capital stock in sector s	variable

Depreciation It is valued at the price index of investment and related to public and private capital stock

$$DEP_s = d_s KSK_s \tag{5.67}$$

Total investment in real terms is given by

$$ITT = \frac{S - \sum_c SV_c}{PI} \tag{5.68}$$

$$SV_c = svr_c X_c \tag{5.69}$$

	Description	Status
ITT	total investment in real terms	variable
SV_c	inventories of commodity c	variable
svr_c	share of inventories in domestic sales of commodity c	parameter

The optimal allocation of investment across different commodities is given by a Leontief function

$$I_c = ioI_c ITT \tag{5.70}$$

	Description	Status
I_c	demand for investment commodity c	variable
ioI_c	Leontief parameter for investment demand by type of investment good	parameter

Price index of investment good is the weighted average of the price of investment goods

$$PI = \sum_c \left\{ (1 + vati_c)[P_c + \sum_{ctm} tcitm_{ctm,c} \times P_{ctm}] ioI_c \right\} \tag{5.71}$$

5.2.8. Price Equations

The domestic prices for imports from the mainland and EU attract no tariffs, while those from the USA and Rest of the World do and are obtained by using the respective exchange rates to convert to domestic currency.

$$PMML_c = PWMML_c \times ERML \tag{5.72}$$

$$PMEU_c = PWMEU_c \times EREU \tag{5.73}$$

$$PMUS_c = PWMUS_c \times ERUS \times (1 + tmus_c) \tag{5.74}$$

$$PMROW_c = PWMROW_c \times ERROW \times (1 + tmrw_c) \tag{5.75}$$

Consumer Prices They are given by

$$PCT_{c,qu} = [P_c + \sum_{ctm} tchtm_{ctm,c,qu} \times P_{ctm}](1 + texc_{c,qu})(1 + tc_{c,qu} + vatc_{c,qu}) \tag{5.76}$$

$$PCTZ_{c,qu} = [PZ_c + \sum_{ctm} tchtmz_{ctm,c,qu} \times PZ_{ctm}](1 + texcz_{c,qu})(1 + tcz_{c,qu} + vatcz_{c,qu}) \tag{5.77}$$

	Description	Status
$PCT_{c,qu}$	consumer prices inclusive of tax paid by household qu for commodity c	variable
$PCTZ_{c,qu}$	benchmark consumer prices inclusive of tax paid by household qu for commodity c	constant

Consumer Price Index used in the model is defined as

$$PCINDEX = \frac{\sum_{c,qu} PCT_{c,qu} \times CZ_{c,qu}}{\sum_{c,qu} PCTZ_{c,qu} \times CZ_{c,qu}} \tag{5.78}$$

	Description	Status
$PZ_c = 1$	benchmark price of commodity c	constant
$tchtmz_{ctm,c,qu}$	benchmark value of trade-transport margin per unit of private consumption	parameter
$texcz_{c,qu}$	benchmark value of excise duties on commodity c paid by household qu	parameter
$tcz_{c,qu}$	benchmark value of other taxes on commodity c paid by household qu	parameter
$vatcz_{c,qu}$	benchmark value of vat on commodity c paid by household qu	parameter
$CZ_{c,qu}$	benchmark value of consumer demand for commodity c by household qu	constant

5.2.9. Labour Market

The relationship between labour supply and labour demand is given by

$$\sum_s LSK_s = LSR - UNEMP \tag{5.79}$$

	Description	Status
LSR	active population	variable

The response of the real wage to the labour market conditions is given by a wage curve [de Galdeano and Turunen (2006)]

$$\ln\left[\frac{PL}{PCINDEX}\right] = elasU \times \ln(UNRATE) + err \tag{5.80}$$

	Description	Status
$UNRATE$	unemployment rate	variable
$elasU$	unemployment elasticity	parameter
err	error	calibrated parameter

Labour supply It is given by

$$LSR = LSRI\left[\frac{PL(1-tyavr)PCINDEXZ}{PLZ(1-tyavrz)PCINDEX}\right]^{elasLS} \tag{5.81}$$

	Description	Status
$LSRI$	benchmark level of labour supply	variable
$PCINDEXZ$	benchmark level of consumer price index	variable
PLZ	benchmark level of wage rate	variable
$elasLS$	elasticity of labour supply	parameter
$tyavr$	average income tax rate	variable
$tyavrz$	benchmark level of average income tax rate	parameter

National Employment It is defined as

$$EMPN = LSR - UNEMPN \tag{5.82}$$

	Description	Status
$EMPN$	national employment	variable

The national average wage including social security contributions is

$$PLAVRT(LSR - UNEMP) = \sum_s PL\left[1 + \frac{tl(sec)}{(1-tl(sec))}\right](1 + premLSK_s)LSK_s \tag{5.83}$$

	Description	Status
$PLAVRT$	national average wage including social security contribution	variable

5.2.10. Market Clearing equations

The equilibrium in the product and factor markets implies demand equals supply at the prevailing market prices and also accounts for unemployment in the labour market. Capital stock is sector specific thus market clearing implies different returns to various sectors.

Each commodity has separate market clearing equations. For trade transport margins, the supply equals the demand from intermediate consumption, household, government, inventory and investment demand besides separate demand for trade transport services.

The demand for trade transport services invoiced separately [Löfgren *et al.* (2002)] is further derived as a demand for these services from private and intermediate consumption and investment goods.

$$X_{nctm} = \sum_{qu} C_{nctm,qu} + CG_{nctm} + I_{nctm} + SV_{nctm} + \sum_{s} io_{nctm,s} XD_s \quad (5.84)$$

$$X_{ctm} = \sum_{qu} C_{ctm,qu} + CG_{ctm} + I_{ctm} + SV_{ctm} + \sum_{s} io_{ctm,s} XD_s + MARGTM_{ctm} \quad (5.85)$$

$$MARGTM_{ctm} = \sum_{c,qu} tchtm_{ctm,c,qu} C_{c,qu} + \sum_{c} tcitm_{ctm,c} I_c + \sum_{s,c} tcictm_{ctm,c,s} io_{c,s} XD_s \quad (5.86)$$

	Description	Status
$MARGTM_{ctm}$	trade transport margins	variable

5.2.11. Other Macroeconomic Indicators

Gross Domestic Product This is derived at both, current (GDPC) and constant prices (GDP)

$$\begin{aligned} GDPC = & \sum_{c,qu} C_{c,qu}(P_c + \sum_{ctm} tchtm_{ctm,c,qu} P_{ctm})(1 + texc_{c,qu})(1 + tc_{c,qu} + vatc_{c,qu}) \\ & + \sum_{c} CG_c P_c + I_c(1 + vati_c)(P_c + \sum_{ctm} tcitm_{ctm,c} P_{ctm}) \\ & + \sum_{c} \Big\{ SV_c P_c + PEML_c EML_c + PEEU_c EEU_c + PEUS_c EUS_c \\ & + PEROW_c EROW_c - PWMML_c ERML \cdot MML_c - PWMEU_c EREU \cdot MEU_c \\ & - PWMUS_c ERUS \cdot MUS_c - PWMROW_c ERROW \cdot MROW_c \Big\} \end{aligned} \quad (5.87)$$

$$\begin{aligned} GDP = & \sum_{c,qu} C_{c,qu}(PZ_c + \sum_{ctm} tchtm_{ctm,c,qu} PZ_{ctm})(1 + texc_{c,qu})(1 + tc_{c,qu} + vatc_{c,qu}) \\ & + \sum_{c} CG_c PZ_c + I_c(1 + vati_c)(PZ_c + \sum_{ctm} tcitm_{ctm,c} PZ_{ctm}) \\ & + \sum_{c} \Big\{ SV_c PZ_c + PEMLZ_c EML_c + PEEUZ_c EEU_c + PEUSZ_c EUS_c \\ & + PEROWZ_c EROW_c - PWMMLZ_c ERML \cdot MML_c - PWMEUZ_c EREU \cdot MEU_c \\ & - PWMUSZ_c ERUS \cdot MUS_c - PWMROWZ_c ERROW \cdot MROW_c \Big\} \end{aligned} \quad (5.88)$$

$$GDPDEF = \frac{GDPC}{GDP} \quad (5.89)$$

	Description	Status
$GDPC$	GDP at current prices	variable
GDP	GDP at constant prices	variable

5.2.12. Incorporation of Dynamics

The model has a recursive dynamic structure composed of several temporary equilibria. Each year the equilibrium is achieved through market clearance of all markets and they are linked through time via investment or capital accumulation. This implies an endogenous determination of investment behavious, which in turn depends on the rate of return. The actual investment and capital accumulation depend on the expected rate of return in year $t+1$, which in turn depends on the actual rate of return in year t.

The normal rate of return in sector s is an inverse logistic function of the proportionate growth in sector s's capital stock [Blundell *et al.* (2002)]

$$ROR_{s,t} = RORH_s + \frac{1}{B_s}\left\{\ln\left[\frac{(KSKg_{s,t} - KSKgmin_s)}{(KSKgmax_s - KSKg_{s,t})}\right]\left[\frac{(KSKgmax_s - KSKtrend_s)}{(KSKtrend_s - KSKgmin_s)}\right]\right\} \quad (5.90)$$

	Description	Status
$ROR_{s,t}$	normal rate of return to capital in sector s in time t	variable
$RORH$	historic normal rate of return to capital	parameter
$KSKg_{s,t}$	growth rate of capital stock in sector s at time t	variable
$KSKgmin_s$	minimum possible growth rate of capital stock in sector s	parameter
$KSKgmax_s$	maximum possible growth rate of capital stock in sector s	parameter
$KSKtrend_s$	historic growth rate in sector s	parameter
B_s	sensitivity parameter of capital growth to variations in rate of return in sector s	parameter

The sensitivity of capital growth rate to variations in rate of return is obtained by differentiating equation 5.90 with respect to $KSKg_{s,t}$

$$\frac{\partial ROR_{s,t}}{\partial KSKg_{s,t}} = \frac{1}{B_s}\left\{\frac{1}{(KSKg_{s,t} - KSKgmin_s)} + \frac{1}{(KSKgmax_s - KSKg_{s,t})}\right\}$$

$$= \frac{1}{B_s}\left\{\frac{KSKgmax_s - KSKgmin_s}{(KSKg_{s,t} - KSKgmin_s)(KSKgmax_s - KSKg_{s,t})}\right\}$$

$$SEA = \left\{\frac{\partial ROR_{s,t}}{\partial KSKg_{s,t}}\right\}^{-1}$$

$$B_s = SEA\left\{\frac{(KSKgmax_{s,t} - KSKgmin_s)}{(KSKgmax_s - KSKtrend_s)(KSKtrend_s - KSKgmin_s)}\right\} \quad (5.91)$$

Evaluating equation 5.91 in the neighbourhood of $KSKg_{s,t} = KSKtrend_s$ provides

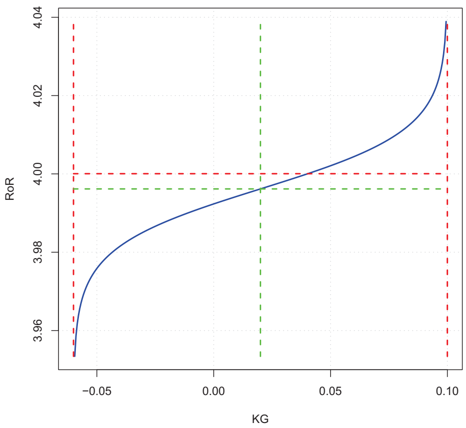
$$SEA = \left\{\frac{\partial ROR_{s,t}}{\partial KSKg_{s,t}}|_{KSKg_{s,t}=KSKtrend_s}\right\}^{-1} \quad (5.92)$$

where SEA is considered the same for all sectors in absence of reliable estimates by sector. In the absence of any reliable information we assume $SEA = 1$

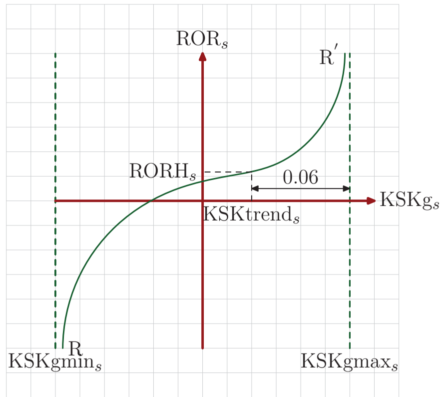
We give a small program in R [Dixon-Rimmer.R] for simulating the inverse logistic relationship between the Rate of Return (RoR) and the capital stock ($\mathrm{KSK_s}$).

```
# Dixon-Rimmer.R
# see reference [4] at the end of this chapter
# R program to generate the inverse logistic relationship
# between KSKg (capital stock) and RoR (rate of return)
SEA <- 1.0
RoRn <- 4.00
KT <- 0.04
KMin <- -0.06
Diff <- 0.06
KMax <- KT + Diff
C <- SEA*(KMax-KMin)/((KMax-KT)*(KT-KMin))
```

```
np <- 300
KG <- matrix(nr=np+1,nc=1)
RoR <- matrix(nr=np+1,nc=1)
KMaxL <- matrix(nr=np+1,nc=1)
KMinL <- matrix(nr=np+1,nc=1)
KMed <- matrix(nr=np+1,nc=1)

RoRb <- matrix(nr=np+1,nc=1)
RoRh <- matrix(nr=np+1,nc=1)

dp <- (KMax-KMin)/np

for (i in 1:(np+1))
{
  KG[i] <- KMin + (i-1)*dp
  KMinL[i] <- KMin
  KMaxL[i] <- KMax
  KMed[i] <- (KMax+KMin)/2
}

for (i in 1:(np+1))
  RoR[i] <- RoRn+(1/C)*(log(KG[i]-KMin)-log(KMax-KG[i])-log(KT-KMin)+log(KMax-KT))

ih <- min(which(KG>4.0e-02)) # for KG = Trend (4%)
ib <- which(KG==(KMax+KMin)/2) # for mid point of KG & RoR

for (i in 1:(np+1))
{
  RoRb[i] <- RoR[ib] # red horizontal dashed line
  RoRh[i] <- RoR[ih] # green horizontal dashed line (X axis)
}

#plot(KG,RoR,type='b',pch=20,col='blue',panel.first=grid())
plot(KG,RoR,type='l',lwd=2,col='blue',panel.first=grid())
lines(KMinL,RoR,type='l',lwd=2,lty=2,col='red')
lines(KMaxL,RoR,type='l',lwd=2,lty=2,col='red')
lines(KMed,RoR,type='l',lwd=2,lty=2,col='green')
lines(KG,RoRb,type='l',lwd=2,lty=2,col='green')
lines(KG,RoRh,type='l',lwd=2,lty=2,col='red')
```



a: generated by Dixon-Rimmer.R



b: illustrative

Fig. 5.5.: Expected Rate of Return $\mathbb{E}[RoR]$ v/s KSK for Industry s

Since capital stock is sector specific and non fungible, the minimum possible growth rate of capital stock is set equal to the depreciation rate in sector s. This ensures a positive investments in sector s. The maximum possible growth rate of capital stock is set at the trend + an investment limit set at 6% for all sectors.

The present value of investing a unit of capital in industry s in year t is defined as

$$PVK_{s,t} = -PI_t + \frac{PK_{s,t+1} + PI_{t+1}d_s + PI_{t+1}(1 - d_s)}{1 + NINT_t} \tag{5.93}$$

	Description	Status
$NINT_t$ parameter	nominal interest rate in time t	parameter

Purchase of one unit of capital in time t by sector s involves an immediate outgo of PI_t and two benefits in year $t+1$ discounted by $NINT_t$ *viz.* rental value of an extra unit of capital in $t+1$ $[PK_{s,t+1} + PT_{t+1}d_s]$ and the value at which the depreciated capital can be sold $[PI_{t+1}(1 - d_s)]$. The expected rate of return on investment in industry s in year t is given by dividing equation 5.93 by PI_t

$$ROR_{s,t} = -1 + \frac{\frac{PK_{s,t+1}}{PI_t} + \frac{PI_{t+1}}{PI_t}}{1 + NINT_t} \tag{5.94}$$

Under static expectations the asset prices (cost of buying a unit of capital) and net rental rates will increase by the current rate of inflation $RINF_f$. The expected rate of return under static expectations is

$$ROR_{s,t} = -1 + \frac{\frac{PK_{s,t}(1+RINF_t)}{PI_t} + \frac{PI_t(1+RINF_t)}{PI_t}}{1+NINT_t} \quad (5.95)$$

$$= -1 + \frac{\frac{PK_{s,t}}{PI_t} + 1}{1+RINT_t} \quad (5.96)$$

$$RINT_t = \frac{1+NINT_t}{1+RINF_t} \quad (5.97)$$

	Description	Status
$RINT_t$	average rate of return to capital in time t	variable
$RINF_t$	current rate of inflation at time t	parameter

The weighted average return to capital is taken as a proxy for the real interest rate in the model. The return to capital is expressed in real terms using the production price

$$RINT_t = \frac{\sum_s \frac{PK_{s,t}}{PD_{s,t}} KSK_{s,t}}{\sum_s KSK_{s,t}} \quad (5.98)$$

The capital stock in the industry s in period $t+1$ is given by

$$KSK_{s,t+1} = (1-d_s)KSK_{s,t} + INV_{s,t} \quad (5.99)$$

The growth rate of capital stock is given by

$$KSKg_{s,t} = \frac{KSK_{s,t+1}}{KSK_{s,}} - 1 \quad (5.100)$$

Combining equations 5.90 and 5.91 we can obtain the actual growth rate of capital in industry s; $KSKg_{s,t}$

$$KSKtMgmin_s = KSKtrend_s - KSKgmin_s$$

$$KSKgmaxMt_s = KSKgmax_s - KSKtrend_s$$

$$B_s = \frac{(KSKgmax_s - KSKgmin_s)}{(KSKgmaxMt_s)(KSKtMgmin_s)}$$

$$\alpha ROR_{s,t} = \exp^{\{(ROR_{s,t} - RORH_s)B_s\}}$$

$$\alpha ROR_{s,t} = \left[\frac{(KSKg_{s,t} - KSKgmin_s)}{(KSKgmax_s - KSKg_{s,t})}\right]\left[\frac{(KSKgmax_s - KSKtrend_s)}{(KSKtrend_s - KSKgmin_s)}\right] \quad (5.101)$$

$$KSKg_{s,t} = \frac{(\alpha ROR_{s,t} KSKgmax_s[KSKtMgmin_s] + KSKgmin_s[KSKgmaxMt_s])}{\alpha ROR_{s,t}[KSKtMgmin_s] + (KSKgmaxMt_s)} \quad (5.102)$$

The parameter $\alpha ROR_{s,t}$ is given by

$$\alpha ROR_{s,t} = \exp^{\left\{(ROR_{s,t} - RORH_s)\frac{(KSKgmax_s - KSKgmin_s)}{(KSKgmaxMt_s)(KSKtMgmin_s)}\right\}} \quad (5.103)$$

A first estimate of investments in the sector s in year t is derived from equations 5.99, 5.100, 5.102

$$INVS_{s,t} = KSK_{s,t}\frac{(\alpha ROR_{s,t} KSKgmax_s * KSKtMgmin_s + KSKgmin_s * KSKgmaxMt_s)}{\alpha ROR_{s,t} * KSKtMgmin_S + KSKgmaxMt_s} + d_s KSK_s \quad (5.104)$$

while the actual level of investments in sector s in year t is given by

$$INV_s = \frac{INVS_s}{\sum_s INVS_s \frac{S - \sum_c SV_c P_c}{PI}} \tag{5.105}$$

The model requires $RORH_s$ and this is obtained by solving equations 5.103 and 5.104 simultaneously for $RORH_s$ and $\alpha ROR_{s,t}$. Subsequently the model uses the calibrated value of $RORH_s$ in solving for other investment variables like $KSKg_{s,t}$, $KSK_{s,t}$, $INVS_{s,t}$ and INV_s.

5.2.13. Closure rules

The closure rules refer to the manner in which demand and supply of commodities, the macroeconomic identities and the factor markets are equilibrated ex-post. Due to the complexity of the model, a combination of closure rules is needed. The particular set of closure rules should also be consistent, to the largest extent possible, with the institutional structure of the economy and with the purpose of the model. In mathematical terms, the model should consist of an equal number of independent equations and endogenous variables. The closure rules reflect the choice of the model builder of which variables are exogenous and which variables are endogenous, so as to achieve ex-post equality. Three macro balances are usually identified in CGE models that can be a potential source of ex-ante disequilibria and must be reconciled ex-post ([Adelman and Robinson (1989)]):

- The savings-investment balance
- The government balance
- The external balance

The most widely used macro closure rule for CGE models is based on the investment and savings balance. In the model, the investment is assumed to adjust to the available domestic and foreign savings. This reflects an economy in which savings form a binding constraint. Additional assumptions are needed with regard to regional government behaviour in AzorMod. First, regional government savings are fixed in real terms while regional government total current consumption adjusts to achieve the target set with respect to the government savings. The allocation between the consumption of different goods and services is provided by a Cobb-Douglas function. Secondly, the transfers received by the regional government from the Mainland government, from the EU, from the US and from the ROW are fixed in real terms. On the expenditure side, the regional government transfers to the households are also fixed in real terms.

For the external balance, the exchange rates are kept unchanged in the simulations, while the balances of the current accounts adjust. An alternative closure is also possible where the balances of the current accounts corresponding to US and ROW are set while the real exchange rates adjust.

The setup of the closure rules is important in determining the mechanisms governing the model. Therefore, the closure rules should be established also taking into account the policy scenario in question.

According to Walras' law if $(n-1)$ markets are cleared the n^{th} one is cleared as well. Therefore, in order to avoid over-determination of the model, the current account balance with respect to ROW has been dropped (see equation 5.44). However, the system of equations guarantees, through Walras' law, that the total imports from ROW less the total exports to ROW and the transfers from ROW equals the current account balance.

5.2.14. Measure of Welfare: Compensating and Equivalent Variation

The Linear Expenditure System (LES) or Stone-Geary utility function has the form

$$\mathbb{U} = \prod_{i=1}^{n} (x_i - \gamma_i)^{\alpha_i} \quad \sum_{i=1}^{n} \alpha_i = 1 \tag{5.106}$$

where γ_i is the minimum consumption of commodity i, whose consumption is x_i.

The consumer maximises the utility subject to budget constraint $\mathbb{I} = \sum_{i=1}^{n} p_i \times x_i$. The demand for commodity i x_i at price p_i is

$$x_i = \gamma_i + \frac{\alpha_i[\mathbb{I} - \sum_{j=1}^{n} p_j \times \gamma_j]}{p_i} \tag{5.107}$$

The indirect utility function is

$$\mathbb{IU} = [\mathbb{I} - \mu] \prod_{i=1}^{n} \left[\frac{\alpha_i}{p_i}\right]^{\alpha_i} \tag{5.108}$$

where $\mu = \sum_{j=1}^{n} p_j \times \gamma_j$

Using the variables in the model the indirect utility VU_{qu} of household qu evaluates to

$$VU_{qu} = CBUD_{qu} - PCT_{c,qu} \times \mu_{c,qu} \prod_{i=1}^{n} \left[\frac{\alpha H_{c,qu}}{PCT_{c,qu}}\right]^{\alpha H_{c,qu}} \tag{5.109}$$

Compensating Variation is given by

$$\begin{aligned} CV &= \mathbb{IU}^{o}{}_{@[\mathbb{U}^o, p^o]} - \mathbb{IU}^{n}{}_{@[\mathbb{U}^o, p^n]} \\ &= [\mathbb{I}^o - \mu^o] \prod_{i=1}^{n} \left[\frac{\alpha_i}{p_i^o}\right]^{\alpha_i} - [\mathbb{I}^n - \mu^n] \prod_{i=1}^{n} \left[\frac{\alpha_i}{p_i^n}\right]^{\alpha_i} \end{aligned} \tag{5.110}$$

Equivalent Variation is given by

$$\begin{aligned} EV &= \mathbb{IU}^{n}{}_{@[\mathbb{U}^n, p^o]} - \mathbb{IU}^{o}{}_{@[\mathbb{U}^n, p^n]} \\ &= [\mathbb{I} - \mu] \prod_{i=1}^{n} \left[\frac{\alpha_i}{p_i}\right]^{\alpha_i} \end{aligned} \tag{5.111}$$

5.2.15. Variables, Parameters & Index Sets

5.2.15.1. *Index Sets*

Name of Set	Denotes: Description
qu	household
c	commodities $c = 1, .., 45$
cc	alias for c
s	sectors $s = 1, .., 45$
ss	alias for s
ctm	7 trade transport margin sectors $s \in 25, 26, 27, 29, 30, 31, 32$
$nctm$	38 NON-trade transport margin sectors
t	time 2002-2036

5.2.15.2. *Endogenous Variables*

Table 5.1.: Endogenous Variables & Their Description

Variable	Description
$C_{c,qu}$	Consumption of commodity c by household qu @ purchasers price
$CBUD_{qu}$	Disposable income of household qu
$CGBUD$	Current public consumption
CG_c	Government Expenditure
DEP_s	Depreciation related to public and private capital stock in sector s
$EDEU_c$	Demand for exports to EU
$EDML_c$	Demand for exports to mainland
$EDROW_c$	Demand for exports to ROW
$EDUS_c$	Demand for exports to USA
EEU_c	Demand for exports to EU
EML_c	Demand for exports to mainland
$EMPN$	national employment
$ERML$	Exchange rate with respect to mainland
$EREU$	Exchange rate with respect to EU
$EROW_c$	Demand for exports to ROW
$ERROW$	Exchange rate with respect to ROW
$ERUS$	Exchange rate with respect to USA
EUS_c	Demand for exports to USA
GDP	GDP at constant prices variable
$GDPC$	GDP at current prices variable
$GEXP$	Government Expenditure
$GREV$	Total Revenue of the Regional Government
I_c	Demand for investment commodity c
ITT	Total investment in real terms
KL_s	Value added in production
KSK_s	Capital stock in sector s
$KSKg_{s,t}$	Growth rate of capital stock in sector s at time t
LSK_s	Number of employees in branch s
LSK_s	Labour demand in sector s
LSR	Active population
$MARGTM_{ctm}$	Trade transport margins
MEU_c	Demand for imports from EU
MML_c	Demand for imports from mainland
MPS_{qu}	Marginal propensity to save of household qu

Continued on next page ...

Table 5.1 ... continued from previous page

Variable	Description
$MROW_c$	Demand for imports from ROW
MUS_c	Demand for imports from USA
$PCINDEX$	consumer price index
P_c	Price of commodity c NET of taxes
P_{ctm}	Price of commodities receiving trade-transport margins $ctm \subset c$
$PCT_{c,qu}$	Consumer prices inclusive of tax paid by household qu for commodity c
PDD_c	Domestic price of good from domestic producers
$PDDE_c$	Price index corresponding to domestic production $XDDE_c$
$PEEU_c$	Domestic price of imported good from EU
$PEML_c$	Domestic price of imported good from mainland
$PEROW_c$	Domestic price of imported good from ROW
$PEUS_c$	Domestic price of imported good from USA
PI	Price index of composite investment good
$PKavr_{qu}$	Real average return to capital qu
PK_s	Price of capital in sector s
PL	Price of labour
$PLAVRT$	National average wage including social security contribution
$PMEU_c$	Domestic price of imported good from EU
$PMML_c$	Domestic price of imported good from mainland
$PMROW_c$	Domestic price of imported good from ROW
$PMUS_c$	Domestic price of imported good from USA
$rCGBUDGDP$	Ratio of regional government current expenditure to GDP
$RINT$	Average rate of return to capital corresponding to firms
$ROR_{s,t}$	Normal rate of return to capital in sector s in time t
$rSGGDP$	Ratio of regional government saving to GDP
$rSUBSIDGDP$	Ratio of total subsidies of regional government to GDP
$rTRANSGDP$	Ratio of total transfers of regional government to GDP
$rTRANSRGDP$	Ratio of total transfers RECEIVED by regional government to GDP
$rTRPRODGDP$	Ratio of total revenues of regional government from taxes on products and production to GDP
$rTRPROPGDP$	Ratio of total revenues of regional government from taxes on income and wealth to GDP
SG	Government saving
SH_{qu}	Savings of household qu
SEU	Current account balance with EU
SF	Savings of firms
$SGML$	Net transfers to Azores from mainland
$SGEC$	Net transfers to Azores from European Commission
SML	Current account balance with mainland
$SROW$	Current account balance with ROW
SUS	Current account balance with USA
$SUBSID$	Government subsidies on products and production
SV_c	Inventories of commodity c
$TRANS$	Government transfers
$TRANSR$	Total transfers received by the regional government
$TRGML$	transfers to regional government from mainland
$TRHG_{qu}$	Transfers to households from local government
$TRHML_{qu}$	Transfers to households from mainland
$TRPROP$	Revenue of Regional Government from taxes on income and wealth
$TRPROD$	Revenue of Regional Government from taxes on products and production
$tyavr$	Average income tax rate for the economy
$UNEMP$	Number of unemployed

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Table 5.1 ... continued from previous page

Variable	Description
$UNRATE$	Unemployment rate
X_c	Demand for composite domestic and imported good
XD_s	Domestic production in sector s
XDD_c	Domestic demand for goods
$XDDE_c$	Demand for domestic production of commodity c
YH_{qu}	Income of household qu
$\alpha ROR_{s,t}$	Parameter is supply of capital function

5.2.15.3. *Parameters*

Table 5.2.: Parameters & Their Description

Parameter	Description
αCG_c	Current public consumption of commodity c
$\alpha H_{c,qu}$	marginal budget shares in Stone-Geary utility function: commodity c of household qu
$\gamma A1_c$	distribution parameter for imports from mainland
$\gamma A2_c$	distribution parameter for imports from EU
$\gamma A3_c$	distribution parameter for imports from USA
$\gamma A4_c$	distribution parameter for imports from ROW
$\gamma A5_c$	distribution parameter for demand from domestic production
γFK_s	CES distribution parameter for capital in production
γFL_s	CES distribution parameter for labour in production
$\gamma T1_c$	distribution parameter for exports from mainland
$\gamma T2_c$	distribution parameter for exports from EU
$\gamma T3_c$	distribution parameter for exports from USA
$\gamma T4_c$	distribution parameter for exports from ROW
$\gamma T5_c$	distribution parameter for demand from domestic production
ρF_s	elasticity of substitution parameter in production
σA_c	elasticity of substitution between imported and domestic goods
σF_s	elasticity of substitution in production
σT_c	elasticity of substitution between imported and domestic goods
aA_c	efficiency parameter in Armington function
aF_s	efficiency parameter in CES production function
aKL_s	fixed share of value added in production
aT_c	efficiency parameter in CET function
B_s	sensitivity parameter of capital growth to variations in rate of return in sector s
d_s	depreciation rate in sector s
$elasE_c$	price elasticity of export demand
$elasLS$	elasticity of labour supply
$elasS_{qu}$	elasticity of savings with respect to after-tax rate of return of household qu
$elasU$	unemployment elasticity
$io_{c,s}$	input-output coefficient corresponding to intermediate consumption
ioI_c	Leontief parameter for investment demand by type of investment good
$KSKgmin_s$	minimum possible growth rate of capital stock in sector s
$KSKgmax_s$	maximum possible growth rate of capital stock in sector s
$KSKtrend_s$	historic growth rate in sector s
$MPSZ_{qu}$	benchmark marginal propensity to save of household qu

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Table 5.2 ... continued from previous page

Parameter	Description
$\mu H_{c,qu}$	minimum consumption of commodity c by household qu
$\mu H_{c,qu}$	marginal budget shares in Stone-Geary utility function: commodity c household qu
$MUtspeu$	scaling parameter to ensure consistency between EU funds and subsidy by sectors
$PKavrZ_{qu}$	benchmark level of $PKavr_{qu}$ qu
$premLSK_s$	wage premium over average wage to labour in sector s
$RORH$	historic normal rate of return to capital
$shUNEMPB_{qu}$	share of unemployment benefits of household qu
$shYKF$	share of savings in capital income of firms
$shYKH_{qu}$	share of household qu in capital income
$shYLH_{qu}$	share of household qu in labour income
svr_c	share of inventories in domestic sales of commodity c
$tc_{c,qu}$	other tax rate on consumption of commodity c by household qu
$tchtm_{ctm,c,qu}$	quantity of commodity ctm as trade-transport service per unit of commodity c
$tchtmz_{ctm,c,qu}$	benchmark value of trade-transport margin per unit of private consumption
$tcictm_{ctm,c,s}$	quantity of commodity ctm used as trade-transport margin per unit of intermediate consumption
$tcitm_{ctm,c}$	quantity of commodity ctm used as trade-transport margin on per unit investment good c
$texc_{c,qu}$	excise duty on consumption of commodity c by household qu
$texcz_{c,qu}$	benchmark value of excise duties on commodity c paid by household qu
$tcz_{c,qu}$	benchmark value of other taxes on commodity c paid by household qu
tk_s	corporate tax rate in sector s
tl_s	social insurance contribution rate in sector s
$tmrw_c$	Tariffs on commodity c imported from ROW
$tmus_c$	Tariffs on commodity c imported from USA
$trep$	replacement rate out of national average wage
tsp_s	subsidy rate on production in sector s
$tspeuea_s$	subsidy rate on production in sector s from the European Agricultural Guidance and Guarantee Fund (EAGGF)
$tspeufi_s$	subsidy rate on production in sector s from Financial Instrument for Fisheries Guidance (FIFG)
$tspeuer_s$	subsidy rate on production in sector s from the European Regional Development Fund (ERDF)
$tspeues_s$	subsidy rate on production in sector s from the European Social Fund (ESF)
$tspusa_s$	subsidy rate on production in sector s
ty_{qu}	rate of income tax on household qu
$tyavrz$	benchmark level of income tax rate
tyz_{qu}	benchmark rate of income tax on household qu
$vatc_{c,qu}$	value added tax on consumption of commodity c by household qu
$vatcz_{c,qu}$	benchmark value of vat on commodity c paid by household qu
$vatic_{c,s}$	value added tax on intermediate consumption of commodity c by sector s
$vati_c$	value added tax on investment of commodity c

5.2.15.4. *Exogenous Variables*

Table 5.3.: Exogenous Variables & Their Description

Variable	Description
CCT_c	Total private consumption of commodity c
$CZ_{c,qu}$	benchmark value of consumer demand for commodity c
$EDIEU_c$	Benchmark demand for exports to EU
$EDIML_c$	Benchmark demand for exports to mainland
$EDIROW_c$	Benchmark demand for exports to ROW
$EDIUS_c$	Benchmark demand for exports to USA
$ERML$	exchange rate mainland
$GDPDEF$	GDP Deflator
KSK_s	capital demand in sector s
$LSRI$	benchmark level of labour supply
$PCINDEXZ$	benchmark level of consumer price index
PLZ	benchmark level of wage rate
$PWEEU_c$	Foreign currency price of exports to EU
$PWEML_c$	Foreign currency price of exports to mainland
$PWEROW_c$	Foreign currency price of exports to ROW
$PWEUS_c$	Foreign currency price of exports to USA
PZ_c	benchmark value of price level of domestic commodity
$TRGEC$	Transfers from the EU as production subsidies
$TRGEU$	Other transfers from the EU
$TRGUS$	Transfers to Regional Govt of Azores from USA
$TRGW$	Transfers to Regional Govt of Azores from ROW

Table 5.4.: Activity and Commodity Disaggregation in AzorMod

No.	Sector Name and Description
1	Agriculture, Hunting & Forestry, Logging
2	Fishing
3	Mining and Quarrying
4	Production of Meat and Meat Products
5	Processing of Fish and Fish Products
6	Manufacture of Dairy Products
7	Prepared Animal Feeds
8	Beverages and Tobacco Products
9	Fruits, Vegetables, Animal oils, Grain mill and Starches
10	Textile and Leather
11	Wood, Products of Wood and Cork
12	Pulp, paper products; publishing and printing
13	Coke, refined petroleum products and nuclear fuel
14	Chemical and Chemical products
15	Rubber and Plastic Products
16	Other non-metallic mineral products
17	Basic metals and fabricated metal products
18	Machinery and equipment n.e.c
19	Electrical and optical equipment
20	Transport Equipment
21	Manufacturing n.e.c
22	Electricity, gas, steam and hot water supply
23	Collection, purification and distribution of water
24	Construction
25	Sale, maintenance, repair of motor vehicles and motorcycles
26	Wholesale trade and commission trade, except for motor vehicles and motorcycles
27	Retail trade, except for motor vehicles and motorcycles
28	Hotels and restaurants
29	Land Transport; transport via pipelines
30	Water transport
31	Air transport
32	Supporting transport activities; activities of travel agencies
33	Post and telecommunications
34	Financial Intermediation, excluding insurance and pension funding
35	insurance and pension funding except compulsory social security
36	Activities auxiliary to financial intermediation
37	Real estate activities
38	Renting of Machinery and equipment without operator
39	Computer and related activities; research and development
40	Other business activities
41	Public Administration and defence; compulsory social security
42	Education
43	Health and social work
44	Other community, social and personal service activities
45	Activities of households as employers of domestic staff

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6

The 2001 SAM

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6.1. Introduction

This chapter deals with the input-output (IO) table and the Social Accounting Matrix (SAM) for the Azores. The foremost requirement for any CGE model is the presence of a SAM. The SAM is a table of incomes and expenditures of all agents in the economy. The SAM is largely based on the IO table. In the case of any country the IO table should be available from the Statistical service. However in the case of the Azores, the Autonomous Region of the Azores as it is termed, is a Portuguese territory and the economic activity is integral to Portugal. For policy evaluation these islands belong to the peripheral regions of Europe and are classified as such by the European Union. This necessitates the creation of a SAM and hence an IO table only for the islands for any meaningful policy evaluation for these islands. This implies separate trade flows with Portugal, the EU, United States on account of the Lajes air base and the Rest of the World via tourism, another important economic activity. The farm subsidies and various transfers from the EU play an important role and affect the economic decision process. Since each island is distinct in terms of its economy and structure, it was important to estimate the IO tables by each island.

The chapter is structured as follows. The initial part deals in brief with the methodology of construction of the IO table. This is important as it outlines in brief the methodology adopted to extract the IO table for the Azores from that of Portugal. Later the structure of the SAM and the methodology adopted to create one for the Azores is outlined. The final part deals with the data exposition and the subsequent calibration of parameters for the model.

6.2. Specialization of the Input-Output Matrix

The input-output models, whether regional or local are usually constructed with a base and coefficients of localization. The coefficient of localization is defined as

$$LQ_i = \frac{\%\text{Regional Employment in sector } i}{\%\text{National Employment in sector } i} \tag{6.1}$$

When using the localization coefficients for regionalisation or specialization, there is a tendency to overvalue the regional multipliers. There exist many alternatives to localization coefficients that attenuate these tendencies. Whatever the case, the utilization of this approach assumes that the technology at the sub-regional level is similar to that at the regional level and the degree of specialization of diverse sectors at the local level.

The other approach is to calculate the margins based on the regional matrix matrix and adjust the technical coefficients through standard procedures.

The approach adopted is a mixture of both the above. One one side we directly calculate the different components of the margins by each island. On the other side when absolutely necessary rerun the coefficients of localization. After deriving the margins matrix and the structure of coefficients we obtain the matrix for each island. In summation

(1) Estimate the margins matrix for each of the islands
(2) Choose the structure of the technology that is representative of each of the islands

When we wish to estimate the regional models we normally assume the same structure as prevailing in the national one and estimate the best possible information available with the total of rows and columns equaling that at the regional level.

6.2.1. Proposal for Specialization

6.2.1.1. *Regional Assymetries in the Islands*

The diverse regions of the country are highly assymetric and how much of the economic structure and technology can be applied to all regions is debatable and can give rise to a statistical bias. In the archipelago of the Azores, there exist significant differences between the islands. These differences go beyond scale and exist as structural differences and variations in labour activity. These differences between the island of São Miguel and the rest of the islands is statistically significant prompting the extraction of the matrix for São Miguel from that of the Azores. An approach of sequential estimation of the islands from the largest to the smallest is adopted as opposed to simultaneously estimating the matrices for all islands on accounts of the diversity.

6.2.1.2. *Estimation of Margin Matrices*

The business enterprises served as the base for calculating the regional accounts and for re-estimating the different variables at the level of each island. The methodology adopted was similar to that of the INE (Instituto Nacional de Estatística). Wherever possible estimates at the level of the island were obtained for various macro-economic indicators like production by sector, intermediate consumption and income. Where all components were difficult to obtain, the coefficients were obtained through local coefficients. The matrix of margins was obtained by grouping the diverse and independently estimated component matrices in a coherent manner. The following methodology was used in the estimation of the different components.

Intermediate Consumption by Sector The vector of Intermediate Consumption by sector @ Market Prices is available at the island level. The same technical coefficients of the reference matrix and the same percentage of trade and transport margins and net taxes were used to obtain the Intermediate Consumption by sector @ Base Prices

Net Indirect Taxes The total net taxes on products were estimated through the difference in market prices and base prices on the intermediate consumption by sector.

Income To obtain the income, it was necessary to know the level of employment and amount of labour hours and total hours worked in each sector and island. The information about the employment in the census and the level of employment as disclosed in the matrix of the Azores, the labour hours on each island were estimated. The mean level of income was obtained using the level of employment.

Other Taxes Due to the unavailability of information at the level of the individual islands, the local coefficients were used.

Gross Value Added (GVA) The GVA was obtained through the information about establishments on each of the islands. Specifically it was necessary to consolidate some sectors of activity. Some sectors that were poorly represented were consolidated using the local coefficients even thought there were some exceptions.

Households The final consumption of the households was estimated through two indicators. First, the number of inhabitants in each island and the index of purchasing power of the municipality. The mean purchasing power is assumed to be 100 and the disparity from the mean gives the extent of variation of each family on each island through the purchasing power of the municipality. In case of absence of data, the aggregate consumption is same for all islands.

Public Administration

Gross Fixed Capital formation (GFCF) The ratio of GFCF to that of production was assumed to be the same for all islands.

Foreign Trade The calculation of foreign trade was the most difficult to obtain. Normally there exists data about trade from diverse regions of a country with the Rest of the World but there does not exist data on trade amongst diverse regions within a country. The trade was distinguished between intra-regional i.e. between the islands and inter-regional comprised trade of the Autonomous Region of the Azores with the rest of the country. The intra-regional trade is not necessarily similar across all islands and there exist specific activities or sectors in specific islands. There exist two solutions to the problem of lack on information on this aspect.

Input-Output Table by Island After estimating the diverse components it was possible to obtain the table of resources and employment for each of the islands. The methodology utilised for deriving the matrices for São Miguel and Terceira is as follows. The IO table of the Azores combined with the information on São Miguel and Terceira enable the estimation of margins in both these islands. The cross-entropy method is then used to obtain the individual cells of the IO table. After assigning the value of intermediate consumption per product it was necessary to rerun the reference input-output matrix. The input-output matrix that served as a reference to estimate that for São Miguel was the reference matrix for the Azores. The reference matrix for Terceira was based on the matrix after extracting the matrix of São Miguel. This sequential process was applied island by island from the largest to the smallest.

6.3. Structure of the SAM

The Social Accounting Matrix (SAM) describes the production process and the flows of transactions of economic agents i.e. Households, Firms, Governments and Rest of the World. It traces the expenditures (in column) and income (in row) of each economic agent. The SAM is a square matrix and it is always balanced i.e. the total of the expenditures (in column) always equals the total of the receipts (in row). In this description, we will mainly follow the European System of Accounts, ESA 1995. However we will slightly adjust the global standard presentation of the SAM since we will not deal with the financial account in an extended way.

The SAM is composed of following accounts or blocks:

- Commodities account
- Production account
- Factors of production account
- Institutional accounts
- Capital account
- Rest of the world account
- Other accounts

Table 6.1 depicts a template of a nation's Social Accounting Matrix (SAM). We had earlier depicted another SAM structure in table 3.1 for a rudimentary GAMS program to build a CGE model. This template is not exhaustive but a reduced version catering to the AzorMod for allowing the reader to be familiar with a structure.

Table 6.1.: A National Social Accounting Matrix (Template)

		Commodities			Activities			Factors		Institutional Accounts			Other Accounts		
		A	I	P	A	I	P	K	L	F	H	G	IVA	ET	TK
Commodities	Agricultural				IO	IO	IO				C	CG			
	Industrial				IO	IO	IO				C	CG			
	Services				IO	IO	IO				C	CG			
Activities	Agricultural	XD													
	Industrial		XD												
	Services			XD											
Factors	Capital				K	K	K								
	Labour				L	L	L								
Institutional Sectors	Firms														
	Household							KSH	LSH	TRFH					
	Government												TRCG	TREG	TRKG
Other Accounts	IVA	TRC	TRC	TRC											
	Excise	TRE	TRE	TRE											
	TKap				TRK	TRK	TRK								
	TLab				TRL	TRL	TRL								
	TProd				TRP	TRP	TRP								
	TProd										TRH				
	Tarrifs	TRM	TRM	TRM											
	Subsidy on Cdty	TRCS	TRCS	TRCS											
	Subsidy on Prod				TRPS	TRPS	TRPS								
ROW		M	M	M				KS_W	LS_W	TRF_W		TRG_W			
Change in Stocks															
Savings-Investment															
Total		Commodity Supply D & M			Net Outlay Prod Sectors			K	L	F	HH	G	IVA	ET	TK

		Other Accounts						ROW	CIS	S-I	Total
		TL	TP	TY	TM	SC	SP				
Commodities	Agricultural						E	SV	I	Total DD	
	Industrial							E	SV	I	Total DD
	Services							E	SV	I	Total DD
Activities	Agricultural										
	Industrial										
	Services										
Factors	Capital								K_W		K Income
	Labour								L_W		L Income
Institutional Sectors	Firms							$TR_W F$			F Income
	Household								$TR_W H$		HH Income
	Government	TRLG	TRPG	TRHG	TRMG	TRCSG	TRPSG	$TR_W G$			G Income
Other Accounts	IVA										IVA
	Excise										Excise
	TKap										Capital Tax
	TLab										Labour Tax
	TProd										Prod Tax
	TProd										HH inc tax
	Tarrifs										Tarrifs
	Subsidy on Cdty										Cdty Subsidy
	Subsidy on Prod										Prod Subsidy
ROW											Forex Outflow
Change in Stocks										CS	CIS
Savings-Investment										CS	Sav-Inv
Total		TL	TP	TY	TM	SC	SP	FX inflow	CIS	Inv	

Table 6.2.: Abbreviations in SAM Structure

No.	Sector Name and Description
C	Final consumption of the households
CG	Final consumption of the government
DEPR	Depreciation
E	Exports
I	Investments
IO	Intermediate consumption
K	Capital use of the branch of activities
KRoW	Capital income received from Rest of the World
KSF	Income from capital received by the firms
KSH	Income from capital received by the households
KSRoW	Income from capital transferred to the Rest of the World
L	Labour use of the branch of activities
LRoW	Labour income received from the Rest of the World
LSH	Income from labour received by the households
LSRoW	Income from labour transferred to the Rest of the World
M	Imports
SC	Total changes in inventories
SF	Firms savings
SG	Government savings
SH	Households savings
SRoW	Foreign savings
SV	changes in inventories by commodity
TRCS	Subsidies on commodities
TRFH	Transfers from the firms to the households
TRFRoW	Transfers of the firms to the Rest of the World
TRGF	Transfers of the government to the firms
TRGH	Transfers of the government to the households
TRGRoW	Transfers of the government to the Rest of the World
TRH	Taxes on the households income
TRHG	Taxes on households' income received by the government
TRK	Taxes on capital
TRKG	Taxes on capital received by the government
TRL	Taxes on labour
TRLG	Taxes on labour received by the government
TRM	Taxes and duties on imports excluding VAT
TRMG	Tariffs received by the government
TRP	Taxes on production
TRPG	Taxes on production received by the government
TRPS	Subsidies on production
TRPSG	Subsidies on production paid by the government
TRRoWF	Transfers of the Rest of the World to the firms
TRRoWG	Transfers of the Rest of the World to the government
TRRoWH	Transfers of the Rest of the World to the households
TRC	Taxes on commodities (VAT)
TRCG	Taxes on commodities (VAT) received by the government
TRE	Taxes on commodities (Excises)
TREG	Taxes on commodities (Excises) received by the government
TTM	Trade and transport margins
XD	Domestic production delivered to the domestic market and exports

6.3.1. The production account

The production account traces the intermediate inputs, factors of production and indirect taxes paid by the firms for the realisation of their production on one hand, and payments received from sales of their output on the other hand. The production block distinguishes firms by branch of activity. Therefore, each column of the production block represents one branch of activity. The column shows the expenditure of the firms of that branch i.e. the inputs or the intermediate consumption used for the production and the value added. The inputs include goods and services delivered by the other branches of activity as well as the auto-consumption of the given branch of activity. The valued added by branch of activity is composed of the capital (net capital, tax on capital and consumption of fixed capital) and labour (net labour and tax on labour) used.

Once the production is done, the row of the corresponding column explains how that domestic production is used. In fact, a share of the domestic production is used to satisfy the domestic demand and another share of it is sold abroad to satisfy the foreign demand. The production is estimated at the basic prices (see European System of Accounts, ESA 1995) i.e. the production prices without the trade and transport margins, the subsidies and taxes on products. However the basic prices include taxes and subsidies on production.

In the sections related to the construction of the database, we only present the accounts following the columns given the symmetry of the Social Account Matrix (SAM).

The intermediate consumption comes from the Input-Output Matrix 2001 of Azores. However the intermediate consumption was valued at the basic prices i.e. without trade and transport margins, subsidies and taxes on products. Given that the intermediate consumption is valued at the market prices in the calculation of the production, we need to transform the basic prices to the market prices by adding trade and transport margins, subsidies and taxes on products to the basic prices. Therefore, the margins available at twenty-five branches of activity have been transformed into forty-five branches of activity for the current version of the model. Net taxes on products on intermediate consumption come from the Input-Output Matrix 2001. They are split by branch of activity. In order to split them by commodity (product), the shares of the intermediate consumptions for each branch of activity have been used. The gross compensation of employees comes from the Input-Output Matrix 2001. This value is split between the actual social contributions (as tax on labour) and the net remuneration of employees. The total value of taxes of labour is split into the branches of activity according to their relative weights to the total compensation of employees.

The gross operating surplus is split between the consumption of fixed capital, taxes on capital and net remuneration of capital. The value of the consumption of fixed capital is obtained by applying its 1998 share in the gross operating surplus to the gross operating surplus of 2001. The total taxes on capital come from the regional government accounts of Azores.

The net taxes of production come from the Input-Output Matrix 2001. The subsidies on production are divided in five groups:

(1) Subsidies on production from the European Union: EAGGF
(2) Subsidies on production from the European Union: FIFG
(3) Subsidies on production from the European Union: ERDF
(4) Subsidies on production from the European Union: ESF
(5) Subsidies on production from the Region Government of Azores

The difference between net taxes on production and subsidies on production gives taxes on production.

6.3.2. The commodities account

The commodity account explains the trade flows at the market level. It describes the supply of commodities from the output (goods and services) from activities (producers) and the rest of the world (imports including tariffs on imported goods) on one hand, and sales of these commodities to activities as intermediate consumption and as final consumption (households, government, gross capital formation, and the rest of the world) on the other hand.

Each column of the commodities block is composed of the production at the basic prices, taxes and subsidies on products, trade and transport margins and imports.

The column total gives the total supply of the commodities in the domestic and foreign markets at the purchasers' price. The row of the corresponding column shows how the total supply is delivered to the domestic and foreign markets. The total supply is delivered to the firms as intermediate consumption, to the households as final consumption expenditure by households, to the government as final consumption expenditure by government, to the rest of the world as exports and the remaining quantity is used as the investments goods (gross fixed capital formation and changes in inventories).

In the SAM, we have distinctly trade and transport margins on intermediate consumption, on final consumption expenditure by households and on gross fixed capital formation (GFCF). The original trade and transport margins data was available at a level of thirteen branches of activity is disaggregated to the level of 21 branches of activity paying the trade and transport margins. The data contains all types of margins except transport margins on final consumption expenditure by households and on GFCF.

The transport margins for the final consumption expenditure by households are estimated by using the weight of trade margins on final consumption expenditure by households to the total trade margins. The transport margins on GFCF were null since there is no investment for the transport services.

Given the lack of split for the trade and transport margins received by the branch of activity (trade and transport branches), they are split into the trade and transport branches by using their intermediate consumption.

The taxes on products are composed of value added tax (VAT) on intermediate consumption, VAT on final consumption expenditure by households, VAT on gross fixed capital formation, import duties, excises, other taxes on products on final consumption expenditure by households. Data comes from "TABELA DE DESTINO AMPLIADA A PREÇOS BÁSICOS - AÇORES 2001" in which the VAT on intermediate and final consumptions over fifty products is provided. After transforming the table to the forty-five products of the current SAM, the shares of the VAT on final consumption expenditure by households and the VAT on the gross fixed capital formation are driven. Given the difference between the values of the VAT from the revenues of the Regional Government of Azores and those from the "TABELA DE DESTINO AMPLIADA A PREÇOS BÁSICOS - AÇORES 2001", the latter is adjusted in order to match the total VAT of the Regional Government.

The subsidies on products on intermediate consumption were obtained after splitting the net taxes on products on intermediate consumption available at branches of activity by product (commodity). Due to the lack of information about splitting of taxes on intermediate consumption on one hand and subsidies on intermediate consumption on the other hand, the positive values are assigned to the taxes and negative values are assigned to the subsidies. The taxes on intermediate consumption represent here the VAT on intermediate consumption. Excises come from the revenues of the Regional Government. They are applied on "beverages and tobacco products" and "refined petroleum products".

The other taxes on products on final consumption expenditure by households include consumption taxes, stamp taxes, taxes on financial and capital transactions, car registration taxes, taxes on entertainment, etc. The total value comes from the revenues of the Regional Government. It is split by commodity according to the final consumption expenditure by households (net of taxes and margins).

The imports come from four origins:

(1) Mainland
(2) European Union
(3) United States
(4) Rest of the World

VAT on final consumption expenditure and other taxes on products on final consumption expenditure are split by households into six income groups following the Households Budget Survey (only expenditures are available).

6.3.3. The factors of production account

The two basic factors of production are:

- Labour
- Capital

The factors of production account shows (in row) the supply of the production factors (labour and capital) to the branches of activity, government and rest of the world for the realisation of their production on one hand and the remunerations of the owner (households, firms, government and rest of the world) of these factors (in column) on the other hand.

According to the data availability, one can split labour by skills, occupation, ages, etc. The commuting problem can also be analysed and quantified. The capital can be further disaggregated, between malleable or non-malleable, new capital or old capital, etc.

Due to the lack of information on ownership of labour and capital, it is assumed that all the income from labour and capital go to the (domestic) households. The labour and capital incomes are split into six income groups following the Households Budget Survey.

6.3.4. The institutional accounts

The institutional accounts trace the income received by each economic agent (firms, households and government) and their respective expenditures.

It decomposes all types of expenditures and incomes of each economic agent. The expenditures contain the consumption of commodities, all types of transfers received from the other agents, all taxes paid to other agents, and savings.

In this block, the firms, and the government can be further disaggregated. Firms can be distinguished by type (e.g. financial and non- financial firms) and government by authority level (e.g. islands, municipalities, etc).

6.3.4.1. *The firms account*

The expenditures of firms are composed of transfers to the households, to the governments and to the rest of the world. It also includes the corporate taxes and the savings of the firms. The consumption of firms, called the intermediate consumption, and corporate taxes are presented in the production account.

The incomes of firms are the remunerations from the ownership of factors of production i.e. the remuneration of capital, and the transfers from the other economic agents i.e. households, government and rest of the world.

The interaction of firms with the other agents is not captured due to lack of data. Firms appear only in the production account.

6.3.4.2. *The households account*

The expenditures of households contain the final consumption expenditure by households, the tax on income and the savings.

The income of households contains the transfers received from the other economic agents and the remunerations from the ownership of factors of production, i.e. the remunerations of capital and labour received from the producers.

The final consumption expenditure by households at basic prices comes from the Input-Output Matrix 2001. In order to express it at the market prices (since the final demand is valued at the market prices):

- VAT on the final consumption expenditure by households
- Excise taxes
- Trade and transport margins on the final consumption expenditure by households
- Other taxes on products on the final consumption expenditure by households

are added to the households' consumption at the basic prices.

In the SAM, the final consumption expenditure by households includes final consumption expenditure by non-profit organisations serving households (NPISH).

The households' income tax comes from the revenues of the Regional Government.

The households' savings is derived from the difference between the households' revenues and the households' expenditures.

The final consumption expenditure by households, household income tax and the household savings are divided into six income groups following the Households Budget Survey.

Due to the splitting of households to six income groups, there were unbalances between the groups. Therefore, the labour income received by households and household savings are used in order to balance these accounts. It is assumed a negligible savings for the lowest group following the other European savings patterns.

6.3.4.3. *The government account*

The expenditures of government contain the final consumption expenditure by government, the transfers to the other agents and the savings. The income of government contains transfers received from the other economic agents, taxes paid by other agents and remunerations of production factors (labour and capital) received from producers.

In the SAM, the Regional Government of Azores and the Mainland are distinguished. The Government account states for the Regional Government of Azores while the Mainland is analysed as one of the agents composing the rest of the world.

The final consumption expenditure by the Regional Government comes from the Input-Output Matrix 2001.

The transfers from the Regional Government to the households are derived as the difference between the Regional Government's revenues and the Regional Government's expenditures. There is no savings by assumption on the Regional Government behaviour.

The transfers from the Regional Government to the households are split into six income groups following the Households Budget Survey.

6.3.5. The capital account

The capital account is composed of the gross fixed capital formation (GFCF) and changes in inventories. The column of GFCF gives the investment carried out by commodity. The row of the GFCF is composed of the depreciation of capital, the domestic savings and the current external balances.

Data on GFCF and changes in inventories comes from Input-Output Matrix 2001.

6.3.6. The rest of the world account

The rest of the world account describes the interaction between the domestic economy and the foreign economy.

In column or on the expenditures side, there are exports of goods and services to the rest of the world, compensation of employees from the rest of the world, capital transfers from the rest of the world, property income and net taxes on production from the rest of the world, current transfers from the rest of the world and current external balance. In row, there are imports of goods and services from the rest of world, compensation of employees to the rest of the world, capital transfers to the rest of the world, property income, net taxes on products to the rest of the world and current transfers to the rest of the world.

In the SAM, we distinguish four agents:

(1) Mainland
(2) European Union
(3) United States
(4) Rest of the World

Data on the total exports comes from the Input-Output Matrix 2001.

The transfers from the Mainland to the Regional Government of Azores come from the revenues of the Regional Government of Azores. They contain fees, fines, penalties, current transfers, transfers from the Central Government Budget State, autonomous public funds, retirement funds, State transfers to municipalities, national subsidy programs, a share of VAT revenues, other current transfers, other capital transfers and other assigned revenues.

The transfers from the European Union to the Regional Government of Azores come from the revenues of the Regional Government of Azores. They contain agriculture fund, regional development fund, social fund, EDF and cohesion fund.

The transfers from the Rest of the World to the Regional Government of Azores come from the revenues of the Regional Government of Azores. They contain property income, sale of investments assets, other assets, other capital receipts and debt.

The current external balance of each foreign agent is derived as the difference of its revenues and its expenditures.

6.3.7. The other accounts

The other accounts are composed of all types of taxes and subsidies received by the government or institutions supposed to collect taxes and to provide subsidies. All the taxes and the subsidies on products and production mentioned in the commodities and activities sections are provided. This account is very important since it gives all the details about the types of taxes and subsidies, respectively received and paid by the government. This is very useful for the analysis of impact of any specific fiscal policy on the economy.

All taxes (subsidies) except the social contributions (tax on labour) are paid (received) to (from) the Regional Government. The social contributions are paid to the Mainland (Central Government).

6.4. Balancing the Social Accounting Matrix (SAM)

[a] A SAM is constructed from different sources of information and the researcher may have valid reasons to believe in some data as opposed to others. For example the row and column totals for the SAM may be more plausible than the individual entries in the SAM. This causes a mismatch between the row and column sums. The SAM by definition has its row and column totals matching, while constructing a SAM it is not always so. Hence procedures to ensure that the row and column totals match.

6.4.1. Entropy Minimisation

Minimising entropy is one way of minimising error in the SAM or maximising the probability that the modified entries in the balanced SAM would be the state that the economy would find itself in when it is in equilibrium. The reader is referred to [Fofana *et al.* (2005)] and [Robinson and El-Said (2000)] who give an overview of this method to balance a SAM along with a GAMS code.

The following discussion can be omitted by the reader without any loss of continuity on the construction of the SAM for Azores. These sections are included only for the sake of completeness of discussion in the event the reader wants to investigate entropy from a view point of thermodynamics and probability theory.

[a]This section is written by Sameer Rege

6.4.2. Entropy and Thermodynamics

[b] Broadly speaking thermodynamics deals with the relationship between the "flow" of heat due to temperature differential leading to either conversion of energy into work or vice-versa.

All elements have phases like gas, liquid and solid. The existence of an element in a particular phase depends the temperature and pressure. The triple point of water is the temperature at which water exists in all three phases, ice, liquid and vapour.

Early physicists carried out empirical observations of relationships between pressure and temperature at constant volume and observed that $\frac{T_s}{T_i} = \lim_{n \to 0} \frac{p_s}{p_i} = 1.36609$, irrespective of the gas. Arbitrarily defining a temperature scale based on the difference between freezing point (ice point T_i) and boiling point (steam point T_s) of water as 100, i.e. $T_s - T_i = 100$, we have $T_s = 373.16$ and $T_i = 273.16$. Absolute temperature is expressed in ^oK degree Kelvin, after William Thompson, who later was called Lord Kelvin.

All bodies have mass m measured in kilograms (say). Length l is measured in metres (m) and time in seconds (sec). Velocity is rate of change of distance $v = \frac{dl}{dt}$ and acceleration is rate of change of velocity $a = \frac{dv}{dt} = \frac{d^2l}{dt^2}$. A force of 1 newton N is required to accelerate a mass of 1 kg to 1 $\frac{m}{sec^2}$. Pressure is defined as force per unit area $\frac{N}{m^2}$. 1 Pascal (Pa) is the pressure acting on a surface of area 1 m^2 under a force of 1N. Work is defined as the product of force times distance. Thus when a force of 1 newton acts through a distance of 1 m, 1 joule of work is done. From the above definitions of pressure, force and work we can infer that work is done when either volume changes with pressure remaining constant or volume and pressure both change. However no work is done if either is zero.

Only an ideal gas follows the equation $PV = nRT$ at all pressures, where P is the pressure (Pa), V the volume (m^3), T the absolute temperature, n the number of moles and R the universal gas constant equals 8314 $\frac{joules}{kg\ mol\ K}$. Compression of any gas can be adiabatic, without any heat interaction or isothermal, keeping temperature constant. Adiabatic compression leads to an increase in temperature of the gas and expansion leads to a decrease.

Formally speaking one needs to have a precise definition of heat. Sensible heat is defined as the amount of heat required to change the temperature of a body by 1^o centigrade, while latent heat is the amount of heat required to change phase and does not affect temperature. Thus ice melting to water at 0^oC and water converting to steam at 100^oC are examples of latent heat.

Steam engines were doing a lot of work (pumping water, running railways) from heat (steam generated in a boiler from heating water), but had no formal theory for the limits of work that could be obtained per unit of input. Sadi Carnot in 1824 was the first to realise that for any engine to do work it was necessary to absorb heat

[b] this section follows [Fenn (2003)]

from a heat source and reject heat to a heat sink. Obviously the temperature of the source was higher than the sink. What Carnot said was that the efficiency of the process depended only on the temperatures of the heat source and sink and greater the difference greater the work obtained. So if heat flowed from the source to the sink the system performed work (engine) and if heat flowed from the sink to the source, it was necessary to perform work on the system (refrigerator). Thus it was not possible to do work without heat rejection to a sink at lower temperature. The **second law of thermodynamics** states that it is not possible to obtain work from heat interaction with a source at constant temperature. Or *there is a particular direction for all spontaneous processes, especially flow of heat; the entropy of any isolated system can only remain constant or increase* [Fenn (2003)], pg292.

Joule discovered that there was a fixed ratio of heat to work to bring about a same change which was 778 ft-lb/BTU. This led to what is the **first law of thermodynamics**, which states that the change in total energy of the system dE equals the sum of work dW and heat dQ interactions. Thus according to the first law, $dE = dQ - dW$. In absence of work, increase in heat leads to increase in energy and in absence of heat (adiabatic) increase in work leads to decrease in energy. In other words *for any system there exists a property energy that is conserved and that can be transferred in and out of a system by either heat or work interactions*[Fenn (2003)], pg 290.

From the first and second law we obtain the following. Second law implies that to obtain work we need to have a temperature differential. First law implies that energy of a system depends on heat and work and if work is zero then all energy is due to heat. So if energy does not change and no work is done then there are no heat changes. This implies that there is no temperature differential. Thus the **zeroth law of thermodynamics** states that *if two bodies are in thermal equilibrium with a third, they are in thermal equilibrium with each other*[Fenn (2003)].

For every system there exists a property called entropy assigned a symbol S. it is linked to the changes in heat denoted by q and absolute temperature denoted by T by the expression $\frac{dq}{T} = dS$, where a small amount of heat interaction dq leads to a small change in entropy dS.

From the three laws we can infer that a temperature differential is necessary for work to be done. Any work done by the system or on the system requires heat to be introduced or removed from the system. Also the total energy of the system is conserved. So in the case where temperature differential exists but no work is done, by the conservation of energy the only solution is an increase in entropy that leads the system to equilibrium. From a work-heat perspective for an ideal gas, isothermal expansion implies temperature T is constant and the volume increases from V_1 to V_2, thus the heat change equals $dQ = RT \ln \left[\frac{V_2}{V_1}\right]$. From the definition of entropy $dS = \frac{dQ}{T} = R \ln \left[\frac{V_2}{V_1}\right]$.

Final link to probability is as follows. Consider a system (volume V_2) with two chambers separated by an insulating material. The chambers are at different temperatures (hot fluid in one, volume V_1, with N molecules and other empty). After the insulating barrier is removed, as per second law we can obtain work from the system that is dependent on the temperature differential between the two chambers. Suppose no work is done then the colder chamber will increase in temperature and the hotter region will reduce in temperature. Before the mixing the probability that the hot fluid is in the hot chamber is 1 but after the barrier is removed, the probability P_1 that the hot fluid is confined to the hot chamber goes to zero $P_1 = \left[\frac{V_1}{V_2}\right]^N \Rightarrow \ln(P_1) = N \ln\left[\frac{V_1}{V_2}\right]$ and the fluid fills up the whole space and achieves a lower temperature. On the contrary the probability P_2 that the fluid now fills the whole chamber goes to $1 \Rightarrow \ln(P_2) = 0$. The difference between the two is $\ln(P_2) - \ln(P_1) = -N \ln\left[\frac{V_1}{V_2}\right] \Rightarrow \frac{R}{N}[\ln(P_2) - \ln(P_1)] = R \ln\left[\frac{V_2}{V_1}\right] = S$. Thus we have $k[\ln(P_2) - \ln(P_1)] = S_2 - S_1 \Rightarrow k \ln P = S$, where P is the thermodynamic probability for the state whose entropy is S

6.4.3. Entropy and Probability

Since the literature on balancing the SAM deals with entropy minimisation and sometimes cross-entropy minimisation is the word used, we now give a brief description about this and its link to information theory. The explanation is entirely based on [Sivia and Skilling (2006)] and the reader is directed to this for further details. For an elegant introduction to the link between entropy and information from a thermodynamic perspective, the reader can also consult, [Ben-Naim (2008a)] and [Ben-Naim (2008b)].

6.4.3.1. *Bayesian Probability*

Denote the information we have as I, for example European names and nationalities. Consider two events A and B which are not mutually exclusive, like A is the event that a person is blue eyed and B is the event that the person is left handed. $A \cap B$ is the set of all blue eyed people who are left handed. The probability that a person is blue eyed given the total population is N is given by $P(A|I) = \frac{n_A}{N}$, where n_A is the population of blue eyed people. The probability that the person is left handed is given by $P(B|I) = \frac{n_B}{N}$, where n_B is the population of left handed people. The probability that the person is blue eyed given that all are left handed is given by $P(A|B) = \frac{P(A \cap B)}{P(B)}$ and the probability that the person is left handed given that they are all blue eyed is given by $P(B|A) = \frac{P(B \cap A)}{P(A)}$. Since $P(A \cap B) = P(B \cap A)$, we have $P(B|A) = \frac{P(A|B)P(B)}{P(A)}$. Here $P(B)$ is called the *prior* probability as it does not account for any information about B. $P(B|A)$ is called the *conditional* probability or *posterior* probability because it depends on the value of A. $P(A|B)$ is called the *likelihood* and $P(B)$ is the normalising constant.

The principle of maximum entropy can be explained as follows. Consider a die with the six faces. Each face has unique 1 to 6 dots. Since there is no reason *a priori* to assume or favour one face over the other, each face is equally probable. Hence the probability of obtaining i dots on the top face is $\frac{1}{6}$. The average value of outcomes of dots X_i obtained after a large number of trials is given by $\sum_{i=1}^{6} ip(X_i|I) = \sum_{i=1}^{6} i\frac{1}{6} = 3.5$. Now consider the case that after a large number of trials the average was 4.5 as opposed to 3.5. Formally we have $\sum_{i=1}^{6} ip(X_i|I) = 4.5$ We need to find a probability distribution that gives us this value of the mean subject to the constraint that it is a probability distribution, or $\sum_{i=1}^{6} p(X_i|I) = 1$.

Define entropy S as

$$S = -\sum_{i=1}^{6} p_i \log_e[p_i] \tag{6.2}$$

where $p_i = p(X_i|I)$ subject to

$$\sum_{i=1}^{6} p_i = 1 \quad \text{and} \quad \sum_{i=1}^{6} ip_i = 4.5$$

Why one needs to maximise equation 6.2 remains to be seen. Reverting back to the description of blue eyed and left handed people or *kangaroos* as described by [Sivia and Skilling (2006)], assume that the information is as follows. A third of all kangaroos have blue eyes and a fourth of all kangaroos are left handed. Based on this information what is the proportion of the kangaroos that are both left handed and blue eyed. Since there is no correlation between being left handed and blue eyed, *a priori* one should expect that being left handed is independent of being blue eyed and hence the probability of being both left handed and blue eyed should be $\frac{1}{3}\frac{1}{4} = \frac{1}{12}$. Formulating the problem as follows we have 4 distinct possibilities

(1) blue eyed and left handed: probability p_1
(2) blue eyed and right handed: probability p_2
(3) not blue eyed and left handed: probability p_3
(4) not blue eyed and right handed: probability p_4

Based on logic we have

(1) $p_1 + p_2 + p_3 + p_4 = 1$
(2) blue eyed are a third: $p_1 + p_2 = \frac{1}{3}$
(3) left handed are a fourth: $p_1 + p_3 = \frac{1}{4}$

denoting p_1 by x we have the following

(1) blue eyed and left handed: probability x
(2) blue eyed and right handed: probability $\frac{1}{3} - x$
(3) not blue eyed and left handed: probability $\frac{1}{4} - x$
(4) not blue eyed and right handed: probability $1 - \left[x + \frac{1}{3} - x + \frac{1}{4} - x\right] = \frac{5}{12} + x$

Using the above definition of entropy we have

$$
\begin{aligned}
S &= -\sum p_i \log_e[x_i] \\
&= -\Big\{ x\log_e[x] + \big[\frac{1}{3}-x\big]\log_e\big[\frac{1}{3}-x\big] + \big[\frac{1}{4}-x\big]\log_e\big[\frac{1}{4}-x\big] + \big[\frac{5}{12}+x\big]\log_e\big[\frac{5}{12}+x\big] \Big\}
\end{aligned}
$$

Maximising S with respect to x we have

$$
\begin{aligned}
\frac{dS}{dx} &= \log_e\big[x\big] + 1 - \log_e\big[\frac{1}{3}-x\big] - 1 - \log_e\big[\frac{1}{4}-x\big] - 1 + \log_e\big[\frac{5}{12}+x\big] + 1 = 0 \\
&= \frac{\big[\frac{5}{12}+x\big]x}{\big[\frac{1}{3}-x\big]\big[\frac{1}{4}-x\big]} = 1 \Rightarrow x = \frac{1}{12}
\end{aligned}
$$

Thus the optimal x matches the expected probability of $\frac{1}{12}$. When using other functions like p_i^2, $\log_e[p_i]$, $\sqrt{p_i}$, the correlation between left handedness and eye colour was different from zero while in case of the function $p_i \log_e[p_i]$ it was zero [Sivia and Skilling (2006)], pg112, table 5.1.

The link to thermodynamics is as follows. We know that 1 gram mole of gas has 6.023×10^{23} molecules. This is an exceptionally large number. Stirling's approximation $\log_e[n!] \approx n\log_e[n] - n$. We will need this approximation later. Now consider the diffusion of the gas in a container. We arbitrarily divide the container in M smaller boxes of the same dimension to begin with. Later we relax the assumption and can have each of the M boxes of different sizes subject to the condition that they all add up to the same volume. We will assume that $N \gg M$, that is the number of molecules are substantially greater than the number of boxes. Since the gas cannot escape we have $N = \sum_{i=1}^{M} n_i$, where n_i are the number of molecules in each box. We can expect the probability of number of molecules in each box to be $p_i = \frac{n_i}{N} \quad \forall i = 1, 2, .., M$. Each molecule can land in any of the M boxes and hence there are M^N ways in which the N molecules can occupy the container. The expected frequency F that corresponds to the probability p_i for n_i molecules to be in container i is $F(\{p_i\}) = \frac{\text{number of ways of obtaining } n_i}{M^N}$. Beginning with n_1, how many ways can n_1 molecules be chosen from N? It is ${}^{N}\mathrm{C}_{\mathrm{n_1}}$. After choosing n_1 we have $N - n_1$ left and we have to choose n_2 in ${}^{N-n_1}\mathrm{C}_{\mathrm{n_2}}$. Thus till n_M we have a total of ${}^{N}\mathrm{C}_{\mathrm{n_1}} \times {}^{\mathrm{N-n_1}}\mathrm{C}_{\mathrm{n_2}} \times {}^{\mathrm{N-n_1-n_2}}\mathrm{C}_{\mathrm{n_3}} \times ... \times {}^{\mathrm{n_M}}\mathrm{C}_{\mathrm{n_M}} = \frac{\mathrm{N!}}{\mathrm{n_1!n_2!...n_M!}}$. Thus we have the expected frequency $F(\{p_i\})$ as

$$
F(\{p_i\}) = \frac{1}{M^N}\frac{N!}{n_1!n_2!...n_M!}
$$

Taking log of both sides and using Stirling's approximation we get

$$\begin{aligned}\log[F] &= -N\log[M] + \log[N!] - \sum_{i=1}^{M}\log[n_i!] \\ &= -N\log[M] + N\log[N] - \sum_{i=1}^{M} n_i \log[n_i] \\ &= -N\log[M] - N\sum_{i=1}^{M} p_i \log[p_i] \quad \because p_i = \frac{n_i}{N} \quad \sum_{i=1}^{M} p_i = 1\end{aligned}$$

Since N and M are constants the expected frequency of allocation of molecules in the M boxes will generate a *pdf* based on maximisation of F and denoting this by S we have

$$S = -\sum_{i=1}^{M} p_i \log[p_i] \tag{6.3}$$

As shown in the earlier section this is equal to entropy of a fluid.

Now relaxing the assumption of all boxes being of the same size, let each box be of size m_i such that $\sum_{i=1}^{M} m_i = 1$. Thus the expected frequency of a molecule in each box of size m_i is

$$F(\{p_i\}) = \frac{1}{M^N}\frac{N!}{n_1!n_2!...n_M!} \times m_1^{n_1} m_2^{n_2} ... m_M^{n_M}$$

As before taking the logarithm and using Stirling's approximation we get

$$\frac{1}{N}\log_e[F] = -\sum_{i=1}^{M} p_i \left[\frac{p_i}{m_i}\right] = S \tag{6.4}$$

The equation 6.4 is called the *cross entropy* or *Shannon-James entropy* or *Kullback* number.

If we drop the negative sign from equation 6.4, then one needs to minimise the cross-entropy as opposed to maximise cross-entropy!

6.5. Data

This section highlights the various components of the SAM and outlines the procedures for obtaining additional information for parameters needed in the model description. It covers external accounts *viz.* exports and imports, components of final demand like household and government consumption and investment, besides various components of value added. Then it tabulates the input-output table and explains the methodology to obtain the trade and transport margins. The model requires various tax and subsidy rates in the benchmark. Their computation is next followed by the specification of the parameters for the demand system.

6.5.1. Exports & Imports

Table 6.3 shows the exports by destination namely Mainland, EU, USA, and RoW for the 45 sectors, while table 6.4 shows the imports from these destinations. Sectors 22,23,24,37,41,42,43 and 45 have neither exports nor imports and hence are non-tradeables.

Table 6.3.: Exports [X] €

	Mainland	EU	USA	RoW	Total X
1	102235271	38915	69053	127779	102471019
2	9731054	2139070	453620	1145927	13469671
3	2206638	0	0	1318	2207956
4	9084904	24539	426	143	9110012
5	16593712	8404945	703960	1530084	27232701
6	169941646	2442688	1098913	647883	174131129
7	120498	0	0	0	120498
8	20541088	1001	481464	246889	21270442
9	3508229	153853	112926	149388	3924396
10	3665812	0	172450	401172	4239434
11	3096203	0	7024	2469	3105696
12	176668	0	5668	9745	192081
13	0	0	0	0	0
14	395899	0	585	28686	425170
15	0	0	0	0	0
16	1203243	0	9103	34809	1247155
17	1143623	0	5591	66079	1215293
18	0	0	0	0	0
19	0	0	0	0	0
20	0	0	0	0	0
21	43553	0	1214	4440	49206
22	0	0	0	0	0
23	0	0	0	0	0
24	0	0	0	0	0
25	1545852	316	410	920	1547499
26	1736797	27382	35481	79659	1879319
27	1442963	0	0	0	1442963
28	6481303	495020	641441	1440108	9057873
29	2402749	284090	368120	826472	3881432
30	8149961	963614	1248639	2803337	13165551
31	90335206	3172886	4111388	9230531	106850011
32	10989673	1299368	1683706	3780111	17752858
33	1595360	188628	244422	548755	2577165
34	0	638939	827929	1858795	3325664
35	0	36758	47630	106935	191323
36	0	0	0	0	0
37	0	0	0	0	0
38	0	0	0	0	0
39	97411	0	731222	70895	899528
40	724836	0	1934627	4033917	6693379
41	0	0	0	0	0
42	0	0	0	0	0
43	0	0	0	0	0
44	2310183	0	35862	20037	2366081
45	0	0	0	0	0
Total	471500336	20312011	15032875	29197281	536042503

Table 6.4.: Imports [M] €

	Mainland	EU	USA	RoW	Total M
1	26206370	31232390	10188814	401634	68029208

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Table 6.4 ... continued from previous page

	Mainland	EU	USA	RoW	Total M
2	250785	371233	17664	283231	922913
3	165255	124132	53	4751	294192
4	27754613	361599	292635	62424	28471272
5	12012601	4780147	213088	3613087	20618924
6	50665461	22978	1054	3040	50692533
7	94196	898149	9003598	1974405	11970348
8	39388747	2430219	1121933	264162	43205061
9	48359774	31221911	15741371	3988969	99312024
10	38194267	28891878	1286684	3962716	72335545
11	1193422	248841	342935	75007	1860205
12	22510712	16751463	820874	399865	40482914
13	29813918	28234160	1185	0	58049263
14	15425736	72370178	750322	735502	89281738
15	12094970	25548328	630319	2322474	40596092
16	34863440	15692129	308975	914318	51778863
17	8765359	1131601	1191217	32807217	43895393
18	14517904	4037545	66650001	11540278	96745729
19	21869183	2509914	45762755	35464371	105606224
20	22239346	10282598	120323639	20979256	173824838
21	34290815	25074367	9691762	8648993	77705936
22	0	0	0	0	0
23	0	0	0	0	0
24	0	0	0	0	0
25	9909	7	3	2	9920
26	1821487	89724	34844	24902	1970958
27	1405609	101	39	28	1405777
28	8121145	623414	242101	173023	9159683
29	398117	10380	4031	2881	415409
30	1806227	336696	130755	93447	2367124
31	80678394	644302	250213	178820	81751729
32	1998889	193254	75050	53636	2320828
33	1958491	270393	105006	75045	2408936
34	16935669	1274306	494873	353672	19058520
35	6454571	577989	224460	160416	7417436
36	9284868	1223830	475270	339663	11323631
37	0	0	0	0	0
38	6870219	1633276	634277	453301	9591073
39	9821884	59076	7250276	1438810	18570047
40	56048452	5931567	6905909	1112691	69998619
41	0	0	0	0	0
42	0	0	0	0	0
43	0	0	0	0	0
44	4093985	96548	555887	427879	5174299
45	0	0	0	0	0
Total	668384789	315180624	301723876	133333915	1418623204

6.5.2. Final Demand [FD]

Comprises of private consumption C, government consumption G, change in stocks and investment I, and net exports or exports X minus imports M. Since the model has six households the private consumption is the total of consumption all individual households. Table 6.5 shows the consumption by household i (Q_i) for each commodity. Table 6.9 shows the final demand for each of the 45 commodities.

Table 6.5.: Household Consumption [C] €

	Q1	Q2	Q3	Q4	Q5	Q6	C
1	3635448	5430667	7943556	9344478	13607267	16377208	56338622
2	885327	1740508	3108693	4333163	6946023	9445488	26459202

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Table 6.5 ... continued from previous page

	Q1	Q2	Q3	Q4	Q5	Q6	C
3	20856	27734	41085	36918	78456	16569	221616
4	2029183	3682615	7269908	9681630	9946949	15612998	48223283
5	485851	907948	1687331	2507227	3537063	6200507	15325928
6	4061742	6402307	9347947	10853315	11208542	13369642	55243496
7	325210	448280	726382	874549	1056797	1208937	4640156
8	556552	3390305	10882596	16520280	17622293	27351178	76323205
9	3842463	8951647	18972544	25338009	23470157	39228021	119802842
10	2043539	5917325	11883184	18159972	20827122	49427423	108258565
11	16435	26604	48830	78373	145293	289852	605387
12	1102844	1306926	1649509	2293196	3826168	4901496	15080139
13	798458	5496452	6124097	6916274	5794095	7416733	32546108
14	5299187	2605124	1926566	2717060	5817934	10470679	28836550
15	360067	1326196	1447586	2215593	4053542	2803759	12206742
16	424206	332738	327351	449330	775729	1036088	3345442
17	104325	325478	531124	1057091	2249386	2836511	7103915
18	118397	596401	1402493	3259620	4230952	5521019	15128881
19	413802	1130138	2249981	4664695	5824964	11840636	26124215
20	12371118	16110531	20436587	27032922	38625825	35465608	150042592
21	3639257	5801569	7908942	10267040	12787999	12962394	53367202
22	2512520	3262853	4112554	5382002	7707359	7004453	29981742
23	254244	401503	541694	689254	859343	823217	3569255
24	60703	119855	235756	629927	513637	923503	2483381
25	14	35703	261579	602081	1801936	770777	3472090
26	24710	40000	73417	117835	218451	435798	910212
27	261958	541178	1418856	747735	1209277	13503708	17682712
28	2206023	5008293	8773653	17273044	16721450	52745644	102728105
29	599711	734019	2282853	2373701	1938235	409036	8337555
30	0	25959	29057	10951	0	555878	621845
31	1797054	988978	1817274	2487466	7507480	18013452	32611704
32	69040	11763	734911	1625813	3944433	8331123	14717083
33	1655556	2631885	3806693	6151566	9564261	11772532	35582493
34	0	35966	124788	751679	1935903	5808620	8656956
35	159921	499181	1148240	2460546	3750356	5058501	13076746
36	25048	40546	74419	119444	221432	441745	922634
37	1620515	3055415	7545146	14955086	25274421	39115880	91566463
38	0	0	105227	67289	5179575	5273258	10625350
39	0	0	0	47347	0	1987434	2034782
40	43088	244916	2277209	2542139	7971751	8101297	21180400
41	5463	129023	784007	3231592	4954150	2316418	11420653
42	8269	4044	499821	1975861	2836812	6335977	11660784
43	3672825	4366147	7722941	11786795	17979387	17856381	63384476
44	3657556	4621329	7493502	11882500	30532990	51152202	109340079
45	391212	307976	413627	996401	5900871	9704178	17714264
Total	61559698	99064027	168193516	247508783	350956065	542223760	1469505848

Table 6.6.: VAT on Household Consumption [TRVATCZ] €

	Q1	Q2	Q3	Q4	Q5	Q6	TRVATCZ
1	99746	149002	217948	256385	373344	449343	1545768
2	30131	59236	105801	147474	236400	321466	900508
3	1738	2311	3423	3076	6537	1381	18466
4	75839	137634	271705	361841	371757	583519	1802294
5	16736	31275	58122	86365	121839	213584	527921
6	116799	184104	268808	312096	322311	384455	1588573
7	5535	7630	12363	14885	17986	20576	78974
8	39135	238397	765233	1161659	1239149	1923257	5366830
9	162134	377717	800552	1069145	990330	1655238	5055116
10	164853	477352	958619	1464968	1680128	3987320	8733239
11	1255	2031	3728	5984	11093	22129	46219
12	80713	95649	120721	167830	280022	358722	1103657
13	64854	446445	497425	561769	470621	602418	2643531
14	424915	208892	154481	217867	466510	839590	2312255
15	29582	108957	118930	182028	333029	230350	1002876
16	34467	27035	26598	36509	63029	84184	271822

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Table 6.6 ... continued from previous page

	Q1	Q2	Q3	Q4	Q5	Q6	TRVATCZ
17	7199	22459	36649	72942	155214	195727	490190
18	9525	47978	112825	262224	340364	444145	1217060
19	32193	87924	175046	362909	453176	921190	2032438
20	1047835	1364563	1730981	2289691	3271611	3003940	12708620
21	280633	447375	609880	791720	986118	999566	4115292
22	53959	70074	88322	115585	165525	150429	643893
23	10756	16986	22916	29159	36355	34826	150997
24	5103	10076	19820	52958	43181	77639	208778
25	1	2866	20998	48331	144646	61872	278714
26	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0
28	67238	152649	267414	526469	509657	1607647	3131075
29	39346	48158	149773	155734	127164	26836	547010
30	0	2172	2431	916	0	46512	52031
31	51765	28488	52348	71653	216258	518890	939403
32	3688	628	39259	86850	210711	445046	786183
33	134890	214439	310159	501212	779269	959192	2899160
34	0	126	438	2639	6796	20392	30391
35	561	1753	4031	8638	13167	17759	45909
36	88	142	261	419	777	1550	3237
37	7385	13924	34384	68153	115180	178257	417283
38	0	0	744	476	36642	37304	75167
39	0	0	0	2637	0	110698	113335
40	2407	13680	127197	141995	445274	452510	1183062
41	0	0	0	0	0	0	0
42	0	0	0	0	0	0	0
43	10373	12331	21812	33290	50779	50432	179017
44	36485	46098	74749	118530	304571	510251	1090683
45	0	0	0	0	0	0	0
Total	3149861	5158555	8286896	11795008	15396518	22550142	66336980

Table 6.7.: Excise Taxes on Household Consumption [TREXCZ] €

	Q1	Q2	Q3	Q4	Q5	Q6	TREXCZ
1	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0
8	148049	901859	2894891	4394577	4687724	7275715	20302815
9	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0
13	347939	2395152	2668656	3013858	2524854	3231940	14182398
14	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0

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Table 6.7 ... continued from previous page

	Q1	Q2	Q3	Q4	Q5	Q6	TREXCZ
31	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0
41	0	0	0	0	0	0	0
42	0	0	0	0	0	0	0
43	0	0	0	0	0	0	0
44	0	0	0	0	0	0	0
45	0	0	0	0	0	0	0
Total	495988	3297010	5563547	7408435	7212578	10507655	34485213

Table 6.8.: Other Taxes on Household Consumption [TRCZ] €

	Q1	Q2	Q3	Q4	Q5	Q6	TRCZ
1	21010	31385	45907	54003	78638	94646	325589
2	3845	7560	13503	18821	30170	41027	114927
3	133	177	263	236	502	106	1418
4	12815	23257	45911	61142	62817	98600	304541
5	3111	5814	10805	16056	22650	39706	98143
6	25635	40406	58997	68498	70740	84379	348654
7	1922	2649	4292	5167	6244	7143	27417
8	2362	14385	46176	70098	74774	116054	323849
9	23711	55239	117076	156356	144830	242069	739281
10	10667	30888	62029	94793	108716	258006	565099
11	98	158	290	466	864	1723	3598
12	8834	10468	13213	18368	30648	39261	120792
13	3333	22946	25566	28873	24189	30963	135870
14	41906	20601	15235	21486	46008	82802	228039
15	2856	10519	11481	17573	32150	22238	96817
16	3368	2641	2599	3567	6158	8225	26558
17	538	1680	2741	5455	11608	14638	36660
18	603	3039	7146	16609	21558	28131	77086
19	2118	5784	11514	23872	29810	60595	133693
20	62953	81982	103996	137562	196555	180474	763522
21	18682	29781	40599	52704	65645	66540	273951
22	21711	28194	35536	46506	66599	60525	259070
23	2150	3396	4581	5829	7267	6962	30185
24	491	969	1907	5095	4154	7469	20086
25	1	2251	16492	37960	113609	48596	218909
26	1532	2480	4551	7305	13542	27016	56425
27	23462	48470	127078	66970	108307	1209442	1583730
28	18887	42878	75115	147881	143159	451577	879497
29	6218	7610	23669	24611	20096	4241	86446
30	0	307	344	130	0	6574	7354
31	18814	10354	19026	26043	78600	188593	341431
32	719	122	7653	16931	41077	86759	153262
33	13428	21347	30876	49896	77576	95488	288612
34	0	316	1098	6614	17034	51110	76172
35	1407	4392	10104	21651	33001	44512	115067
36	220	357	654	1050	1947	3885	8114
37	14245	26858	66324	131460	222171	343842	804900
38	0	0	923	590	45412	46233	93157
39	0	0	0	394	0	16518	16912
40	359	2041	18980	21188	66444	67524	176537
41	48	1139	6923	28537	43748	20455	100851
42	73	36	4414	17448	25051	55950	102971
43	32342	38447	68005	103790	158320	157237	558141
44	40265	50875	82494	130811	336128	563119	1203691

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Table 6.8 ... continued from previous page

	Q1	Q2	Q3	Q4	Q5	Q6	TRCZ
45	3455	2720	3653	8799	52108	85693	156427
Total	450325	696919	1249741	1779195	2740624	5166646	12083449

Table 6.9.: Final Demand [FD = C+G+I+X-M] €

	C	G	I	CIS	X	M	FD
1	56338622	0	6894060	4112849	102471019	68029208	101787342
2	26459202	0	0	519598	13469671	922913	39525558
3	221616	0	0	920818	2207956	294192	3056199
4	48223283	0	0	1374985	9110012	28471272	30237007
5	15325928	0	0	1478281	27232701	20618924	23417985
6	55243496	0	0	2303824	174131129	50692533	180985915
7	4640156	0	0	202997	120498	11970348	-7006697
8	76323205	0	0	1911987	21270442	43205061	56300572
9	119802842	0	0	4996984	3924396	99312024	29412198
10	108258565	0	723842	2311256	4239434	72335545	43197552
11	605387	0	0	416749	3105696	1860205	2267628
12	15080139	0	0	324836	192081	40482914	-24885857
13	32546108	0	0	298305	0	58049263	-25204849
14	28836550	6357986	0	477946	425170	89281738	-53184086
15	12206742	0	6778136	1890620	0	40596092	-19720594
16	3345442	0	0	1275891	1247155	51778863	-45910375
17	7103915	0	14058667	3954503	1215293	43895393	-17563016
18	15128881	0	128736845	166531	0	96745729	47286527
19	26124215	0	82451518	76696	0	105606224	3046204
20	150042592	0	89937170	631663	0	173824838	66786588
21	53367202	0	38264412	3944522	49206	77705936	17919406
22	29981742	0	0	0	0	0	29981742
23	3569255	390967	0	0	0	0	3960222
24	2483381	0	297298757	0	0	0	299782138
25	3472090	0	3920601	179730	1547499	9920	9110000
26	910212	0	231731	0	1879319	1970958	1050304
27	17682712	2665274	0	0	1442963	1405777	20385171
28	102728105	92683	0	0	9057873	9159683	102718977
29	8337555	0	0	0	3881432	415409	11803578
30	621845	0	0	0	13165551	2367124	11420272
31	32611704	0	0	0	106850011	81751729	57709985
32	14717083	0	0	0	17752858	2320828	30149113
33	35582493	0	0	0	2577165	2408936	35750722
34	8656956	0	0	0	3325664	19058520	-7075901
35	13076746	0	0	0	191323	7417436	5850632
36	922634	0	0	0	0	11323631	-10400997
37	91566463	0	34032193	0	0	0	125598656
38	10625350	0	0	0	0	9591073	1034277
39	2034782	0	1762730	0	899528	18570047	-13873007
40	21180400	0	39638169	1133718	6693379	69998619	-1352952
41	11420653	390450034	0	0	0	0	401870687
42	11660784	164976564	0	0	0	0	176637348
43	63384476	163923780	0	0	0	0	227308256
44	109340079	8744951	2355773	0	2366081	5174299	117632584
45	17714264	0	0	0	0	0	17714264
Total	1469505848	737602238	747084604	34905289	536042503	1418623204	**2106517278**

6.5.3. Value Added

The payment to factors, taxes and subsidies all make a part of the value added. Due to the different taxes, subsidies levied and given by different set of agents like the local government, mainland government, the EU, USA the components of value added by sector as quite large. We break them into three for ease of aggregation and data disclosure. Table 6.10 shows the value added due to payment and taxes on factors besides taxes and subsidies by regional government and depreciation. Table 6.11 shows the different subsidies given by the EU and the USA. The EU subsidies are specific like on fishing (FIFG). Table 6.12 shows the remaining taxes like Value Added Tax known as IVA:-Imposto Valor Acrescentado, on households, tariffs etc. The total Value Added by sector is shown in table 6.13 is the sum of all individual components of value added by each sector.

Table 6.10.: Value Added: Production €

	LZ	TRLZ	KZ	TRKZ	TRPZ	TRSPZ	DEPZ	VAPROD
1	12917426	715697	70384358	4803303	63748	-5100063	29239646	113024115
2	14100671	781255	15865575	302683	9999	-1528908	6287656	35818932
3	4071203	225567	1677899	745	11677	-236731	652806	6403167
4	5031742	278786	1901279	2229	57243	-19049	740253	7992481
5	6150247	340758	1305584	1050	20369	-11495	508136	8314648
6	9984824	553214	5261480	17138	31191	-173880	2052796	17726765
7	2033838	112686	7387883	33880	20893	-5384	2886241	12470036
8	6669281	369515	7310995	57199	42928	-20087	2865409	17295241
9	12492766	692168	714014	314	61030	-183544	277794	14054543
10	1069394	59250	1836434	1827	141815	-36963	714879	3786637
11	3538891	196074	1534558	1006	26694	-28779	597164	5865608
12	3149262	174486	630377	267	42315	-114567	245251	4127392
13	0	0	0	0		0	0	0
14	399983	22161	131816	0	61753	-17299	51262	649677
15	511283	28328	132682	0	41157	-3618	51599	761431
16	8798054	487461	5146714	9886	122703	-28910	2005344	16541252
17	5091044	282072	4088245	433	48393	-16052	1590041	11084177
18	1520860	84264	2733903	492	454495	-10631	1063376	5846759
19	551173	30538	56356		74614	-595	21916	734002
20	552253	30598	90176	4	71862	-746	35070	779216
21	683548	37872	614789	253	46626	-4437	239183	1617834
22	38165866	2114599	3657760	13766	104254	-44639	1427816	45439421
23	5339877	295859	128713		4227	-19324	50055	5799407
24	98745202	5471029	29177347	711134	308005	-403066	11623298	145632948
25	20105226	1113940	5696068	31928	92268	-63785	2227554	29203200
26	19141885	1060566	31568987	903756	242190	-134754	12628289	65410919
27	61721944	3419736	33261214	871234	148310	-97277	13273730	112598890
28	28875266	1599849	2199220	2908	133002	-149395	856383	33517234
29	10304150	570907	8872369	559	162246	-75085	3450583	23285729
30	2782427	154162	19475359	182529	4258	-5245096	7644734	24998373
31	46259473	2563030	13534304	11946	11218	-4353	5267986	67643605
32	24973417	1383665	24086810	354150	41990	-44243	9504818	60300606
33	26104691	1446344	9493171	44449	89127	-31718	3709074	40855139
34	37901904	2099974	27620204	505953	122862	-39098	10937950	79149749
35	4743921	262839	845700	701	50875	-14151	329156	6219041
36	70857	3926	94079	7	104966	-613	36589	309810
37	312586	17319	105758161	1126919	5278626	-1448490	41566420	152611541
38	1763878	97729	9029549	15572	126645	-3839	3517547	14547081
39	172298	9546	868753	646	2992	-981	338100	1391354
40	20620152	1142470	29677977	1017804	194696	-44406	11937248	64545942
41	269434491	14928157	31587092	0	0	-814432	12283869	327419177
42	143486942	7949968	5579787	983	47860	-46644	2170300	159189196
43	122446331	6784202	18261894	208876	66648	-57556	7183077	154893472
44	55682357	3085110	5248845	19665	220785	-385207	2048865	65920420
45	16636105	921732	0	0	0	0	0	17557837
Total	1155108991	63999408	544528483	11258194	9009553	-16709888	216139263	1983334003

LZ:	labour demand by sector from the SAM (labour outlays)
TRLZ:	social security contributions
KZ:	capital demand by sector (capital services)
TRKZ:	corporate income taxes
TRPZ:	taxes on production
TRSPZ:	subsidies on production by regional government
DEPZ:	depreciation by branch of activity
VAPROD:	TOTAL of above VA items

Table 6.11.: Value Added: Production Subsidies/Transfers €

	TRSPEUEA	TRSPEUFI	TRSPEUER	TRSPEUES	TRSPUSA	PRDSUB
1	-7179307	0	-4460886	0	0	-11640193
2	0	-2700911	-1337294	0	0	-4038205
3	0	0	-656064	0	0	-656064
4	-119209	0	-74071	0	0	-193279
5	0	-90272	-44696	0	0	-134968
6	-1088119	0	-676106	0	0	-1764225
7	-33695	0	-20936	0	0	-54631
8	0	0	-78107	0	0	-78107
9	0	0	-713683	0	0	-713683
10	0	0	-62835	0	0	-62835
11	0	0	-48922	0	0	-48922
12	0	0	-194756	0	0	-194756
13	0	0	0	0	0	0
14	0	0	-29407	0	0	-29407
15	0	0	-6150	0	0	-6150
16	0	0	-49144	0	0	-49144
17	0	0	-27288	0	0	-27288
18	0	0	-18071	0	0	-18071
19	0	0	-1012	0	0	-1012
20	0	0	-1269	0	0	-1269
21	0	0	-7543	0	0	-7543
22	0	0	-138545	0	0	-138545
23	0	0	-59974	0	0	-59974
24	0	0	-363008	0	0	-363008
25	0	0	-161557	0	0	-161557
26	0	0	-341311	0	0	-341311
27	0	0	-246387	0	0	-246387
28	0	0	-66277	0	0	-66277
29	0	0	-228617	0	0	-228617
30	0	0	-15970176	0	0	-15970176
31	0	0	-13254	0	0	-13254
32	0	0	-134711	0	0	-134711
33	0	0	-96573	0	0	-96573
34	0	0	-92952	0	0	-92952
35	0	0	-33643	0	0	-33643
36	0	0	-1458	0	0	-1458
37	0	0	-4679548	0	0	-4679548
38	0	0	-12401	0	0	-12401
39	0	0	-3168	0	0	-3168
40	0	0	-143459	0	0	-143459
41	0	0	-955963	0	0	-955963
42	0	0	-15844	-50123	0	-65966
43	0	0	-152680	0	0	-152680
44	0	0	-278425	0	0	-278425
45	0	0	0	0	0	0
Total	-8420329	-2791183	-32698171	-50123	0	-43959805

TRSPEUEA: EU subsidies on production - EAGGF
TRSPEUFI: EU subsidies on production - FIFG
TRSPEUER: EU subsidies on production - ERDF
TRSPEUES: EU subsidies on production - ESF
TRSPUSA: subsidies on production from USA
PRDSUB: TOTAL of above subsidies on production

Table 6.12.: Value Added: Production Taxes €

	TRVATICZ	TRVATIZ	TRMZ	TRSICZ	TRVATCZ	TREXCZ	TRCZ	PRDTAX
1	0	0	7680	-5034250	1545768	0	325589	-3155213

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Table 6.12 ... continued from previous page

	TRVATICZ	TRVATIZ	TRMZ	TRSICZ	TRVATCZ	TREXCZ	TRCZ	PRDTAX
2	79719	0	218	0	900508	0	114927	1095372
3	132453	0	3	0	18466	0	1418	152340
4	0	0	257	-137797	1802294	0	304541	1969295
5	327297	0	2775	0	527921	0	98143	956135
6	0	0	3	-71990	1588573	0	348654	1865239
7	974106	0	7961	0	78974	0	27417	1088458
8	0	0	1005	-84897	5366830	20302815	323849	25909603
9	0	0	14308	-1038253	5055116	0	739281	4770451
10	178043	0	3807	0	8733239	0	565099	9480188
11	256185	0	303	0	46219	0	3598	306306
12	336678	0	885	0	1103657	0	120792	1562013
13	911374	0	1	0	2643531	14182398	135870	17873173
14	1762703	0	1077	0	2312255	0	228039	4304075
15	1251985	116520	2142	0	1002876	0	96817	2470339
16	2897376	0	887	0	271822	0	26558	3196643
17	2089494	0	24654	0	490190	0	36660	2640998
18	1179483	2310559	56700	0	1217060	0	77086	4840889
19	2310882	1912754	62563	0	2032438	0	133693	6452329
20	322408	2103921	102467	0	12708620	0	763522	16000938
21	511588	581754	13300	0	4115292	0	273951	5495885
22	1265705	0	0	0	643893	0	259070	2168668
23	130295	0	0	0	150997	0	30185	311478
24	7499680	8535018	0	0	208778	0	20086	16263562
25	215248	0	0	0	278714	0	218909	712870
26	1339938	0	43	0	0	0	56425	1396406
27	8856	0	0	0	0	0	1583730	1592586
28	1094131	0	301	0	3131075	0	879497	5105004
29	976159	0	5	0	547010	0	86446	1609620
30	289882	0	163	0	52031	0	7354	349430
31	1676685	0	311	0	939403	0	341431	2957829
32	1132479	0	93	0	786183	0	153262	2072017
33	964120	0	131	0	2899160	0	288612	4152022
34	3593836	0	615	0	30391	0	76172	3701014
35	167449	0	279	0	45909	0	115067	328705
36	553313	0	591	0	3237	0	8114	565255
37	1750033	0	0	0	417283	0	804900	2972216
38	268492	0	789	0	75167	0	93157	437605
39	786528	0	6301	0	113335	0	16912	923076
40	3713299	0	5815	0	1183062	0	176537	5078713
41	0	0	0	0	0	0	100851	100851
42	168369	0	0	0	0	0	102971	271340
43	656975	0	0	0	179017	0	558141	1394133
44	951707	0	713	0	1090683	0	1203691	3246794
45	0	0	0	0	0	0	156427	156427
Total	44724954	15560526	319146	-6367188	66336980	34485213	12083449	167143080

TRVATICZ:	value added tax on intermediate consumption
TRVATIZ:	value added tax on gross fixed capital formation
TRMZ:	total tariffs
TRSICZ:	subsidies on intermediate consumption ∴ negative sign
TRVATCZ:	value added tax on households consumption
TREXCZ:	excise taxes
TRCZ:	other taxes on consumption paid by household
PRDTAX:	TOTAL of the above taxes

Table 6.13.: Value Added €

	VAPROD	PRDSUB	PRDTAX	VA
1	113024115	-11640193	-3155213	98228708
2	35818932	-4038205	1095372	32876099
3	6403167	-656064	152340	5899443
4	7992481	-193279	1969295	9768497
5	8314648	-134968	956135	9135815
6	17726765	-1764225	1865239	17827780
7	12470036	-54631	1088458	13503863

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Table 6.13 ... continued from previous page

	VAPROD	PRDSUB	PRDTAX	VA
8	17295241	-78107	25909603	43126736
9	14054543	-713683	4770451	18111311
10	3786637	-62835	9480188	13203990
11	5865608	-48922	306306	6122993
12	4127392	-194756	1562013	5494649
13	0	0	17873173	17873173
14	649677	-29407	4304075	4924345
15	761431	-6150	2470339	3225620
16	16541252	-49144	3196643	19688751
17	11084177	-27288	2640998	13697887
18	5846759	-18071	4840889	10669577
19	734002	-1012	6452329	7185319
20	779216	-1269	16000938	16778886
21	1617834	-7543	5495885	7106177
22	45439421	-138545	2168668	47469545
23	5799407	-59974	311478	6050911
24	145632948	-363008	16263562	161533502
25	29203200	-161557	712870	29754513
26	65410919	-341311	1396406	66466014
27	112598890	-246387	1592586	113945090
28	33517234	-66277	5105004	38555960
29	23285729	-228617	1609620	24666732
30	24998373	-15970176	349430	9377627
31	67643605	-13254	2957829	70588180
32	60300606	-134711	2072017	62237911
33	40855139	-96573	4152022	44910587
34	79149749	-92952	3701014	82757811
35	6219041	-33643	328705	6514104
36	309810	-1458	565255	873607
37	152611541	-4679548	2972216	150904209
38	14547081	-12401	437605	14972284
39	1391354	-3168	923076	2311262
40	64545942	-143459	5078713	69481197
41	327419177	-955963	100851	326564065
42	159189196	-65966	271340	159394570
43	154893472	-152680	1394133	156134925
44	65920420	-278425	3246794	68888789
45	17557837	0	156427	17714264
Total	1983334003	-43959805	167143080	**2106517278**

6.5.4. Input-Output Table

The use of commodity i in sector j denoted by a_{ij} in €s is shown in table 6.14. The matrix is 45 x 45.

Table 6.14.: Input-Output Flows a_{ij} €

i	1	2	3	4	5	6	7	8
1	8726954	0	1306	16762953	65372	101577997	18161661	7373267
2	0	1614052	0	3473	23867396	0	0	0
3	33967	300	563428	7844	85748	25940	371350	2917
4	17855	0	0	2643298	56531	82399	1154120	0
5	0	410845	0	145513	3581423	0	2144195	0
6	203	0	0	121	6052	8524499	161556	184
7	74985337	114678	0	0	0	0	1396957	0
8	1256108	1304	819	5827	718	4944	466778	5991665
9	15505	3680	328	180680	794819	5039913	18624593	1371270
10	970353	192860	45971	3219	3977	30059	5100	21995
11	342096		3372	1486	12871	6440	3400	192017
12	1151814	297335	102273	127595	1020787	13394554	1903116	1318471
13	8269421	1668631	1717296	149669	140665	919920	370306	93713
14	13883385	38169	535744	6792	3511	473164	1206912	662409

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Table 6.14 ... continued from previous page

15	448257	28200	0	195767	75164	3074056	527122	1925823
16	4067905	0	54624	17164	0	654845	21426	2206145
17	1360914	76917	33336	262597	2392071	384008	49146	983722
18	981981	96421	118447	17573	33899	470542	18379	231648
19	95355	27492	2668	2063	4351	19912	6970	7350
20	16271	172265	5818	0	0	0	0	0
21	4362	0	0	0	0	2383	0	0
22	4545841	294487	980604	218039	348551	1962605	733125	187293
23	221945	31925	9910	38784	55332	847087	31312	131010
24	548808	189752	138929	124302	149843	1399133	271605	286630
25	426381	118516	14270	50779	61988	666488	210584	1803
26	317230	97605	31622	149379	335699	3571978	208158	27705
27	2637	0	37	51	0	1255	211	3
28	598299	231143	107444	57332	133031	471202	120198	162415
29	1284042	137802	906362	178752	297156	2168575	980726	439103
30	371176	36695	174058	49872	86097	609472	278957	113335
31	1833773	2023480	195689	168733	408218	836454	465213	285164
32	651402	2262459	118331	60936	190220	630096	148415	151906
33	490501	71752	70303	91060	150029	430160	148850	93257
34	15182875	3376472	443977	396981	489526	2918692	1582455	1541143
35	124369	163432	49102	47666	56879	351705	111660	70812
36	395727	121475	51930	47412	172989	1237999	142312	90156
37	261313	205121	120260	227229	292473	1271859	461568	396748
38	3523861	74116	0	0	0	0	0	0
39	287560	13929	31878	16230	29422	253209	57969	104127
40	2055684	353270	421275	267427	361241	7897079	425793	2180448
41	0	0	0	0	0	0	0	0
42	22733	48331	11242	18574	22994	179279	31348	111732
43	1748373	37419	6952	32588	58588	72434	49340	22446
44	137821	80982	13575	18816	34942	217996	49975	118820
45	0	0	0	0	0	0	0	0

i	9	10	11	12	13	14	15	16
1	12218520	216773	2899252	0	0	42417	98549	2377
2	17854	0	0	0	0	0	0	0
3	23944	271	746	0	0	92	0	4905027
4	77529	2982	0	0	0	4781	0	0
5	129491	0	0	0	0	0	0	0
6	364844	0	0	0	0	2371	0	0
7	0	0	0	0	0	0	0	0
8	33903	283	321	3895	0	4149	0	2104
9	12249696	92	74	16	0	25991	0	1752
10	7331	2436988	8954	17217	0	31963	120355	114873
11	15620	1227	2480556	0	0	2730	10162	333447
12	907955	27974	114266	1966164	0	65975	54810	4693
13	209955	27506	63238	14728	0	384690	17773	4852557
14	135026	390816	323295	241212	0	1554809	1533115	2544513
15	774053	49164	107783	111003	0	69149	440941	
16	407858	1373	55364	127357	0	91175	66043	11470395
17	579448	25910	152289	63012	0	52731	296495	16955
18	66881	11095	34296	1941	0	18013	120977	9697
19	5770	1631	1714	2513	0	5298	25602	4274
20	0	1895	1741	0	0	5474	118263	0
21	0	44380	54014	16889	0	4128	48702	0
22	461783	103599	105715	38269	0	57509	91730	2567620
23	187716	4069	4797	4747	0	13231	0	518264
24	230865	23345	72734	46174	0	31247	21102	670265
25	58006	291	4988	48823	0	4783	2853	61595
26	451260	2506	24119	1605693	0	19862	13049	0
27	169	2	17	353	0	33	126	0
28	176085	31351	47106	42417	0	33449	28326	374261
29	405769	47272	141642	51029	0	59629	80774	132134
30	114653	13171	44291	14492	0	16106	24312	5675784
31	312914	56350	124776	178456	0	63663	71580	282183
32	174745	25304	46636	17833	0	25100	25964	236335
33	153583	16042	32678	107339	0	17108	18599	182105
34	261958	391176	398174	161031	0	62027	50044	1256192

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Table 6.14 ... continued from previous page

35	73807	14934	40863	15322	0	16996	17654	104852
36	89839	25955	33503	14466	0	27805	13972	0
37	386271	27611	50262	80914	0	49341	63290	0
38	0	0	0	42	0	0	0	0
39	53177	7247	13632	32841	0	41460	18426	5584
40	2054759	56758	90397	319250	0	210228	113157	932172
41	0	0	0	0	0	0	0	0
42	40655	6356	7739	8745	0	5839	6169	67269
43	9732	2570	10325	4777	0	1141	2348	25573
44	63093	5112	6870	38266	0	10112	3084	79847
45	0	0	0	0	0	0	0	0

i	17	18	19	20	21	22	23	24
1	23655	29497	0	2326	4596	0	30266	0
2	0	0	0	0	0	0	0	0
3	0	57	0	1	0	1359509	1411	2700783
4	0	0	0	0	3821	0	0	0
5	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0
8	3280	0	0	0	0	0	0	8869
9	0	0	0	0	0	0	0	6803
10	38579	0	82336	2431	387222	23593	3932	485413
11	73641	44194	7004	5638	171600	7009	13733	6748356
12	186136	171705	23369	12756	21715	269807	68427	1025885
13	89260	146394	7765	9233	23437	7912609	152133	332669
14	543286	363985	84838	31793	62644	77658	318803	30941706
15	379245	1056560	341061	24283	104963	7023	64031	8908287
16	847658	1400812	37813	21844	81292	48017	299767	83692959
17	7217822	6332203	480880	434223	211700	187208	200229	5923896
18	169231	4260687	13971	82539	7373	69934	227399	2264360
19	84960	2192739	675211	44029	4551	2074680	18213	479190
20	151469	227453	28246	514167	14739	39526	1177	2213560
21	22400	35260	0	0	390330	0	8665	911562
22	174256	314599	17863	19777	17627	35053798	300454	1190116
23	11691	0	0	11810		45459	926159	24618
24	458993	4602302	15379	9682	13831	1798760	450379	95409541
25	467	138759	6163	428	520	1671	5913	197335
26	7843	1038925	70140	7936	4609	8926	0	902724
27	64	7787	3700	71	148	20	0	8698
28	215172	396116	24389	13498	18931	292819	114454	388710
29	185980	399884	43047	10333	18675	615138	14244	1718850
30	53843	117392		2534	5899	5508	3379	517356
31	411656	926907	43898	30139	34756	205246	69926	1523204
32	168207	244604	19643	9173	11679	170836	516826	552508
33	102423	217147	7710	8543	12152	591927	102161	422861
34	583499	1859964	46390	30215	110576	1120095	40109	7286264
35	58879	126184	5604	12427	5593	163573	23271	1122628
36	23931	569908	6569	8987	9225	341845	6784	888478
37	223764	267404	30147	107451	15836	509315	263886	2387350
38	20389	0	0	0	0	8582	11738	0
39	30761	153587	22517	6779	3757	421067	44151	695048
40	288660	850259	97811	63052	31462	3075816	270645	4268356
41	0	0	0	0	0	0	0	0
42	29637	51860	4485	6854	2652	121332	17777	33166
43	1184	72331	0	1364	16	23244	24296	104818
44	11420	0	0	5747	4885	34417	7233	189287
45	0	0	0	0	0	0	0	0

i	25	26	27	28	29	30	31	32
1	1411	660733	691545	6148964	6778	2380	0	0
2	0	8898		1617539	0	0	0	0
3	884	31659	6393	42757	0	0	0	37074
4	0	17386	13823	7668196	0	0	0	0
5	0	228033	5996	11964889	0	0	0	0
6	0	0	16985	1705412	0	0	0	0
7	0	14843	7443	0	0	0	0	0
8	3585	223838	29721	25530839	4792	4043	0	2613

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Table 6.14 ... continued from previous page

9	29715	112462	115642	16139848	0	0	0	0
10	24116	487319	270457	1150781	25627	4714	264605	142660
11	6656	436636	514163	45378	5350	0	0	54605
12	385193	2889292	1989474	112526	177030	14667	123343	1182858
13	458681	1614958	616374	53961	4871512	468328	7172878	696955
14	608272	1388777	315626	328438	36527	1359	0	68993
15	1645548	491985	1074366	229268	350857	0	0	74376
16	717440	910167	594712	3181245	62712	0	0	343371
17	1384023	3066570	2017568	4895446	59207	55914	2586	233244
18	816806	241688	193038	102307	17484	0	79546	4323
19	1277155	3960370	1155548	51361	234962	15611	253408	26922
20	4400957	25907	158360	91569	215629	350134	3005995	104717
21	93522	283439	150142	13542	7346	630	366957	3342
22	354920	1120387	3255662	2422924	478885	11475	550934	1133505
23	101444	176140	378563	162557	67778	7976	18145	292909
24	481900	2239532	1049894	323505	1041261	3475	372295	2712117
25	31883	349734	736351	89563	24702	109	0	8403
26	37847	3590429	1320675	1504399	24490	3953	1951451	2586
27	0	4495	33937	12279	233	0	0	111
28	786606	4320146	2344404	659809	504099	34136	2270747	2743268
29	901638	4147142	1569284	548386	3817299	7153	73097	459937
30	90035	1062935	224384	158763	11947	6357348	0	56044
31	967974	5868646	2405353	1213820	759195	51720	34011352	18112900
32	2223232	9072506	6790892	495979	2524437	6734953	28463550	13146119
33	503972	2246430	2710177	382958	274889	28604	676409	974402
34	1555183	7512431	8014557	538989	1940981	3841195	3212700	4849704
35	191510	519471	214312	225086	338800	31684	207213	149141
36	269054	798343	1034529	91485	66267	3946	0	67395
37	2924031	7737548	11860750	2425588	1272150	380938	1529673	1235311
38	0	4848084	3155866	221020	150004	2547199	166027	691195
39	146006	707322	900130	333445	67526	28709	0	807616
40	2750057	13751041	9402330	3129055	846245	167341	2596398	2643365
41	0	0	0	0	0	0	0	0
42	39141	298911	120998	87250	106257	21345	0	186767
43	11653	139162	58908	60563	7820	840	0	44049
44	196515	434633	1220891	750332	102768	32745	5778	185557
45	0	0	0	0	0	0	0	0

i	33	34	35	36	37	38	39	40
1	12629	0	0	0	58742	468	232	481125
2	0	0	0	0	0	0	0	0
3	0	0	0	0	6395	0	0	7871
4	0	0	0	0	0	0	0	74740
5	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0
8	16476	0	0	0	1348	427	1363	50215
9	0	0	0	0	0	0	0	5167
10	29303	6332	0	13	3771	842	714	140273
11	0	0	0	0	11788	0	0	77174
12	914249	1412942	75174	2664	156759	37807	30908	5182201
13	90340	6116	8299	0	214103	38075	4873	567516
14	28467	33988	0	308	20643	1086	5277	312687
15	11367	0	0	0	1569	63127	3375	1340035
16	6077	0	0	0	41299	3433	417	213300
17	74819	0	0	0	33372	53723	8165	1313703
18	0	0	0	0	114271	1950	0	84516
19	5457449	202611	185165	1419	3362	37385	36306	757143
20	0	0	0	0	2099	461	0	13117
21	6391	132428	0	4301	6309	384	2508	255105
22	265458	1049563	47159	922	486750	4012	15507	423628
23	17015	109566	8575	0	96569	1876	395	133453
24	895006	674528	0	6683	6602997	9634	10770	514184
25	3344	14778	0	426	8300	83209	314	10436
26	1915535	0	0	0	977147	25480	0	77420
27	139	0	0	0	0	1578	0	150
28	118084	1668489	139766	4379	399723	27315	38759	2201140

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Table 6.14 ... continued from previous page

29	153647	208123	0	291	43963	18218	3228	286058
30	40755	0	0	0	6267	6305	399	41450
31	1266175	0	0	0	564862	37851	71534	5575457
32	59509	0	0	0	206642	63945	6569	939131
33	10742102	2152609	199919	4282	267519	30691	26450	2387122
34	2047768	33694	313504	19202	21356054	1852758	181060	7701666
35	101499	101907	42690	79	27617	75727	2917	226970
36	123530	195062	2614504	0	85679	39225	3128	714698
37	1319527	5413887	643852	56710	3014401	184595	87532	1118770
38	0	710735	87545	1078	3794	130157	1012	58609
39	161452	5398263		13341	223104	82073	121100	1696219
40	1141076	10148455	784284	62245	2664021	87047	145788	26527328
41	0	0	0	0	0	0	0	0
42	43478	278280	0	161	11816	5729	3246	359039
43	27260	0	0	0	717	0	141	77028
44	1182558	220609	17423	1660	34543	3619	538	2867435
45	0	0	0	0	0	0	0	0

i	41	42	43	44	45
1	401468	50187	4006531	509309	0
2	0	5632	2483	0	0
3	290820	14891	688	2170	0
4	648110	603647	3776708	547	0
5	723922	83444	10265247	7948	0
6	814871	64350	3131286	272	0
7	1562	793		122	0
8	121273	13397	1210344	68611	0
9	151669	25794	3896503	5690	0
10	585585	40757	2142380	506111	0
11	107994	18596	2259	2236086	0
12	182023	36807	558627	2096006	0
13	60483	26642	602909	2976765	0
14	660811	91739	1611530	5500183	0
15	2734458	70070	2828810	470312	0
16	2192360	206139	75688	21980	0
17	3823527	1192964	5831890	1447775	0
18	108819	238168	189682	88438	0
19	5352774	127047	117080	2338666	0
20	220013	1413647	379166	18045	0
21	2683	1538	11070307	843107	0
22	6912468	1434295	2099857	2320201	0
23	830989	856511	115145	217189	0
24	1772703	496341	337970	1719177	0
25	39419	4771	37675	28573	0
26	448591	88412	334076	35578	0
27	23667	795	10696	1254	0
28	3904465	553178	2945809	3006537	0
29	5311405	657708	742463	234782	0
30	1552596	161289	234535	10799	0
31	5416385	1918421	1381758	1251975	0
32	2423604	583903	464316	608698	0
33	953485	535977	495592	8300469	0
34	6827545	1185288	3928262	3504299	0
35	167695	52482	104239	272082	0
36	196578	37956	171927	620196	0
37	3291418	3634619	2570945	4662209	0
38	0	0	0	537166	0
39	2458311	229323	609981	670515	0
40	10678772	2132787	5107617	14197173	0
41	0	0	0	0	0
42	613869	1166875	100769	50183	0
43	241519	48012	12062018	9939	0
44	2055916	1488469	719052	12864481	0
45	0	0	0	0	0

Each sector produces only one commodity and the output table is also a 45 x 45 diagonal matrix with outputs on the diagonal and rest zeros. Table 6.15 shows the outputs of each sector (we have just shown the diagonal elements to save space).

Table 6.15.: Output Q of Sector i

i	1	2	3	4	5	6	7	8
Q	253044315	46494039	12830283	30593780	44060254	178642871	65518264	46115789
i	9	10	11	12	13	14	15	16
Q	47327374	7825183	13415854	9329864	0	0	4373629	53926806
i	17	18	19	20	21	22	23	24
Q	23950233	34446154	2980940	2330010	3423106	101986844	10361405	411746153
i	25	26	27	28	29	30	31	32
Q	55460210	153110034	181092729	130362980	43560957	30242823	155005440	113644353
i	33	34	35	36	37	38	39	40
Q	69031049	109229762	11353259	488514	185690308	17544892	2202711	129215761
i	41	42	43	44	45			
Q	401769836	180716890	241015611	139903614	17557837			

6.5.5. Trade and Transport Margins

Table 6.16 shows the total trade margins of the trade sectors ctd: 25,26,27 paid by each of the 45 sectors j. The values in the intersection of the column COICTZ and rows 25,26,27 shows the total trade margins by each of these sectors as they appear in the SAM. The last column COITZ shows the trade margins paid by commodities in the Gross Fixed Capital formation. This is needed for computing the margins to be paid during investment.

Table 6.16.: Trade Margins [COITDZ] $ctd \times j$

i	1	2	3	4	5	6	7	8
1	171801	0	101	633708	5988	9704910	717991	530731
2	0	133912	0	429	6021099	0	0	0
3	824	9	45545	378	8141	2776	17075	334
4	911	0	0	268906	12329	19366	120932	0
5	0	27305	0	14535	751798	0	218182	0
6	3	0	0	4	399	669474	5408	15
7	802200	1579	0	0	0	0	28812	0
8	71057	94	156	640	172	1270	53819	632064
9	411	123	29	9556	83420	596302	948945	169964
10	33442	8772	5529	224	583	4845	355	3529
11	20236	0	665	169	3220	1715	407	47256
12	27646	7662	7293	7787	195017	2832038	145205	193976
13	198487	42999	122460	9135	26873	194501	28254	13787
14	333236	984	38204	415	671	100042	92086	97455
15	10759	727	0	11948	14360	649954	40219	283330
16	156842	0	7156	1294	0	115065	1633	380922
17	32665	1982	2377	16027	456994	81192	3750	144727
18	23570	2485	8446	1073	6476	99488	1402	34080
19	2289	708	190	126	831	4210	532	1081
20	391	4439	415	0	0	0	0	0
21	105	0	0	0	0	504	0	0
22	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0
25	-1078095	-128199	-74122	-247634	-1182806	-2370220	-1218914	-154772
26	-802111	-105580	-164250	-728469	-6405565	-12702967	-1204874	-2378191
27	-6668	0	-194	-250	0	-4464	-1219	-289
total	0	0	0	0	0	0	0	0
i	9	10	11	12	13	14	15	16

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Table 6.16 ... continued from previous page

1	794513	19245	258036	0	0	3487	8333	26
2	3589	0	0	0	0	0	0	0
3	1932	25	69	0	0	9	0	55164
4	13092	550	0	0	0	924	0	0
5	21407	0	0	0	0	0	0	0
6	21714	0	0	0	0	188	0	0
7	0	0	0	0	0	0	0	0
8	6177	56	65	898	0	964	0	55
9	1085316	9	7	2	0	3149	0	22
10	856	304005	1183	2041	0	3822	14790	1896
11	2945	250	520068	0	0	493	1882	9330
12	100723	2659	13294	184883	0	6165	5348	54
13	23291	2614	7357	1385	0	17817	1734	55326
14	14979	37147	37613	22682	0	104498	41314	29011
15	85869	4673	12540	10438	0	6462	11443	0
16	51641	193	7909	13918	0	10053	7485	206365
17	64280	2463	17718	5925	0	4928	10884	193
18	7419	1055	3990	183	0	1683	11805	111
19	640	155	199	236	0	495	2498	49
20	0	180	203	0	0	511	11540	0
21	0	4218	6284	1588	0	386	4752	0
22	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0
25	-261931	-39392	-151831	-7204	0	-32181	-23816	-357601
26	-2037692	-339792	-734181	-236923	0	-133632	-108946	0
27	-762	-314	-524	-52	0	-222	-1050	0
total	0	0	0	0	0	0	0	0

i	17	18	19	20	21	22	23	24
1	2970	1058	0	219	496	0	3159	0
2	0	0	0	0	0	0	0	0
3	0	2	0	0	0	160859	164	174528
4	0	0	0	0	934	0	0	0
5	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0
8	1070	0	0	0	0	0	0	1320
9	0	0	0	0	0	0	0	473
10	6926	0	3924	331	26342	4098	673	46534
11	19217	3709	529	1148	8502	2048	3963	1066849
12	26888	7129	869	1384	2692	26960	6744	95044
13	12894	6078	289	1002	2906	777159	14993	30820
14	78479	15112	3154	3451	7767	7760	31419	2866608
15	54783	43867	3051	2636	8634	702	6311	825312
16	141077	68244	1651	2749	8252	9287	44991	8703149
17	289745	186225	8249	47128	13109	18707	19733	548822
18	24446	237538	519	8958	914	6988	22411	209783
19	12273	91039	13548	4779	564	207310	1795	44395
20	21880	9444	1050	55805	1828	3950	116	205076
21	3236	1464	0	0	13357	0	854	84452
22	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0
25	-38802	-78529	-2838	-6580	-9486	-192928	-157326	-2652448
26	-651760	-587972	-32291	-121914	-84116	-1030586	0	-12133804
27	-5320	-4407	-1704	-1097	-2693	-2314	0	-116911
total	0	0	0	0	0	0	0	0

i	25	26	27	28	29	30	31	32
1	99	46709	48821	284153	1634	600	0	0
2	0	1508	0	275592	0	0	0	0
3	61	2198	443	2871	0	0	0	9148
4	0	2629	2087	1089628	0	0	0	0
5	0	31686	832	1665099	0	0	0	0
6	0	0	940	83767	0	0	0	0
7	0	486	244	0	0	0	0	0
8	552	34599	4588	3947539	2131	1858	0	1199

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Table 6.16 ... continued from previous page

9	2380	9036	9279	1189947	0	0	0	0
10	2492	50516	27999	112300	8417	1612	91231	15771
11	1135	74700	87858	7217	2398	0	0	3757
12	38660	290077	199748	10666	43985	3816	32379	26248
13	46036	162137	61885	5115	383673	84872	1882998	10952
14	61050	139429	31690	31131	9075	354	0	17934
15	165158	49394	107869	21731	64627	0	0	19333
16	83098	105753	69010	336979	21212	0	0	20122
17	138909	307875	202569	464015	14710	14549	679	20666
18	81980	24265	19382	9697	4344	0	20882	1124
19	128183	397610	116020	4868	43347	4062	66524	6998
20	441708	2601	15900	8679	38544	61515	789123	8719
21	9386	28457	15075	1284	1825	164	96332	869
22	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0
25	-549086	-156189	-356579	-532631	-319828	-4665	0	-123270
26	-651804	-1603469	-639539	-8946627	-317075	-168738	-2980148	-37942
27	0	-2007	-26119	-73020	-3022	0	0	-1629
total	0	0	0	0	0	0	0	0
i	33	34	35	36	37	38	39	40
1	3206	0	0	0	3990	32	16	33267
2	0	0	0	0	0	0	0	0
3	0	0	0	0	390	0	0	489
4	0	0	0	0	0	0	0	11571
5	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0
8	7608	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	422
10	10078	306	0	0	408	92	78	15423
11	0	0	0	0	1925	0	0	12802
12	239884	70933	0	135	16430	4050	0	559946
13	23704	307	0	0	22440	4079	0	61321
14	7469	1708	0	10	2164	116	0	33786
15	2983	0	0	0	164	6763	362	144793
16	2151	0	0	0	4584	386	47	24085
17	19631	0	0	0	3498	5756	876	141948
18	0	0	0	0	11977	209	0	9132
19	1431945	10132	0	72	352	4005	0	81811
20	0	0	0	0	220	49	0	1417
21	1677	6537	0	218	661	41	269	27565
22	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0
25	-3050	-89923	0	-434	-583	-19302	-1647	-137531
26	-1747158	0	0	0	-68620	-5911	0	-1020275
27	-127	0	0	0	0	-366	0	-1972
total	0	0	0	0	0	0	0	0
i	41	42	43	44	45	COICTZ	COITZ	
1	19788	5845	166207	21905	0	13493045	93196	
2	0	2618	393	0	0	6439140	0	
3	27718	3258	46	100	0	514557	0	
4	117752	253763	496017	54	0	2411445	0	
5	130285	33898	1336465	720	0	4232212	0	
6	51494	9482	142939	8	0	985835	0	
7	66	79	0	2	0	833468	0	
8	23959	6168	172357	6659	0	4979095	0	
9	15427	6033	273308	275	0	4403834	0	
10	80200	13355	208289	34216	0	1147483	61865	
11	22966	9264	351	246014	0	2184988	0	
12	20620	12493	37714	116381	0	5621628	0	
13	6852	9043	40704	165286	0	4563564	0	
14	74859	31138	108798	305399	0	4820196	0	
15	309769	23783	190980	26114	0	3221840	0	
16	320686	72462	7886	1578	0	11009915	0	

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Table 6.16 ... continued from previous page

17	433143	365284	393725	80388	0	4586042	1913890
18	12327	80839	12806	4911	0	1008697	13450425
19	606382	43123	7904	129855	0	3473137	8565353
20	24924	433583	25598	1002	0	2170409	9399354
21	304	522	747383	46814	0	1106581	4422868
22	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0
25	-177154	-71894	-430481	-518859	0	-13960762	
26	-2016006	-1332153	-3817170	-646055	0	-68724302	
27	-106361	-11986	-122218	-22767	0	-522048	
total	0	0	0	0	0		

Table 6.17 shows the total transport margins of the trade sectors ctd: 29,30,31,32 paid by each of the 45 sectors j. The values in the intersection of the column COICTTZ and rows 29,30,31,32 shows the total transport margins by each of these sectors as they appear in the SAM.

Table 6.17.: Transport Margins [COITTZ] $ctt \times j$

i	1	2	3	4	5	6	7	8
1	111198	0	20	159028	451	840810	165593	106010
2	0	10188	0	16	72799	0	0	0
3	156	1	3020	25	203	73	1167	14
4	270	0	0	26102	392	661	12000	0
5	0	3355	0	849	14682	0	13031	0
6	2	0	0	1	33	58256	1198	2
7	617455	861	0	0	0	0	7927	0
8	18693	18	14	56	5	38	4850	87569
9	259	56	6	2116	6788	51509	214565	23306
10	1735	317	95	4	3	31	6	37
11	6180	0	67	17	104	60	42	2835
12	3697	718	359	288	1685	26376	4604	4367
13	26546	4028	6027	337	232	1811	896	310
14	44567	92	1880	15	6	932	2920	2194
15	1439	68	0	441	124	6053	1275	6379
16	5820	0	90	16	0	529	21	2908
17	4369	186	117	592	3947	756	119	3258
18	3152	233	416	40	56	927	44	767
19	306	66	9	5	7	39	17	24
20	52	416	20	0	0	0	0	0
21	14	0	0	0	0	5	0	0
22	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0
29	-262338	-637	-7891	-74087	-30729	-505214	-225260	-106494
30	-75834	-169	-1515	-20670	-8903	-141989	-64073	-27487
31	-374652	-9347	-1704	-69934	-42214	-194869	-106853	-69159
32	-133086	-10450	-1030	-25256	-19671	-146794	-34089	-36841
total	0	0	0	0	0	0	0	0

i	9	10	11	12	13	14	15	16
1	112075	2599	31283	0	0	355	848	25
2	72	0	0	0	0	0	0	0
3	75	1	3	0	0	0	0	17989
4	694	35	0	0	0	42	0	0
5	689	0	0	0	0	0	0	0
6	2737	0	0	0	0	21	0	0
7	0	0	0	0	0	0	0	0

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Table 6.17 ... continued from previous page

8	294	3	3	36	0	39	0	25
9	137772	1	1	0	0	349	0	23
10	8	3612	12	17	0	32	122	166
11	162	16	29964	0	0	23	88	4972
12	1984	83	298	1040	0	1035	31	11
13	459	81	165	8	0	6350	10	11556
14	295	1154	842	128	0	25103	935	6059
15	1692	145	281	59	0	1085	269	0
16	363	2	58	84	0	61	45	13295
17	1266	77	397	33	0	827	179	40
18	146	33	89	1	0	283	68	23
19	13	5	4	1	0	83	14	10
20	0	6	5	0	0	86	67	0
21	0	131	141	9	0	65	28	0
22	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0
29	-104975	-2656	-25188	-276	0	-12991	-1079	-1132
30	-29661	-740	-7876	-78	0	-3509	-325	-48621
31	-80953	-3166	-22188	-966	0	-13870	-956	-2417
32	-45207	-1422	-8293	-96	0	-5469	-347	-2025
total	0	0	0	0	0	0	0	0
i	17	18	19	20	21	22	23	24
1	302	108	0	22	50	0	385	0
2	0	0	0	0	0	0	0	0
3	0	0	0	0	0	6283	6	10438
4	0	0	0	0	42	0	0	0
5	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0
8	43	0	0	0	0	0	0	105
9	0	0	0	0	0	0	0	95
10	57	0	32	3	549	42	7	715
11	900	174	25	54	2272	119	230	96169
12	228	51	4	10	65	868	42	2924
13	109	43	1	7	70	25495	93	948
14	665	108	13	26	186	250	195	88194
15	464	312	55	19	327	23	39	25392
16	854	413	10	17	74	67	433	97947
17	9914	1896	78	348	675	602	123	16885
18	207	2475	2	66	22	225	139	6454
19	104	648	109	35	14	6672	11	1366
20	185	67	4	413	44	127	1	6309
21	27	10	0	0	1281	0	5	2598
22	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0
29	-3191	-1493	-135	-202	-1492	-25163	-40	-142127
30	-924	-438	0	-50	-471	-225	-10	-42779
31	-7063	-3462	-137	-590	-2776	-8396	-198	-125949
32	-2886	-913	-62	-179	-933	-6988	-1462	-45685
total	0	0	0	0	0	0	0	0
i	25	26	27	28	29	30	31	32
1	19	8828	9227	59623	63	23	0	0
2	0	51	0	7337	0	0	0	0
3	4	138	28	144	0	0	0	119
4	0	238	189	77560	0	0	0	0
5	0	1730	45	71603	0	0	0	0

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Table 6.17 ... continued from previous page

6	0	0	191	13700	0	0	0	0
7	0	123	62	0	0	0	0	0
8	45	2843	377	253701	22	19	0	12
9	494	1876	1927	197411	0	0	0	0
10	40	818	453	1437	23	4	248	179
11	108	7108	8359	537	30	0	0	540
12	620	12986	6213	272	504	27	289	1674
13	738	7258	1925	131	17010	962	16826	993
14	979	6242	986	794	104	3	0	74
15	2649	2211	3355	555	1085	0	0	80
16	985	1254	818	3153	41	0	0	334
17	2228	13783	6300	11841	169	104	6	308
18	1315	1086	603	247	50	0	187	5
19	2056	17800	3609	124	726	29	594	29
20	7084	116	495	221	671	724	7052	139
21	151	1274	469	33	21	1	861	4
22	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0
29	-4206	-18062	-6516	-158921	-11012	-1	-30	-65
30	-420	-4629	-932	-46009	-34	-916	0	-8
31	-4516	-25559	-9987	-351761	-2190	-7	-14172	-2558
32	-10372	-39513	-28196	-143733	-7282	-971	-11860	-1857
total	0	0	0	0	0	0	0	0
i	33	34	35	36	37	38	39	40
1	123	0	0	0	687	6	3	5729
2	0	0	0	0	0	0	0	0
3	0	0	0	0	21	0	0	27
4	0	0	0	0	0	0	0	957
5	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0
8	79	0	0	0	21	7	21	782
9	0	0	0	0	0	0	0	80
10	27	1	0	0	6	1	1	227
11	0	0	0	0	163	0	0	1085
12	45	4496	0	0	2439	253	77	12391
13	4	23	0	0	3332	254	12	1357
14	1	71	0	109	321	7	13	748
15	1	0	0	0	24	422	8	3204
16	4	0	0	0	48	4	0	250
17	4	0	0	0	519	359	18	3141
18	0	0	0	0	1778	13	0	202
19	269	1432	0	0	52	250	91	1810
20	0	0	0	0	33	3	0	31
21	0	2632	0	0	98	3	6	610
22	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0
29	-56	-8655		-109	-511	-228	-10	-1364
30	-15	0		0	-73	-79	-1	-198
31	-465	0		0	-6560	-474	-219	-26589
32	-22	0		0	-2400	-800	-20	-4479
total	0	0	0	0	0	0	0	0
i	41	42	43	44	45	COICTTZ		
1	4344	489	40633	6750	0	1667709		
2	0	23	12	0	0	90498		
3	1337	58	2	10	0	41342		

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Table 6.17 ... continued from previous page

4	8721	5897	41496	8	0	175304
5	5784	491	66725	68	0	179052
6	8752	615	27121	3	0	112632
7	14	6	0	1	0	626449
8	1529	120	12622	976	0	384969
9	2516	361	51477	94	0	693082
10	981	54	2870	900	0	15873
11	1628	189	28	38155	0	202403
12	564	88	1475	6807	0	101987
13	188	64	1592	9667	0	147921
14	2049	220	4255	17862	0	211598
15	8479	168	7469	1527	0	77179
16	2867	209	79	30	0	133184
17	11856	3010	15399	4702	0	120430
18	337	572	501	287	0	23053
19	16598	305	309	7595	0	63242
20	682	3564	1001	59	0	29677
21	8	4	29230	2738	0	42456
22	0	0	0	0	0	0
23	0	0	0	0	0	0
24	0	0	0	0	0	0
25	0	0	0	0	0	0
26	0	0	0	0	0	0
27	0	0	0	0	0	0
28	0	0	0	0	0	0
29	-28621	-3269	-80029	-10962	0	-1867416
30	-8366	-802	-25280	-504	0	-564614
31	-29187	-9535	-148938	-58386	0	-1832923
32	-13060	-2902	-50048	-28387	0	-875086
total	0	0	0	0	0	

Consumer prices include excise taxes (*texc*), VAT (*vatc*), other taxes (*tc*) and also include trade and transport margins. The SAM gives the trade and transport margins paid by the households. The trade margins are given in table 6.18 while the transport margins are given in table 6.19.

Table 6.18.: Trade Margins on Household Consumption

	Q1	Q2	Q3	Q4	Q5	Q6	COHTZ
1	1092125	1631427	2386324	2807175	4087760	4919878	16924688
2	396527	779552	1392344	1940769	3111036	4230515	11850743
3	3789	5038	7464	6707	14253	3010	40260
4	474409	860971	1699656	2263500	2325530	3650214	11274279
5	110253	206038	382901	568958	802656	1407065	3477872
6	984437	1551716	2265646	2630499	2716595	3240377	13389270
7	95904	132197	214208	257902	311647	356512	1368369
8	96303	586641	1883069	2858585	3049272	4732709	13206580
9	939792	2189400	4640317	6197187	5740346	9594415	29301456
10	633588	1834635	3684317	5630401	6457336	15324705	33564983
11	3882	6284	11534	18513	34320	68467	143001
12	20505	24300	30669	42637	71140	91133	280385
13	7737	53263	59345	67022	56147	71871	315385
14	121435	59699	44149	62264	133323	239945	660814
15	6629	24416	26651	40791	74628	51619	224734
16	7817	6131	6032	8280	14294	19092	61647
17	33819	105511	172177	342682	729193	919524	2302906
18	37896	190895	448907	1043332	1354234	1767156	4842420
19	133027	363312	723315	1499587	1872586	3806479	8398306
20	3954580	5149929	6532806	8641406	12347220	11337017	47962958
21	1173535	1870807	2550362	3310768	4123691	4179927	17209091

Table 6.19.: Transport Margins on Household Consumption

	Q1	Q2	Q3	Q4	Q5	Q6	COHTTZ
1	63873	95414	139565	164178	239073	287740	989844
2	23191	45592	81432	113506	181950	247423	693093
3	222	295	437	392	834	176	2355
4	27746	50354	99405	132381	136009	213483	659378
5	6448	12050	22394	33276	46944	82292	203404
6	57575	90752	132507	153845	158881	189514	783074
7	5609	7732	12528	15083	18227	20851	80029
8	5632	34310	110132	167185	178337	276793	772389
9	54964	128047	271390	362444	335725	561131	1713701
10	37056	107299	215478	329295	377659	896268	1963055
11	227	368	675	1083	2007	4004	8363
12	1199	1421	1794	2494	4161	5330	16398
13	453	3115	3471	3920	3284	4203	18445
14	7102	3491	2582	3642	7797	14033	38648
15	388	1428	1559	2386	4365	3019	13144
16	457	359	353	484	836	1117	3605
17	1978	6171	10070	20042	42647	53779	134686
18	2216	11165	26254	61019	79203	103352	283210
19	7780	21248	42303	87704	109519	222623	491177
20	231284	301195	382072	505394	722130	663048	2805123
21	68634	109415	149158	193631	241175	244464	1006477

Figure 6.1 shows the typical setup of margins in the model. The data is available for consumption ($c_1..c_{45}$) by type of household ($q_1..q_6$) or consumption ($c_1..c_{45}$) by each sector ($s_1..s_{45}$). What we need is the trade and transportation paid on each of the commodity by the type of end user viz. sector or household.

There are trade ctd: 25,26,27 and transportation margins ctt: 29,30,31,32 paid by 45 sectors and 6 households on each of the 45 commodities. Of the total 45 commodities in the model only the first 21 have trade and transport margins to be paid while the remaining commodities (22-45) have zero margins. Since the input-output flows and the consumption of each commodity by each household is known to obtain the 3rd dimension we disaggreage the margins on each of the cells using the proportions of each trade and transport commodities as shown in table 6.21 for trade margins by sectors, table 6.20 for trade margins by households, table 6.23 for transport margins by sectors and table 6.22 for transport margins by households.

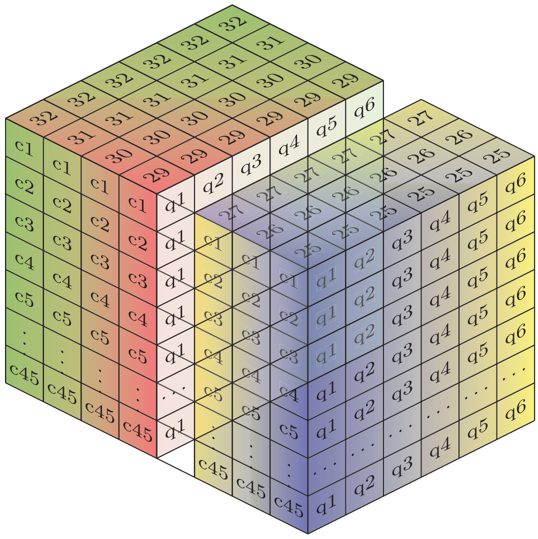

Fig. 6.1.: Trade [ctd:25,26,27] & Transport [ctt:29,30,31,32] Margins by commodity by type

Table 6.20.: Aggregate Trade Margins Households

ctd	Q1	Q2	Q3	Q4	Q5	Q6	ch
25	1028819	1756422	2904982	4008391	4923674	6974185	21596473
26	1597803	2727802	4511568	6225212	7646688	10831224	33540297
27	7701369	13147938	21745644	30005361	36856844	52206222	161663377
Total	10327992	17632162	29162194	40238963	49427206	70011630	216800147
% share of each sector ctd in total ctd							
25	0.099615	0.099615	0.099615	0.099615	0.099615	0.099615	
26	0.154706	0.154706	0.154706	0.154706	0.154706	0.154706	
27	0.745679	0.745679	0.745679	0.745679	0.745679	0.745679	
Total	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	

Table 6.21.: Aggregate Trade Margins

ctd	ch	cic	ci	%ch	%cic	%ci
25	21596473	13960762	7950449	0.099615	0.167783	0.209736
26	33540297	68724302	29956502	0.154706	0.825943	0.790264
27	161663377	522048	0	0.745679	0.006274	0.000000
Total	216800147	83207112	37906951	1.000000	1.000000	1.000000

Table 6.22.: Aggregate Transport Margins Households

ctt	Q1	Q2	Q3	Q4	Q5	Q6	ch
29	95221	162564	268868	370992	455705	645488	1998839
30	12535	21400	35394	48838	59990	84974	263132
31	333116	568702	940588	1297855	1594210	2258134	6992606
32	163162	278554	460706	635698	780854	1106048	3425023
Total	604035	1031220	1705557	2353384	2890760	4094644	12679600
% share of each sector ctt in total ctt							
29	0.157642	0.157642	0.157642	0.157642	0.157642	0.157642	
30	0.020752	0.020752	0.020752	0.020752	0.020752	0.020752	
31	0.551485	0.551485	0.551485	0.551485	0.551485	0.551485	
32	0.270121	0.270121	0.270121	0.270121	0.270121	0.270121	
Total	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	

Table 6.23.: Aggregate Transport Margins

ctt	ch	cic	ci	%ch	%cic	%ci
29	1998839	1867416	0	0.157642	0.363308	0.000000
30	263132	564614	0	0.020752	0.109846	0.000000
31	6992606	1832923	0	0.551485	0.356597	0.000000
32	3425023	875086	0	0.270121	0.170249	0.000000
Total	12679600	5140039	0	1.000000	1.000000	0.000000

6.5.6. Computation of Various Rates

To run counter factual policy simulations one needs to obtain the various tax/subsidy and trade/transportation rates that will be imposed *ad-valorem* on the basic technology. The input-output flows are at market prices and to obtain the values at basic prices we need to subtract trade/transportation margins to obtain the flows at producer values. From the flows at producer values subtracting taxes and adding subsidies gives us the flows at basic prices which forms the base for levying taxes or subsidies.

We have one column for VAT (TRVATICZ) and subsidy (TRSICZ) paid by each sector on intermediate consumption and is shown in Table 6.12. We disaggregate this column into a matrix for VAT on each commodity c_i used by sector s_j by distributing this value in the same proportion as flows a_{ij}. We follow the same procedure for the subsidies (SIC). Formally speaking we define $shio_{ij}$ as

$$shio_{ij} = \frac{a_{ij}}{\sum_{j=1}^{45} a_{ij}} \tag{6.5}$$

and we obtain the table of VAT (TRVAT) and subsidies (TRSIC) by commodity by sector as

$$\begin{aligned} TRVAT_{ij} &= VAT_i * shio_{ij} \\ TRSIC_{ij} &= SIC_i * shio_{ij} \end{aligned} \tag{6.6}$$

We already have the trade ($COITDZ_{ij}$) table 6.16 and transport ($COITTZ_{ij}$) margins in table 6.17 and so to obtain the input-output flows at basic prices (IO_{ij}^{bp})

from the IO table at market prices (IO_{ij}^{mp}) we do the following

$$IO_{ij}^{bp} = IO_{ij}^{mp} - TRVAT_{ij} + TRSIC_{ij} - COITDZ_{ij} - COITTZ_{ij} \qquad (6.7)$$

To calculate the subsidy rates on each commodity ($tsic_{ij}$) in intermediate consumption we use the IO table at basic prices **plus** the trade and transport margins as the base. To obtain the VAT rates by commodity ($vatic_{ij}$) in intermediate consumption we again use the IO table at basic prices **plus** trade and transport margins **plus** the subsidies ($TRSIC_{ij}$).

6.5.7. Parameter Values

This part deals with the values of the parameters used in the model for households and producers.

Table 6.24 shows the different parameters for the six households q_1 to q_6. It also shows other data like transfers, unemployment benefits as obtained in the base year in €.

Table 6.24.: Household Parameters

	Parameter	q1	q2	q3	q4	q5	q6
1	elasY	0.42	0.42	0.43	0.43	0.44	0.44
2	elasY	0.42	0.42	0.43	0.43	0.44	0.44
3	elasY	1.20	1.21	1.22	1.23	1.25	1.26
4	elasY	0.42	0.42	0.43	0.43	0.44	0.44
5	elasY	0.42	0.42	0.43	0.43	0.44	0.44
6	elasY	0.42	0.42	0.43	0.43	0.44	0.44
7	elasY	0.42	0.42	0.43	0.43	0.44	0.44
8	elasY	0.42	0.42	0.43	0.43	0.44	0.44
9	elasY	0.42	0.42	0.43	0.43	0.44	0.44
10	elasY	0.86	0.87	0.88	0.89	0.90	0.91
11	elasY	1.20	1.21	1.22	1.23	1.25	1.26
12	elasY	1.20	1.21	1.22	1.23	1.25	1.26
13	elasY	1.20	1.21	1.22	1.23	1.25	1.26
14	elasY	1.20	1.21	1.22	1.23	1.25	1.26
15	elasY	1.20	1.21	1.22	1.23	1.25	1.26
16	elasY	1.20	1.21	1.22	1.23	1.25	1.26
17	elasY	1.20	1.21	1.22	1.23	1.25	1.26
18	elasY	1.20	1.21	1.22	1.23	1.25	1.26
19	elasY	1.20	1.21	1.22	1.23	1.25	1.26
20	elasY	1.20	1.21	1.22	1.23	1.25	1.26
21	elasY	1.20	1.21	1.22	1.23	1.25	1.26
22	elasY	1.11	1.12	1.13	1.15	1.16	1.17
23	elasY	1.11	1.12	1.13	1.15	1.16	1.17
24	elasY	1.20	1.21	1.22	1.23	1.25	1.26
25	elasY	1.20	1.21	1.22	1.23	1.25	1.26
26	elasY	1.20	1.21	1.22	1.23	1.25	1.26
27	elasY	1.20	1.21	1.22	1.23	1.25	1.26
28	elasY	1.26	1.28	1.29	1.30	1.32	1.33
29	elasY	1.12	1.13	1.14	1.16	1.17	1.18
30	elasY	1.12	1.13	1.14	1.16	1.17	1.18
31	elasY	1.12	1.13	1.14	1.16	1.17	1.18
32	elasY	1.12	1.13	1.14	1.16	1.17	1.18
33	elasY	1.12	1.13	1.14	1.16	1.17	1.18
34	elasY	1.20	1.21	1.22	1.23	1.25	1.26
35	elasY	1.20	1.21	1.22	1.23	1.25	1.26
36	elasY	1.20	1.21	1.22	1.23	1.25	1.26
37	elasY	1.11	1.12	1.13	1.15	1.16	1.17
38	elasY	1.20	1.21	1.22	1.23	1.25	1.26

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Table 6.24 ... continued from previous page

	Parameter	q1	q2	q3	q4	q5	q6
39	elasY	1.20	1.21	1.22	1.23	1.25	1.26
40	elasY	1.20	1.21	1.22	1.23	1.25	1.26
41	elasY	1.20	1.21	1.22	1.23	1.25	1.26
42	elasY	1.02	1.03	1.04	1.05	1.06	1.07
43	elasY	1.21	1.22	1.23	1.24	1.26	1.27
44	elasY	1.20	1.21	1.22	1.23	1.25	1.26
45	elasY	1.11	1.12	1.13	1.15	1.16	1.17
	elasS	0.4	0.4	0.4	0.4	0.4	0.4
	elasY:	income elasticities of demand for commodities					
	elasS:	elasticity of average propensity to save					

Table 6.25 shows the domestic and external trade elasticity values used in Value Added, Armington Function and CET transformation equations.

Table 6.25.: Parameters for Sectors & Commodities

	sigmaF	LSKZ	limINV	sigmaA	sigmaT	elasE
1	0.60	6454	0.06	2.90	-4.00	3.00
2	0.60	7046	0.06	2.90	-4.00	3.00
3	0.60	1300	0.06	2.90	-0.95	3.00
4	0.60	455	0.06	2.95	-3.00	3.00
5	0.60	556	0.06	2.95	-3.00	3.00
6	0.60	903	0.06	2.95	-3.00	3.00
7	0.60	184	0.06	2.95	-3.00	3.00
8	0.60	603	0.06	2.95	-3.00	3.00
9	0.60	1129	0.06	2.95	-3.00	3.00
10	0.60	97	0.06	3.75	-3.00	3.00
11	0.60	320	0.06	3.20	-3.00	3.00
12	0.60	285	0.06	3.20	-3.00	3.00
13	0.60	0	0.06	2.10	-2.50	3.00
14	0.60	36	0.06	3.30	-3.00	3.00
15	0.60	46	0.06	3.30	-3.00	3.00
16	0.60	795	0.06	2.90	-3.00	3.00
17	0.60	460	0.06	2.95	-3.00	3.00
18	0.60	137	0.06	4.05	-3.00	3.00
19	0.60	50	0.06	4.40	-3.00	3.00
20	0.60	50	0.06	4.30	-3.00	3.00
21	0.60	62	0.06	2.80	-2.50	3.00
22	0.60	3450	0.06	2.80	-2.50	3.00
23	0.60	483	0.06	2.80	-2.50	3.00
24	0.60	18100	0.06	1.90	-2.00	3.00
25	0.60	2200	0.06	1.90	-2.00	3.00
26	0.60	1200	0.06	1.90	-2.00	3.00
27	0.60	8700	0.06	1.90	-2.00	3.00
28	0.60	4900	0.06	1.90	-2.00	3.00
29	0.60	539	0.06	1.90	-2.00	3.00
30	0.60	146	0.06	1.90	-2.00	3.00
31	0.60	2420	0.06	1.90	-2.00	3.00
32	0.60	1307	0.06	1.90	-2.00	3.00
33	0.60	1366	0.06	1.90	-2.00	3.00
34	0.60	1983	0.06	1.90	-2.00	3.00
35	0.60	248	0.06	1.90	-2.00	3.00
36	0.60	4	0.06	1.90	-2.00	3.00
37	0.60	16	0.06	1.90	-2.00	3.00
38	0.60	92	0.06	1.90	-2.00	3.00
39	0.60	9	0.06	1.90	-2.00	3.00
40	0.60	1079	0.06	1.90	-2.00	3.00
41	0.60	14096	0.06	1.90	-2.00	3.00
42	0.60	7507	0.06	1.90	-2.00	3.00
43	0.60	6406	0.06	1.90	-2.00	3.00
44	0.60	2913	0.06	1.90	-2.00	3.00
45	0.60	870	0.06	1.90	-2.00	3.00
sigmaF:	CES capital-labour substitution elasticities by sector					

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Table 6.25 ... continued from previous page

	sigmaF LSKZ limINV sigmaA sigmaT elasE
LSKZ:	number of employees by sector
limINV:	parameter to limit investments carried out in different branches of activity to reasonable values
sigmaA:	substitution elasticities for the ARMINGTON function
sigmaT:	elasticities of transformation in the CET function
elasE:	elasticity of export demand

Table 6.26 shows the last set of parameters that deal with the economy is general and are not specific to any particular sector or commodity.

Table 6.26.: Other Parameters in the Model

Parameter Name	Description	Value
unempz	unemployment level (total number of unemployed BIT)	2286
elasU	unemployment elasticity	-0.1
frisch	Frisch parameter in the LES utility function	-1.5
elasLS	income elasticity of labour supply	0.2
growthz	national average growth rate of the economy	0.027
TRGECZ	transfers received by government from the European Commission to be used as subsidies on production	43959805

The Linear Expenditure System (LES) has a utility function of the form

$$\mathbb{U} = \sum_{i=1}^{n} \beta_i \ln(x_i - \gamma_i) \tag{6.8}$$

where x_i is the consumption of commodity i and γ_i is the committed consumption, with $x_i > \gamma_i$, $0 \leq \beta_i \leq 1$ and $\sum_{i=1}^{n} \beta_i = 1$. Maximising 6.8 with respect to the budget constraint $y = \sum_{i=1}^{n} p_i x_i$ gives rise to the Linear Expenditure functions

$$p_i x_i = \gamma_i p_i + \beta_i \left[y - \sum_{j=1}^{n} p_j \gamma_j \right] \tag{6.9a}$$

$$y^* = y - \sum_{j=1}^{n} p_j \gamma_j \tag{6.9b}$$

$$x_i^* = x_i - \gamma_i \tag{6.9c}$$

$$x_i^* = \frac{\beta_i y^*}{p_i} \tag{6.9d}$$

where y^* is called the *supernumerary income*. The elasticity of the marginal utility of income ζ is given by $\zeta = -\frac{y}{y^*}$ and equals the ration of the income to the supernumerary income and has a maximum value of -1. The model uses a value of -1.5.

The income (total expenditure) elasticity is given by

$$e_i = \frac{\beta_i y}{p_i x_i}$$

$$\therefore \beta_i = e_i \frac{p_i x_i}{y} = e_i w_i \text{ where } w_i = \frac{p_i x_i}{y} \tag{6.10}$$

6.6. Calibration

This section explains in brief the methods used to calibrate the parameters of the model to enable the replication of the benchmark SAM.

6.6.1. Calibration of Wage Curve

Equation 5.80 outlines the response of the real wage to the labour market based on the elasticity of unemployment $elasU$. The equation reproduced here, with benchmark values of variables needs err which is constant. PL is the wage rate in € per employed person.

$$\ln\left[\frac{PLZ}{PCINDEXZ}\right] = elasU \times \ln(UNRATEZ) + err \tag{6.11}$$

We reproduce a part of the data for calculating err

Table 6.27.: Calibration of Wage Curve

No.	LSKZ	LZ	LZ/LSKZ	premLSK
1	6454	12917426	2001.46	-0.8250
2	7046	14100671	2001.23	-0.8250
3	1300	4071203	3131.69	-0.7262
4	455	5031742	11058.77	-0.0330
5	556	6150247	11061.60	-0.0328
6	903	9984824	11057.39	-0.0331
7	184	2033838	11053.47	-0.0335
8	603	6669281	11060.17	-0.0329
9	1129	12492766	11065.34	-0.0325
10	97	1069394	11024.68	-0.0360
11	320	3538891	11059.04	-0.0330
12	285	3149262	11050.04	-0.0338
13	0	0	0.00	-1.0000
14	36	399983	11110.65	-0.0285
15	46	511283	11114.85	-0.0281
16	795	8798054	11066.73	-0.0323
17	460	5091044	11067.49	-0.0323
18	137	1520860	11101.17	-0.0293
19	50	551173	11023.46	-0.0361
20	50	552253	11045.07	-0.0342
21	62	683548	11024.97	-0.0360
22	3450	38165866	11062.57	-0.0327
23	483	5339877	11055.65	-0.0333
24	18100	98745202	5455.54	-0.5230
25	2200	20105226	9138.74	-0.2009
26	1200	19141885	15951.57	0.3948
27	8700	61721944	7094.48	-0.3797
28	4900	28875266	5892.91	-0.4847
29	539	10304150	19117.16	0.6716
30	146	2782427	19057.72	0.6664
31	2420	46259473	19115.48	0.6714
32	1307	24973417	19107.43	0.6707
33	1366	26104691	19110.32	0.6710
34	1983	37901904	19113.42	0.6713
35	248	4743921	19128.71	0.6726
36	4	70857	17714.13	0.5489
37	16	312586	19536.64	0.7083

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No.	LSKZ	LZ	LZ/LSKZ	premLSK
38	92	1763878	19172.58	0.6764
39	9	172298	19144.26	0.6740
40	1079	20620152	19110.43	0.6710
41	14096	269434491	19114.25	0.6713
42	7507	143486942	19113.75	0.6713
43	6406	122446331	19114.32	0.6713
44	2913	55682357	19115.12	0.6714
45	870	16636105	19121.96	0.6720
Total	101002	1155108991	PLZ : 11436.50	

Given $PLZ = 11436.5$ and $PCINDEXZ = 1$, from table 6.26 the number of unemployed are 2286 and elasticity of unemployment is -0.1, we get the unemployment rate as $UNRATEZ = \frac{2286}{[2286+101002]} = 0.022132$ and the value of err as

$$\begin{aligned} err &= \ln\left[\frac{PL}{PCINDEX}\right] - elasU \times \ln(UNRATE) \\ &= \ln\left[\frac{11436.5}{1}\right] - (-0.1) \times \ln(0.022132) = 9.344565 - 0.38107 \\ &= 8.963493 \end{aligned} \tag{6.12}$$

The wage premium by sector is denoted by $premLSK$ and given in table 6.27. The wage in each sector $\frac{LZ}{LSKZ}$, wage payments divided by the number of employees when divided by the average wage in the economy will denote the premium or discount to the average wage by sector. Thus we have

$$premLSK = \left[\frac{LZ}{LSKZ}\right] \times \left[\frac{1}{PLZ}\right] - 1 \tag{6.13}$$

6.6.2. Calibrating Household Parameters

Table 6.28.: Household Data

Data	q1	q2	q3	q4	q5	q6	Total
unempbz	644204	957988	1731056	2354145	3905004	8617500	18209897
TRHMLZ	6657300	957988	1731056	2354145	3905004	8617500	23578789
TRHGZ	3944780	5866240	10600119	14415608	23912289	52769260	111508296
YLHZ	34363784	68768090	116806112	189330579	267706238	478134189	1155108992
YKHZ	19263545	28646609	51763562	70395741	116770887	257688138	544528482
TRYHZ	2229925	3316097	5992087	8148926	13517257	29829667	63033959
SHZ	439788	1858803	6715245	20838364	47821097	225155660	302828957
CBUDZ	61559698	99064027	168193516	247508783	350956065	542223760	1469505849
MPSZ	0.00709	0.01842	0.03839	0.07765	0.11992	0.29341	

unempbz:	unemployment benefits
TRHMLZ:	transfers received by the household from the Mainland (excluding unemployment benefits)
TRHGZ:	total transfers received by the household from the government
YLHZ:	labor income received by household
YKHZ:	capital income received by the household
TRYHZ:	personal income taxes paid by the households

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Table 6.28 ... continued from previous page

Parameter	q1	q2	q3	q4	q5	q6	Total
SHZ:	household savings						
CBUDZ:	household disposable budget for consumption						
MPSZ:	average propensity to save						

6.6.3. Demand System

To calibrate the demand system we need the values of tax rate by commodity c on household qu for: excise tax $texc_{c,qu}$, other taxes $tc_{c,qu}$, vat $vatc_{c,qu}$. We also need the minimum expenditure on each commodity by each type of household $\mu H_{c,qu}$.

Table 6.5 gives the consumption by commodity c for household qu, gross of all taxes and margins. Tables 6.6 6.7 6.8 give the values of taxes for each household by commodity while tables 6.18 6.19 give the trade and transport margins respectively. Thus we obtain the consumption net of all taxes and margins by subtracting tables 6.6 6.7 6.8 6.18 6.19 from table 6.5. The net consumption is given below

Table 6.29.: Net Household Consumption [C]

	Q1	Q2	Q3	Q4	Q5	Q6	Cnet
1	2358694	3523439	5153812	6062736	8828452	10625601	36552734
2	431633	848568	1515614	2112592	3386467	4605058	12899931
3	14974	19913	29498	26506	56330	11896	159118
4	1438374	2610400	5153232	6862767	7050836	11067182	34182791
5	349303	652770	1213108	1802573	2542974	4457860	11018588
6	2877297	4535329	6621990	7688377	7940016	9470917	39133925
7	216241	298074	482991	581512	702693	803855	3085366
8	265071	1614713	5183095	7868176	8393036	13026649	36350741
9	2661862	6201244	13143209	17552878	16258925	27175169	82993287
10	1197376	3467152	6962741	10640514	12203283	28961123	63432189
11	10973	17763	32603	52328	97009	193528	404205
12	991593	1175088	1483112	2061866	3440197	4407050	13558907
13	374142	2575531	2869634	3240833	2715001	3475338	15250478
14	4703829	2312441	1710118	2411801	5164295	9294309	25596794
15	320612	1180876	1288965	1972816	3609369	2496533	10869171
16	378097	296572	291770	400490	691411	923471	2981811
17	60790	189657	309488	615970	1310724	1652844	4139472
18	68156	343325	807360	1876435	2435593	3178234	8709104
19	238683	651870	1297802	2690623	3359874	6829749	15068602
20	7074466	9212862	11686732	15458869	22088310	20281129	85802369
21	2097773	3344192	4558942	5918217	7371371	7471897	30762391
22	2436850	3164586	3988696	5219912	7475236	6793499	29078779
23	241338	381122	514197	654266	815721	781429	3388073
24	55109	108809	214029	571874	466301	838395	2254517
25	12	30586	224090	515790	1543681	660308	2974467
26	23179	37520	68866	110531	204909	408783	853787
27	238496	492708	1291778	680765	1100970	12294266	16098982
28	2119898	4812766	8431124	16598693	16068634	50686420	98717534
29	554147	678251	2109410	2193356	1790975	377959	7704099
30	0	23480	26282	9905	0	502793	562460
31	1726474	950136	1745900	2389770	7212621	17305969	31330870
32	64633	11012	687999	1522031	3692646	7799318	13777639
33	1507237	2396099	3465659	5600458	8707416	10717852	32394721
34	0	35523	123252	742426	1912073	5737119	8550393
35	157952	493036	1134105	2430256	3704188	4996231	12915769
36	24740	40047	73504	117974	218708	436311	911283
37	1598885	3014633	7444437	14755473	24937071	38593781	90344280
38	0	0	103560	66223	5097522	5189721	10457026
39	0	0	0	44316	0	1860218	1904535

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	Q1	Q2	Q3	Q4	Q5	Q6	Cnet
40	40322	229195	2131032	2378955	7460033	7581264	19820801
41	5415	127884	777084	3203055	4910402	2295962	11319802
42	8196	4008	495407	1958413	2811761	6280027	11557812
43	3630110	4315369	7633124	11649715	17770288	17648712	62647318
44	3580806	4524356	7336259	11633159	29892291	50078833	107045704
45	387757	305256	409974	987602	5848763	9618484	17557837
Total	46531498	71248160	122225582	183933798	273288379	429893043	1127120460

Table 6.29 is used as the base for calculating all rates. First the rates for trade and transportation margins are obtained as a fraction of the net consumption. Then the tax rates are obtained after augmenting the base consumption net of taxes in table 6.29 with the trade and transportation values as in tables 6.18 6.19. This is base for excise taxes $texc_{c,qu}$ and finally adding the excise taxes to the above gives the base for vat and other taxes. The product of all these tax rates and margin rates gives the consumer prices including taxes and margins. This is shown in table 6.30

Table 6.30.: Benchmark Consumer Prices

	Q1	Q2	Q3	Q4	Q5	Q6
1	1.5413	1.5413	1.5413	1.5413	1.5413	1.5413
2	2.0511	2.0511	2.0511	2.0511	2.0511	2.0511
3	1.3928	1.3928	1.3928	1.3928	1.3928	1.3928
4	1.4107	1.4107	1.4107	1.4107	1.4107	1.4107
5	1.3909	1.3909	1.3909	1.3909	1.3909	1.3909
6	1.4117	1.4117	1.4117	1.4117	1.4117	1.4117
7	1.5039	1.5039	1.5039	1.5039	1.5039	1.5039
8	2.0996	2.0996	2.0996	2.0996	2.0996	2.0996
9	1.4435	1.4435	1.4435	1.4435	1.4435	1.4435
10	1.7067	1.7067	1.7067	1.7067	1.7067	1.7067
11	1.4977	1.4977	1.4977	1.4977	1.4977	1.4977
12	1.1122	1.1122	1.1122	1.1122	1.1122	1.1122
13	2.1341	2.1341	2.1341	2.1341	2.1341	2.1341
14	1.1266	1.1266	1.1266	1.1266	1.1266	1.1266
15	1.1231	1.1231	1.1231	1.1231	1.1231	1.1231
16	1.1219	1.1219	1.1219	1.1219	1.1219	1.1219
17	1.7161	1.7161	1.7161	1.7161	1.7161	1.7161
18	1.7371	1.7371	1.7371	1.7371	1.7371	1.7371
19	1.7337	1.7337	1.7337	1.7337	1.7337	1.7337
20	1.7487	1.7487	1.7487	1.7487	1.7487	1.7487
21	1.7348	1.7348	1.7348	1.7348	1.7348	1.7348
22	1.0311	1.0311	1.0311	1.0311	1.0311	1.0311
23	1.0535	1.0535	1.0535	1.0535	1.0535	1.0535
24	1.1015	1.1015	1.1015	1.1015	1.1015	1.1015
25	1.1673	1.1673	1.1673	1.1673	1.1673	1.1673
26	1.0661	1.0661	1.0661	1.0661	1.0661	1.0661
27	1.0984	1.0984	1.0984	1.0984	1.0984	1.0984
28	1.0406	1.0406	1.0406	1.0406	1.0406	1.0406
29	1.0822	1.0822	1.0822	1.0822	1.0822	1.0822
30	0.0000	1.1056	1.1056	1.1056	0.0000	1.1056
31	1.0409	1.0409	1.0409	1.0409	1.0409	1.0409
32	1.0682	1.0682	1.0682	1.0682	1.0682	1.0682
33	1.0984	1.0984	1.0984	1.0984	1.0984	1.0984
34	0.0000	1.0125	1.0125	1.0125	1.0125	1.0125
35	1.0125	1.0125	1.0125	1.0125	1.0125	1.0125
36	1.0125	1.0125	1.0125	1.0125	1.0125	1.0125
37	1.0135	1.0135	1.0135	1.0135	1.0135	1.0135
38	0.0000	0.0000	1.0161	1.0161	1.0161	1.0161
39	0.0000	0.0000	0.0000	1.0684	0.0000	1.0684
40	1.0686	1.0686	1.0686	1.0686	1.0686	1.0686

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Table 6.30 ... continued from previous page

	Q1	Q2	Q3	Q4	Q5	Q6
41	1.0089	1.0089	1.0089	1.0089	1.0089	1.0089
42	1.0089	1.0089	1.0089	1.0089	1.0089	1.0089
43	1.0118	1.0118	1.0118	1.0118	1.0118	1.0118
44	1.0214	1.0214	1.0214	1.0214	1.0214	1.0214
45	1.0089	1.0089	1.0089	1.0089	1.0089	1.0089

Given the data on expenditure elasticities we use formula 6.10 to obtain the shares, $\alpha H_{c,qu}$, of expenditures on commodity c by household qu. To obtain $\alpha H_{c,qu}$, we use table 6.5 to obtain w_i and table 6.24 for the elasticities. We need to renormalise them so that they sum up to unity. The shares are given in table 6.31.

Table 6.31.: Share of Expenditure of commodity c by Household

	Q1	Q2	Q3	Q4	Q5	Q6
1	0.0253	0.0247	0.0222	0.0173	0.0167	0.0130
2	0.0062	0.0079	0.0087	0.0080	0.0085	0.0075
3	0.0004	0.0004	0.0003	0.0002	0.0003	0.0000
4	0.0141	0.0167	0.0203	0.0179	0.0122	0.0124
5	0.0034	0.0041	0.0047	0.0046	0.0043	0.0049
6	0.0282	0.0291	0.0261	0.0200	0.0138	0.0106
7	0.0023	0.0020	0.0020	0.0016	0.0013	0.0010
8	0.0039	0.0154	0.0304	0.0305	0.0216	0.0217
9	0.0267	0.0407	0.0530	0.0468	0.0288	0.0311
10	0.0294	0.0556	0.0687	0.0693	0.0528	0.0811
11	0.0003	0.0003	0.0004	0.0004	0.0005	0.0007
12	0.0220	0.0170	0.0132	0.0121	0.0134	0.0111
13	0.0159	0.0716	0.0490	0.0366	0.0204	0.0168
14	0.1055	0.0339	0.0154	0.0144	0.0204	0.0238
15	0.0072	0.0173	0.0116	0.0117	0.0142	0.0064
16	0.0084	0.0043	0.0026	0.0024	0.0027	0.0024
17	0.0021	0.0042	0.0043	0.0056	0.0079	0.0064
18	0.0024	0.0078	0.0112	0.0172	0.0149	0.0125
19	0.0082	0.0147	0.0180	0.0247	0.0205	0.0269
20	0.2462	0.2098	0.1636	0.1429	0.1357	0.0806
21	0.0724	0.0755	0.0633	0.0543	0.0449	0.0294
22	0.0464	0.0394	0.0306	0.0264	0.0251	0.0148
23	0.0047	0.0049	0.0040	0.0034	0.0028	0.0017
24	0.0012	0.0016	0.0019	0.0033	0.0018	0.0021
25	0.0000	0.0005	0.0021	0.0032	0.0063	0.0018
26	0.0005	0.0005	0.0006	0.0006	0.0008	0.0010
27	0.0052	0.0070	0.0114	0.0040	0.0042	0.0307
28	0.0463	0.0688	0.0741	0.0964	0.0620	0.1265
29	0.0112	0.0089	0.0171	0.0118	0.0064	0.0009
30	0.0000	0.0003	0.0002	0.0001	0.0000	0.0012
31	0.0335	0.0121	0.0136	0.0123	0.0247	0.0383
32	0.0013	0.0001	0.0055	0.0081	0.0130	0.0177
33	0.0309	0.0321	0.0285	0.0305	0.0315	0.0250
34	0.0000	0.0005	0.0010	0.0040	0.0068	0.0132
35	0.0032	0.0065	0.0092	0.0130	0.0132	0.0115
36	0.0005	0.0005	0.0006	0.0006	0.0008	0.0010
37	0.0299	0.0369	0.0561	0.0734	0.0825	0.0825
38	0.0000	0.0000	0.0008	0.0004	0.0182	0.0120
39	0.0000	0.0000	0.0000	0.0003	0.0000	0.0045
40	0.0009	0.0032	0.0182	0.0134	0.0280	0.0184
41	0.0001	0.0017	0.0063	0.0171	0.0174	0.0053
42	0.0001	0.0000	0.0034	0.0089	0.0085	0.0122
43	0.0737	0.0573	0.0623	0.0628	0.0637	0.0409
44	0.0728	0.0602	0.0600	0.0628	0.1073	0.1162
45	0.0072	0.0037	0.0031	0.0049	0.0193	0.0205
total	1	1	1	1	1	1

We need the minimum expenditure $\mu H_{c,qu}$, which equals $\sum_{i=1}^{n} p_i\gamma_i$ in equation 6.9a

$$p_i x_i = \gamma_i p_i + \beta_i\left[y - \sum_{j=1}^{n} p_j\gamma_j\right]$$

$$\therefore \sum_{j=1}^{n} p_j\gamma_j = y - \frac{p_i(x_i-\gamma_i)}{\beta_i}$$

$$= y - \frac{p_i x_i^*}{\beta_i} \quad \because x_i^* = x_i - \gamma_i$$

$$= y - y^* \quad \because x_i^* = \frac{\beta_i y^*}{p_i}$$

$$= y - \frac{y}{\zeta} \quad \because \frac{y}{y^*} = \zeta \text{ Frisch Parameter}$$

$$\therefore \sum_{i=1}^{n} p_i\gamma_i = y\left[1 - \frac{1}{\zeta}\right] \tag{6.14}$$

Using the relation in 6.14 we obtain the minimum expenditure for each commodity for each household. $\mu H_{c,qu}$ is given in table 6.32

Table 6.32.: Minimum Expenditure of commodity c by Household

	Q1	Q2	Q3	Q4	Q5	Q6
1	1685895	2465469	3538177	4215698	6294323	7579045
2	308513	593772	1040494	1468982	2414412	3284703
3	2743	2791	3018	3382	10028	2129
4	1028090	1826585	3537779	4771996	5026956	7894016
5	249667	456765	832818	1253411	1813036	3179709
6	2056571	3173522	4546105	5346081	5660905	6755430
7	154560	208572	331581	404352	500991	573375
8	189462	1129869	3558281	5471104	5983889	9291668
9	1902587	4339219	9023030	12205321	11591945	19383545
10	491005	1314032	2448512	3936137	4958764	11787584
11	2010	2489	3335	6676	17269	34630
12	181631	164684	151718	263056	612419	788609
13	68532	360951	293556	413470	483321	621886
14	861603	324080	174941	307701	919341	1663147
15	58727	165495	131858	251694	642535	446736
16	69256	41563	29847	51095	123084	165248
17	11135	26580	31660	78586	233333	295764
18	12484	48116	82591	239398	433581	568721
19	43720	91357	132762	343273	598120	1222132
20	1295834	1291148	1195522	1972263	3932130	3629156
21	384250	468676	466368	755054	1312241	1337040
22	588537	637868	663795	991245	1769624	1614064
23	58287	76821	85572	124243	193107	185659
24	10094	15249	21895	72960	83010	150024
25	2	4286	22924	65805	274804	118157
26	4246	5258	7045	14102	36478	73149
27	43685	69051	132145	86853	195993	2199967
28	292103	444587	442002	1313182	2126731	6757930
29	130243	132083	336017	401325	412297	87336
30	0	4572	4187	1812	0	116182
31	405778	185030	278112	437263	1660402	3998945
32	15191	2144	109594	278491	850076	1802213
33	354250	466617	552060	1024732	2004516	2476608
34	0	4978	12608	94720	340385	1026614
35	28932	69097	116016	310055	659414	894038

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Table 6.32 ... continued from previous page

	Q1	Q2	Q3	Q4	Q5	Q6
36	4532	5612	7519	15051	38934	78075
37	386156	607643	1238896	2802019	5903391	9169477
38	0	0	10594	8449	907454	928662
39	0	0	0	5654	0	332872
40	7386	32121	217999	303510	1328025	1356610
41	992	17922	79494	408650	874143	410845
42	2511	1081	117741	507496	849062	1901296
43	641396	575334	726465	1405624	3047514	3043100
44	655898	634071	750480	1484174	5321383	8961233
45	93649	61529	68227	187543	1384587	2285251
Total	14782142	22548690	37555341	56103688	83823954	130472581

6.6.4. Calibration of Production Function

We first obtain the share of value added, net of taxes and subsidies, but gross of depreciation, in output. This is used in the Leontief production function. The calculated values are shown in table 6.33.

Table 6.33.: Share in Output of Value Added Net of Taxes and Subsidies

	LZ	TRLZ	KZ	TRKZ	DEPZ	TOTAL	XDZ	aKL
1	12917426	715697	70384358	4803303	29239646	118060430	253044315	0.4666
2	14100671	781255	15865575	302683	6287656	37337840	46494039	0.8031
3	4071203	225567	1677899	745	652806	6628220	12830283	0.5166
4	5031742	278786	1901279	2229	740253	7954288	30593780	0.2600
5	6150247	340758	1305584	1050	508136	8305775	44060254	0.1885
6	9984824	553214	5261480	17138	2052796	17869453	178642871	0.1000
7	2033838	112686	7387883	33880	2886241	12454527	65518264	0.1901
8	6669281	369515	7310995	57199	2865409	17272400	46115789	0.3745
9	12492766	692168	714014	314	277794	14177056	47327374	0.2996
10	1069394	59250	1836434	1827	714879	3681785	7825183	0.4705
11	3538891	196074	1534558	1006	597164	5867693	13415854	0.4374
12	3149262	174486	630377	267	245251	4199643	9329864	0.4501
13	0	0	0	0	0	0	0	0.0000
14	399983	22161	131816	0	51262	605223	3752877	0.1613
15	511283	28328	132682	0	51599	723892	4373629	0.1655
16	8798054	487461	5146714	9886	2005344	16447459	53926806	0.3050
17	5091044	282072	4088245	433	1590041	11051836	23950233	0.4615
18	1520860	84264	2733903	492	1063376	5402895	34446154	0.1569
19	551173	30538	56356	0	21916	659983	2980940	0.2214
20	552253	30598	90176	4	35070	708101	2330010	0.3039
21	683548	37872	614789	253	239183	1575645	3423106	0.4603
22	38165866	2114599	3657760	13766	1427816	45379807	101986844	0.4450
23	5339877	295859	128713	0	50055	5814504	10361405	0.5612
24	98745202	5471029	29177347	711134	11623298	145728009	411746153	0.3539
25	20105226	1113940	5696068	31928	2227554	29174717	55460210	0.5260
26	19141885	1060566	31568987	903756	12628289	65303483	153110034	0.4265
27	61721944	3419736	33261214	871234	13273730	112547858	181092729	0.6215
28	28875266	1599849	2199220	2908	856383	33533626	130362980	0.2572
29	10304150	570907	8872369	559	3450583	23198568	43560957	0.5326
30	2782427	154162	19475359	182529	7644734	30239210	30242823	0.9999
31	46259473	2563030	13534304	11946	5267986	67636740	155005440	0.4364
32	24973417	1383665	24086810	354150	9504818	60302859	113644353	0.5306
33	26104691	1446344	9493171	44449	3709074	40797729	69031049	0.5910
34	37901904	2099974	27620204	505953	10937950	79065985	109229762	0.7239
35	4743921	262839	845700	701	329156	6182318	11353259	0.5445
36	70857	3926	94079	7	36589	205457	488514	0.4206
37	312586	17319	105758161	1126919	41566420	148781406	185690308	0.8012
38	1763878	97729	9029549	15572	3517547	14424274	17544892	0.8221
39	172298	9546	868753	646	338100	1389343	2202711	0.6307
40	20620152	1142470	29677977	1017804	11937248	64395651	129215761	0.4984
41	269434491	14928157	31587092	0	12283869	328233609	401769836	0.8170

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	LZ	TRLZ	KZ	TRKZ	DEPZ	TOTAL	XDZ	aKL
42	143486942	7949968	5579787	983	2170300	159187980	180716890	0.8809
43	122446331	6784202	18261894	208876	7183077	154884380	241015611	0.6426
44	55682357	3085110	5248845	19665	2048865	66084842	139903614	0.4724
45	16636105	921732	0	0	0	17557837	17557837	1.0000

The CES function form for the Value Added is

$$VA_s = a_s X_s$$
$$VA_s = aF_s\big[\gamma_s K_s^{\frac{\sigma_s - 1}{\sigma_s}} + (1-\gamma_s) L_s^{\frac{\sigma_s - 1}{\sigma_s}}\big]^{\frac{\sigma_s}{\sigma_s - 1}}$$

	Description	Status
γ_s	CES distribution parameter for capital in production	given parameter
σ_s	elasticity of substitution in production	given parameter
aF_s	efficiency parameter in CES production function	to be computed
a_s	fixed share of value added in production	to be computed aKL
VA_s	value added in production	given in benchmark KLZ
K_s	capital stock in sector s	given in benchmark KSKZ
L_s	number of employees in branch s	given in benchmark LSKZ
XD_s	domestic production in sector s	given in benchmark XDZ

We need to obtain γ_s and aF_s. We begin with the producer's cost minimisation problem to derive the expressions for calibrating the parameters.

$$\min wL + rK$$

subject to

$$VA = a\big[\gamma K^{\frac{\sigma-1}{\sigma}} + (1-\gamma) L^{\frac{\sigma-1}{\sigma}}\big]^{\frac{\sigma}{\sigma-1}}$$

The lagrangian is

$$\mathcal{L} = wL + rK + \lambda\Big\{VA - a\big[\gamma K^{\frac{\sigma-1}{\sigma}} + (1-\gamma) L^{\frac{\sigma-1}{\sigma}}\big]^{\frac{\sigma}{\sigma-1}}\Big\}$$

Differentiating with respect to L and K we have

$$\frac{\partial \mathcal{L}}{\partial L} = 0 \Rightarrow$$
$$w = \lambda a \mathcal{C}^{\frac{1}{\sigma}} (1-\gamma) L^{-\frac{1}{\sigma}}$$
$$\frac{\partial \mathcal{L}}{\partial K} = 0 \Rightarrow$$
$$r = \lambda a \mathcal{C}^{\frac{1}{\sigma}} \gamma K^{-\frac{1}{\sigma}}$$
$$\mathcal{C} = \left[\gamma K^{\frac{\sigma-1}{\sigma}} + (1-\gamma) L^{\frac{\sigma-1}{\sigma}}\right]$$
$$\therefore \frac{w}{r} = \frac{1-\gamma}{\gamma}\left[\frac{K}{L}\right]^{\frac{1}{\sigma}} \Rightarrow \frac{1}{\gamma} - 1 = \frac{w}{r}\left[\frac{L}{K}\right]^{\frac{1}{\sigma}}$$
$$\therefore \gamma = \frac{1}{1 + \frac{w}{r}\left[\frac{L}{K}\right]^{\frac{1}{\sigma}}} \qquad (6.15)$$
$$a = \frac{VA}{\left[\gamma K^{\frac{\sigma-1}{\sigma}} + (1-\gamma) L^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}} \qquad (6.16)$$

We need to know or obtain the values of L, K, w and r from the data set. The model sets a growth rate of 2.7% and to achieve that we need to find the total capital stock. The information on Investments (IZ) by sector is gross of tax and trade, transport margins. We first convert it to net of tax and margins and then obtain a composite price index of capital goods (PIZ). This index will be used to value the composite capital good. The total capital stock consists of the fresh investment and the surviving capital stock after depreciation. So we proceed as follows. Obtain (PIZ), the price index of investment, the new deprecaition by sector based on this price index and the rescaled capital stock by sector such that it equals the total capital stock necessary to achieve the 2.7% growth rate.

Table 6.34.: Investment and Price Index of Investment

	IZ	TRVATIZ	COITZ	IZnew	vati	tcitm	iol	PIZ
1	6894060	0	93196	6800864	0	0.0137	0.0098	0.0099
2	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0
10	723842	0	61865	661977	0	0.0935	0.0010	0.0010
11	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0
15	6778136	116520	0	6661616	0.0172	0	0.0096	0.0098
16	0	0	0	0	0	0	0	0
17	14058667	0	1913890	12144776	0	0.1576	0.0175	0.0203
18	128736845	2310559	13450425	112975861	0.0179	0.1191	0.1629	0.1855
19	82451518	1912754	8565353	71973410	0.0232	0.1190	0.1038	0.1188
20	89937170	2103921	9399354	78433895	0.0234	0.1198	0.1131	0.1296
21	38264412	581754	4422868	33259790	0.0152	0.1330	0.0480	0.0552
22	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0
24	297298757	8535018	0	288763738	0.0287	0	0.4163	0.4283
25	3920601	0	0	3920601	0	0	0.0057	0.0057
26	231731	0	0	231731	0	0	0.0003	0.0003
27	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0
31	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0
37	34032193	0	0	34032193	0	0	0.0491	0.0491
38	0	0	0	0	0	0	0	0
39	1762730	0	0	1762730	0	0	0.0025	0.0025
40	39638169	0	0	39638169	0	0	0.0571	0.0571
41	0	0	0	0	0	0	0	0
42	0	0	0	0	0	0	0	0
43	0	0	0	0	0	0	0	0
44	2355773	0	0	2355773	0	0	0.0034	0.0034
45	0	0	0	0	0	0	0	0
TOTAL	747084604	15560526	37906952	693617126				1.0765

IZnew equals IZ - TRVATIZ - COITZ and vati equals TRVATIZ/IZnew while tcitm equals COITZ/IZnew. iol is the vector of shares of IZnew. PIZ, the price index is obtained by multiplying the price of investment of each commodity by its share and equals sum of (1+vati)×(1+tcitm)×iol.

The total capital stock K_{tot} is given by

$$K_{tot} = \frac{\sum_{i=1}^{n} IZ_i - \sum_{i=1}^{n} DEP_i}{growth} \tag{6.17}$$

Using data from tables 6.10 6.34 we obtain the total capital stock as

$$K_{tot} = \frac{693617126 - \frac{216139263}{1.0765}}{0.027} = 18257276725.4$$

Table 6.35.: Depreciation and Investment

	DEPZ	IZnew	KZ	KSKZ	δ	PKZ
1	27161231	6800864	70384358	2389516949	0.0114	0.0295
2	5840716	0	15865575	531726257	0.0110	0.0298
3	606403	0	1677899	55957208	0.0108	0.0300
4	687634	0	1901279	63419087	0.0108	0.0300
5	472017	0	1305584	43544838	0.0108	0.0300
6	1906879	0	5261480	175599076	0.0109	0.0300
7	2681081	0	7387883	246653491	0.0109	0.0300
8	2661730	0	7310995	244295976	0.0109	0.0299
9	258048	0	714014	23812027	0.0108	0.0300
10	664064	661977	1836434	61253227	0.0108	0.0300
11	554716	0	1534558	51179725	0.0108	0.0300
12	227818	0	630377	21022700	0.0108	0.0300
13	0	0	0	0	0	0
14	47618	0	131816	4395494	0.0108	0.0300
15	47931	6661616	132682	4424376	0.0108	0.0300
16	1862800	0	5146714	171707943	0.0108	0.0300
17	1477018	12144776	4088245	136328966	0.0108	0.0300
18	987789	112975861	2733903	91168101	0.0108	0.0300
19	20358	71973410	56356	1879222	0.0108	0.0300
20	32577	78433895	90176	3007012	0.0108	0.0300
21	222181	33259790	614789	20502771	0.0108	0.0300
22	1326324	0	3657760	122092172	0.0109	0.0300
23	46497	0	128713	4292016	0.0108	0.0300
24	10797090	288763738	29177347	979230277	0.0110	0.0298
25	2069215	3920601	5696068	190221568	0.0109	0.0299
26	11730644	231731	31568987	1060685603	0.0111	0.0298
27	12330206	0	33261214	1116826204	0.0110	0.0298
28	795510	0	2199220	73360128	0.0108	0.0300
29	3205308	0	8872369	295859762	0.0108	0.0300
30	7101330	0	19475359	651033536	0.0109	0.0299
31	4893527	0	13534304	451415732	0.0108	0.0300
32	8829196	0	24086810	806324042	0.0109	0.0299
33	3445425	0	9493171	316949187	0.0109	0.0300
34	10160458	0	27620204	925490660	0.0110	0.0298
35	305759	0	845700	28206612	0.0108	0.0300
36	33988	0	94079	3137186	0.0108	0.0300
37	38611792	34032193	105758161	3536545872	0.0109	0.0299
38	3267512	0	9029549	301233885	0.0108	0.0300
39	314067	1762730	868753	28974850	0.0108	0.0300
40	11088724	39638169	29677977	998637897	0.0111	0.0297
41	11410706	0	31587092	1053291780	0.0108	0.0300
42	2016031	0	5579787	186070277	0.0108	0.0300

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Table 6.35 ... continued from previous page

	DEPZ	IZnew	KZ	KSKZ	δ	PKZ
43	6672489	0	18261894	610802920	0.0109	0.0299
44	1903227	2355773	5248845	175200113	0.0109	0.0300
45	0	0	0	0	0	0
total	200775634	693617126	544528480	18257276725		
growth	0.027					
KSKTZ	18257276725					

The expression for labour price w and price of capital r are given by

$$w = PLZ \times (1 + premLSK) \times (1 + \frac{tl}{(1-tl)})$$
$$r = PKZ \times (1 + tk) + \delta \times PIZ \qquad (6.18)$$

The calibrated values for an elasticity of substitution $\sigma = 0.6$ are given in table 6.36.

Table 6.36.: Calibration of Share Parameter-Production Function

	r	w	$\frac{w}{r}$	$LSKZ$	$KSKZ$	KLZ	γ	aF
1	0.04357	2112.35	48477.52	6454	2389516949	118060430	0.999975	0.0594
2	0.04222	2112.11	50024.57	7046	531726257	37337840	0.999629	0.1505
3	0.04166	3305.20	79328.29	1300	55957208	6628220	0.998502	0.5665
4	0.04169	11671.49	279978.31	455	63419087	7954288	0.999253	0.6538
5	0.04168	11674.47	280125.49	556	43544838	8305775	0.998050	1.8621
6	0.04175	11670.03	279517.98	903	175599076	17869453	0.999572	0.3870
7	0.04179	11665.89	279149.74	184	246653491	12454527	0.999983	0.0671
8	0.04189	11672.96	278668.42	603	244295976	17272400	0.999874	0.1550
9	0.04166	11678.42	280295.07	1129	23812027	14177056	0.982889	31.3385
10	0.04168	11635.50	279151.45	97	61253227	3681785	0.999940	0.1041
11	0.04167	11671.77	280091.15	320	51179725	5867693	0.999406	0.5227
12	0.04166	11662.27	279910.79	285	21022700	4199643	0.997847	2.0905
13	0	0	0	0	0	0	1	0.0000
14	0.04165	11726.23	281533.50	36	4395494	605223	0.999064	0.8264
15	0.04165	11730.68	281640.13	46	4424376	723892	0.998607	1.2712
16	0.04171	11679.89	280026.61	795	171707943	16447459	0.999640	0.3332
17	0.04165	11680.69	280418.25	460	136328966	11051836	0.999787	0.2200
18	0.04166	11716.24	281256.29	137	91168101	5402895	0.999945	0.1006
19	0.04165	11634.22	279324.41	50	1879222	659983	0.993420	8.5142
20	0.04165	11657.03	279862.90	50	3007012	708101	0.996978	3.1511
21	0.04166	11635.81	279278.12	62	20502771	1575645	0.999823	0.1925
22	0.04177	11675.50	279546.12	3450	122092172	45379807	0.992725	9.7599
23	0.04165	11668.19	280140.06	483	4292016	5814504	0.931547	225.9401
24	0.04237	5757.81	135877.53	18100	979230277	145728009	0.998247	0.9767
25	0.04182	9645.08	230624.23	2200	190221568	29174717	0.998638	1.0749
26	0.04250	16835.37	396155.11	1200	1060685603	65303483	0.999951	0.1073
27	0.04243	7487.55	176479.54	8700	1116826204	112547858	0.999460	0.3685
28	0.04169	6219.41	149176.27	4900	73360128	33533626	0.983862	16.1949
29	0.04165	20176.36	484388.91	539	295859762	23198568	0.999868	0.2025
30	0.04193	20113.62	479640.99	146	651033536	30239210	0.999996	0.0541
31	0.04168	20174.58	484055.23	2420	451415732	67636740	0.999206	1.0201
32	0.04209	20166.09	479083.41	1307	806324042	60302859	0.999893	0.1771
33	0.04179	20169.14	482587.70	1366	316949187	40797729	0.999449	0.6952
34	0.04220	20172.41	478027.73	1983	925490660	79065985	0.999830	0.2460
35	0.04168	20188.54	484409.28	248	28206612	6182318	0.998189	2.6362
36	0.04165	18695.62	448836.08	4	3137186	205457	0.999933	0.1291
37	0.04197	20619.08	491245.65	16	3536545872	148781406	0.999999	0.0422
38	0.04170	20234.85	485202.34	92	301233885	14424274	0.999993	0.0589
39	0.04167	20204.93	484832.71	9	28974850	1389343	0.999993	0.0592
40	0.04266	20169.25	472819.73	1079	998637897	64395651	0.999946	0.1197
41	0.04165	20173.28	484337.58	14096	1053291780	328233609	0.996360	6.3426
42	0.04166	20172.76	484262.25	7507	186070277	159187980	0.977534	76.9580

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Table 6.36 ... continued from previous page

	r	w	$\frac{w}{r}$	$LSKZ$	$KSKZ$	KLZ	γ	aF
43	0.04200	20173.36	480359.97	6406	610802920	154884380	0.997592	3.7486
44	0.04177	20174.20	483036.35	2913	175200113	66084842	0.994795	10.1577
45	0	20181.42	0	870	0	17557837	1	0.0000

6.6.5. Calibration of the CET function

The calibrated values for CET function and elasticities σT are given in table 6.37. From table 6.3 and table 6.15 we get the values of exports X_i by destination (mainland, European Union, USA and Rest of the World) and the Domestic Output Q_i respectively. The difference between Q_i and X_i is the domestic output sold in domestic markets $XDDZ_i$.

To calibrate the share and scale parameters we use the following formula

$$\gamma T[j]_i = \frac{PE[j]_i \times X[j]_i^{\frac{1}{\sigma T}}}{\sum_{j=1}^{5} PE[j]_i \times X[j]_i^{\frac{1}{\sigma T}}}$$
$$aT_i = \frac{XDDZ}{\left[\sum_{j=1}^{5} \gamma T_j \times X[j]_i^{\frac{\sigma T-1}{\sigma T}}\right]^{\frac{\sigma T}{\sigma T-1}}} \quad (6.19)$$

where $[j] \in ML, EU, USA, RoW, XD$ is the share of output to each destination for each $i \in 1,2,..,45$ commodity and aT is the scale parameter in the CET function

Table 6.37.: Calibration of Parameters-CET Function

	XDDZ	σT	$\gamma T1$	$\gamma T2$	$\gamma T3$	$\gamma T4$	$\gamma T5$	aT
1	150573296	-4.00	0.0486	0.3477	0.3013	0.2583	0.0441	13.4778
2	33024368	-4.00	0.1417	0.2070	0.3050	0.2419	0.1044	6.5270
3	10622327	-0.95	0.0004	0	0	0.9995	0.0001	111.0286
4	21483768	-3.00	0.0131	0.0938	0.3623	0.5210	0.0098	35.0543
5	16827553	-3.00	0.1200	0.1506	0.3442	0.2657	0.1195	6.2594
6	4511742	-3.00	0.0494	0.2033	0.2653	0.3164	0.1657	9.6601
7	65397766	-3.00	0.8908	0	0	0	0.1092	5.2664
8	24845347	-3.00	0.0269	0.7365	0.0940	0.1174	0.0252	18.4305
9	43402978	-3.00	0.0973	0.2760	0.3059	0.2787	0.0421	10.9990
10	3585749	-3.00	0.1456	0	0.4033	0.3044	0.1467	5.1282
11	10310158	-3.00	0.0498	0	0.3793	0.5375	0.0334	13.6774
12	9137783	-3.00	0.1420	0	0.4469	0.3730	0.0381	11.6540
13	0	-2.50	0	0	0	0	0	0
14	3327707	-3.00	0.0789	0	0.6929	0.1893	0.0388	11.7823
15	4373629	-3.00	0	0	0	0	1	1
16	52679651	-3.00	0.1038	0	0.5287	0.3381	0.0294	14.1502
17	22734940	-3.00	0.1015	0	0.5983	0.2627	0.0375	11.8922
18	34446154	-3.00	0	0	0	0	1	1
19	2980940	-3.00	0	0	0	0	1	1
20	2330010	-3.00	0	0	0	0	1	1
21	3373900	-2.50	0.1273	0	0.5330	0.3173	0.0223	15.1681
22	101986844	-2.50	0	0	0	0	1	1
23	10361405	-2.50	0	0	0	0	1	1
24	411746153	-2.00	0	0	0	0	1	1
25	53912711	-2.00	0.0058	0.4030	0.3540	0.2363	0.0010	102.5823
26	151230715	-2.00	0.0482	0.3841	0.3374	0.2252	0.0052	33.5932
27	179649766	-2.00	0.9177	0	0	0	0.0823	5.3015
28	121305107	-2.00	0.0985	0.3565	0.3132	0.2090	0.0228	12.7488
29	39679525	-2.00	0.1188	0.3456	0.3036	0.2026	0.0292	10.8677

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Table 6.37 ... continued from previous page

	XDDZ	σT	$\gamma T1$	$\gamma T2$	$\gamma T3$	$\gamma T4$	$\gamma T5$	aT
30	17077272	-2.00	0.1129	0.3283	0.2884	0.1925	0.0780	6.6284
31	48155429	-2.00	0.0644	0.3438	0.3020	0.2016	0.0882	7.4493
32	95891495	-2.00	0.1176	0.3419	0.3003	0.2004	0.0398	9.0787
33	66453884	-2.00	0.1201	0.3494	0.3070	0.2049	0.0186	14.4184
34	105904098	-2.00	0	0.3933	0.3455	0.2306	0.0306	10.3386
35	11161936	-2.00	0	0.3965	0.3483	0.2325	0.0228	12.5249
36	488514	-2.00	0	0	0	0	1	1
37	185690308	-2.00	0	0	0	0	1	1
38	17544892	-2.00	0	0	0	0	1	1
39	1303183	-2.00	0.3558	0.0000	0.1299	0.4171	0.0973	5.6318
40	122522382	-2.00	0.4733	0.0000	0.2897	0.2006	0.0364	9.2671
41	401769836	-2.00	0	0	0	0	1	1
42	180716890	-2.00	0	0	0	0	1	1
43	241015611	-2.00	0	0	0	0	1	1
44	137537533	-2.00	0.0503	0	0.4035	0.5398	0.0065	28.8315
45	17557837	-2.00	0	0	0	0	1	1

6.6.6. Calibration of the Armington function

The calibrated values for Armington function between imports and domestic goods and elasticities σA are given in table 6.38. From table 6.4 we get the values of imports M_i by source (mainland, European Union, USA and Rest of the World) and the Domestic Output sold in domestic markets X_i is obtained as shown in table 6.37.

To calibrate the share and scale parameters we use the following formula

$$\gamma A[j]_i = \frac{PM[j]_i \times M[j]_i^{\frac{1}{\sigma A}}}{\sum_{j=1}^{5} PM[j]_i \times M[j]_i^{\frac{1}{\sigma T}}}$$
$$aF_i = \frac{XZ}{\left[\sum_{j=1}^{5} \gamma A_j \times M[j]_i^{\frac{\sigma A - 1}{\sigma A}}\right]^{\frac{\sigma A}{\sigma A - 1}}} \tag{6.20}$$

where $[j] \in ML, EU, USA, RoW, XD$ is the share of use from each source for each $i \in 1, 2, .., 45$ commodity and aT is the scale parameter in the CET function

Table 6.38.: Calibration of Parameters-Armington Function

	XZ	tm	σA	$\gamma A1$	$\gamma A2$	$\gamma A3$	$\gamma A4$	$\gamma A5$	aF
1	218610183.9	0.000725158	2.90	0.2062	0.2191	0.1490	0.0489	0.3769	3.6448
2	33947498.93	0.000725158	2.90	0.1115	0.1276	0.0447	0.1163	0.5999	2.1500
3	10916522.29	0.000725158	2.90	0.1547	0.1401	0.0097	0.0455	0.6500	1.9025
4	49955297.89	0.000725158	2.95	0.4021	0.0923	0.0860	0.0509	0.3687	2.9355
5	37449251.87	0.000725158	2.95	0.2650	0.1939	0.0676	0.1765	0.2971	4.1623
6	55204278.14	0.000725158	2.95	0.6341	0.0466	0.0164	0.0235	0.2793	1.9063
7	77376074.96	0.000725158	2.95	0.0504	0.1083	0.2367	0.1415	0.4632	2.9392
8	68051413.3	0.000725158	2.95	0.3667	0.1426	0.1098	0.0673	0.3136	3.4468
9	142729309.7	0.000725158	2.95	0.2538	0.2188	0.1736	0.1090	0.2447	4.5687
10	75925100.29	0.000725158	3.75	0.2930	0.2720	0.1187	0.1603	0.1559	4.1535
11	12170666.03	0.000725158	3.20	0.2140	0.1311	0.1450	0.0902	0.4197	3.2782
12	49621581.85	0.000725158	3.20	0.3025	0.2758	0.1076	0.0859	0.2282	3.9744
13	58049263.29	0.000509524	2.10	0.5044	0.4915	0.0040	0	0	2.0152
14	92610522.89	0.000725158	3.30	0.2485	0.3970	0.0995	0.0989	0.1561	3.3815
15	44971862.58	0.000725265	3.30	0.2497	0.3132	0.1021	0.1516	0.1835	4.1362
16	104459401.6	0.000725158	2.90	0.2947	0.2238	0.0578	0.0840	0.3397	3.6235

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Table 6.38 ... continued from previous page

	XZ	tm	σA	$\gamma A1$	$\gamma A2$	$\gamma A3$	$\gamma A4$	$\gamma A5$	aF
17	66654987.47	0.000725158	2.95	0.2018	0.1008	0.1027	0.3159	0.2788	3.9780
18	131248583.3	0.000725158	4.05	0.1862	0.1358	0.2715	0.1761	0.2305	4.5274
19	108649726.9	0.000770221	4.40	0.2199	0.1344	0.2603	0.2456	0.1398	4.4311
20	176257314.9	0.000725158	4.30	0.2042	0.1707	0.3026	0.2016	0.1208	4.2319
21	81093135.46	0.000725158	2.80	0.2793	0.2498	0.1780	0.1709	0.1220	4.5079
22	101986844	0	2.80	0	0	0	0	1	1
23	10361405	0	2.80	0	0	0	0	1	1
24	411746153	0	1.90	0	0	0	0	1	1
25	53922630.84	0.000725158	1.90	0.0107	0.0002	0.0001	0.0001	0.9888	1.0238
26	153201716.4	0.000725158	1.90	0.0857	0.0176	0.0107	0.0090	0.8771	1.3001
27	181055543.2	0.000725158	1.90	0.0722	0.0005	0.0003	0.0002	0.9268	1.1639
28	130465091.5	0.000725158	1.90	0.1755	0.0454	0.0276	0.0232	0.7283	1.8014
29	40094939.12	0.000725158	1.90	0.0795	0.0117	0.0071	0.0059	0.8958	1.2470
30	19444558.15	0.000725158	1.90	0.1947	0.0804	0.0489	0.0410	0.6350	2.2578
31	129907469.3	0.000725158	1.90	0.5184	0.0408	0.0248	0.0208	0.3951	2.3574
32	98212416.44	0.000725158	1.90	0.1077	0.0315	0.0191	0.0160	0.8257	1.4592
33	68862950.8	0.000725158	1.90	0.1229	0.0433	0.0264	0.0221	0.7853	1.6009
34	124963233.8	0.000725158	1.90	0.2400	0.0615	0.0374	0.0313	0.6298	2.2085
35	18579651.6	0.000725158	1.90	0.3414	0.0959	0.0583	0.0489	0.4555	2.9860
36	11812735.96	0.000725158	1.90	0.5151	0.1773	0.1079	0.0904	0.1093	3.1046
37	185690308	0	1.90	0	0	0	0	1	1
38	27136753.67	0.000725158	1.90	0.2753	0.1293	0.0786	0.0659	0.4509	3.3094
39	19879531.18	0.000725158	1.90	0.3802	0.0258	0.3243	0.1384	0.1313	3.5191
40	192526815.5	0.000725158	1.90	0.3053	0.0936	0.1015	0.0388	0.4608	3.1070
41	401769836	0	1.90	0	0	0	0	1	1
42	180716890	0	1.90	0	0	0	0	1	1
43	241015611	0	1.90	0	0	0	0	1	1
44	142712545.4	0.000725158	1.90	0.1227	0.0171	0.0429	0.0374	0.7799	1.6220
45	17557837	0	1.90	0	0	0	0	1	1

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PART 3
Policy Simulations

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Road Construction Project Under Public-Private Partnership

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In an attempt to circumvent budget restrictions, some governments have resorted to public/private partnerships in order to construct infrastructure. In some cases users are called on to pay the services but in others a virtual toll is established and the government assures payment. This model has been used extensively in recent years in Portugal and has also been adopted in the island of São Miguel, part of the Portuguese archipelago of the Azores, by the autonomous government of this region. The Portuguese government as per the directive of the European Union, of which it is a member, has to maintain its deficit under 3% of the GDP. This has implied a cap on capital expenditure on the development of infrastructure and on transfers to sub national governments as well as a total prohibition of new loan at these levels. The Azorean government has, consequently, embarked on a programme to solicit initial investment from the private sector to build roads and other infrastructure and amortise the payment over a span of time. Since there is a cap on new debt, payment of the project over time will have to come either from increased taxes, reduced government expenditures or the increases in income derived from the improved transport infrastructure. To analyse the welfare impacts of this project, we use a sequentially dynamic general equilibrium model built with the latest data on the Azorean economy at the 45 sector level. The following scenarios are considered: An increase in the income tax levied; a reduction in the transfers to the households equal to the amortisation. and, finally; a reduction in the transport margins that should improve the efficiency of the economy. It is found that the benefits derived from the construction of the road in the short term tend to fall short of the costs incurred in the future,

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making the road a poor investment.

Contents

7.1. Introduction

In an attempt to circumvent budget restrictions, some governments have resorted to public/private partnerships in order to construct infrastructure. This approach to anticipate amenities that are not executable within a certain government budget has, in the past, been mostly associated to projects that lend themselves to the application of user fees, such as major roads, bridges and the like. Users are called upon to pay the services. An innovation of this approach has been to levy no fees to user and have the government assume the cost of what will become a virtual toll. This approach has been used extensively in recent years in Portugal and has also been adopted in the island of São Miguel, part of the Portuguese archipelago of the Azores. Bound by the stability and growth pact associated to the single currency, adopted by some European Union countries, the Portuguese government has resorted to operations off the national budget.

The Azorean government has embarked on a similar programme to solicit initial investment from the private sector to build a road and amortise the payment over a span of twenty five years, after completion.

If in the short-run, during construction, one expects a positive welfare effect from the inflow of investment expenditure, without any additional taxes or any cuts in other projects, in the longer run the impacts will have to take into consideration the reimbursement of the implicit loan associated to the construction and maintenance of the road.

To analyse these welfare impacts of this project, we use a sequentially dynamic general equilibrium model built with the latest data on the Azorean economy at the 45 sector level. The main issues are:

(1) What economic benefits are derived from the improved road?
(2) What is the impact of increasing taxes to pay for the road?
(3) What are the welfare impacts of reducing transfers to compensate the extra expenditure?

The following scenarios are considered: a reduction in the transfers to the households equal to the amortisation. and, finally; a reduction in the transport margins that should improve the efficiency of the economy.

In what follows, section 7.2 describes, in detail, the set-up of the project and its implications on the regional economy and on the public finances. Section 7.3 analyses the scenarios considered and the results obtained. Section 7.4 reviews the main conclusions and advances some suggestions for future work.

7.2. A Public/Private Partnership for the Construction of a Major Road

In 2006, to start in 2007, the regional government of the Azores contracted the construction/ repair of a major road in one of the islands of the archipelago - S. Miguel. This island accounts for about 60% of the economic activity of this region and for about 60% of its GDP. The project affects about 50% of the stock of major roads in the island and will impact on about 80% of the traffic.

The project is as follows: on a first, immediate phase, the government gives concession of existing roads to a private company that will assume its' upkeep and gets a payment of €17624608 on the first year and a payment of €846972 on the second year; simultaneously, for a period of five years, the company will construct/repair the predetermined road sections; as of the sixth year, the government will start payment of the accumulated debt on a schedule that is expected to imply outlays of around €325 million.

Debt, comprising the initial payments by the private company, construction during five years, maintenance costs and interest on outstanding debt, accumulates during the first five years to determine the total that will be reimbursed as of the sixth year. Given the payment schedule, it is estimated that the base value of the project, consisting of new construction, will be €100 million, spread evenly along the five years.

Until the end of the fifth year there are no outlays by the government. In fact, government expenditures are potentially increased as there advances from the private concession on the first two years. The investment and amortization schedule is presented in table 7.1.

Table 7.1.: Amortization Schedule at interest rate of 5.5%

Time	Investment		Maintenance		Advances	
	Present Value	Future Value	Present Value	Future Value	Present Value	Future Value
0	20000000	26139200	0	0	17624608	23034658
1	20000000	24776493	750000	929118	846972	1049250
2	20000000	23484828	750000	880681		
3	20000000	22260500	750000	834769		
4	20000000	21100000	750000	791250		
Total (0-4)		117761021		3435818		24083908
Total (5-29)			22956133			
5		1250000	1250000			
6		1250000	1184834			
7		1250000	1123066			
8		1250000	1064517			
9		1250000	1009021			
10		1250000	956418			
11		1250000	906557			
12		1250000	859296			
13		1250000	814499			
14		1500000	926444			
15		1500000	878146			
16		1500000	832366			
17		2000000	1051963			
18		2000000	997121			
19		2000000	945139			
20		2250000	1007849			
21		2250000	955307			
22		2250000	905505			
23		2250000	858298			
24		2250000	813553			
25		2250000	771140			
26		2250000	730939			
27		2250000	692833			
28		2500000	729682			
29		2500000	691641			

Source: computed

Table 7.2.: Reimbursement Schedule of the Project

T	K	EYI	@ 5.5%	△K	K-△K
0	168236880	12541951	9253028	3288922	164947958
1	164947958	12541951	9072138	3469813	161478145
2	161478145	12541951	8881298	3660653	157817492
3	157817492	12541951	8679962	3861988	153955504
4	153955504	12541951	8467553	4074398	149881106
5	149881106	12541951	8243461	4298490	145582616
6	145582616	12541951	8007044	4534907	141047710
7	141047710	12541951	7757624	4784327	136263383
8	136263383	12541951	7494486	5047464	131215919
9	131215919	12541951	7216876	5325075	125890844
10	125890844	12541951	6923996	5617954	120272890
11	120272890	12541951	6615009	5926942	114345948
12	114345948	12541951	6289027	6252923	108093025
13	108093025	12541951	5945116	6596834	101496190
14	101496190	12541951	5582290	6959660	94536530
15	94536530	12541951	5199509	7342441	87194089
16	87194089	12541951	4795675	7746276	79447813
17	79447813	12541951	4369630	8172321	71275492
18	71275492	12541951	3920152	8621798	62653694
19	62653694	12541951	3445953	9095997	53557697
20	53557697	12541951	2945673	9596277	43961419
21	43961419	12541951	2417878	10124072	33837347
22	33837347	12541951	1861054	10680896	23156451
23	23156451	12541951	1273605	11268346	11888105
24	11888105	12541951	653846	11888105	0

T: time [0: 2012; 24: 2036]
EYI: equated yearly installments, int: interest payment
△K: change in capital, K-△K: balance capital remaining

Table 7.3.: Summary values at stake, assuming an average interest rate of 5,5%

	Heading	Value
1	Future value of investments from 2007 to 2011 in 2012	117761021
2	Future value of maintenance from 2007 to 2011 in 2012	3435818
3	Present value of maintenance from 2012 to 2036 in 2012	22956133
4	Future value of advances in 2007, 2008 in 2012	24083908
5	Total Liabilities in 2012 (5=1+2+3+4)	168236880

Source: computed

The accumulated obligations, at the end of the fifth year, amount to about €168236880, associated to new construction of about 100 million Euros. The reimbursement schedule, presented as table 7.2, is based on a repayment schedule that implies an annual outlay by the government of €12541951.

The main question is what welfare gains and losses can we identify that are associated to this project. The gains will certainly come from the initial additional expenditures that, we will assume, trickles through the economy according to its current structure. Once the roads are completed, one should also expect that travel costs will be lower. In our model this will imply that transport margins are lower. The costs will be associated to the reimbursement of the debt that is accumulated. Since it is assumed that the government will not be able to increase its debt stock, total payment will have to be made out of additional taxes from economic growth prompted from better and more economical roads, additional taxes on the citizens, or less transfers to the citizens, from the government. The scenarios considered are, therefore, based on:

(1) the implicit reduction in transport margins on the land transport sector;
(2) imposition of an additional income tax on the households;
(3) Reduction in the amount of transfers of the households.

7.3. Simulation Results

The model was calibrated using a SAM constructed for the Azorean economy based on 2001 data. It comprises 45 sectors with a time horizon of 35 years, from 2002-2036.

For the simulation, attention was focused only on the welfare impact as measured by equivalent variation, desegregated by household group. As mentioned above, three main simulations were run to assess the impact of an income tax increase, of a transfer reduction and of the fall in the transport margins. In addition to these, two other simulations considered the joint impact of the transport margins' fall and the recourse to taxes or and to expenditure reductions for financing the additional expenditures. All results are to be read as an impact relative to the base situation, the reference scenario, which would imply no action by the government. Effectively an attempt is being made to isolate the pure modelling and closure effects of an increased expenditure on road outlay arising from the project.

The results are largely driven by the closure used in the model. Given that the nature of economy of these islands is extremely sensitive to external support from the mainland, the closure adopted was that investments adapt to the savings and thus is a binding constraint. To prevent free lunch from the rest of the world and the government, the foreign transfers are fixed in real terms along with the transfers to the households from the regional government. Also, the simulations for the road project are operationalised as a transfer to the government by the firm and to the firm by the government. Thus the model behaves through a positive and negative shock to government revenues.

7.3.1. Decrease in transport Margins by 10%

The first simulation that was run involved a reduction on the transport margins by 10%. This assumes the hypothesis that the investment made transport in the Azorean economy that much more efficient. Other values could have been simulated.

Repayment of the debt is assumed to occur through less government expenditures, according to its current structure.

Table 7.4 shows the results obtained as of 2012. There is a negative impact for all household groups, for most of the period. Only in the final years does the welfare impact become positive. At that time the repayment burden becomes very small relative to the overall economy and current benefits exceed current outlays. All impacts during construction are positive.

Table 7.4.: Equivalent Variation: Decrease in transport Margins by 10%

t	q1	q2	q3	q4	q5	q6
2012	-0.2595	-0.5810	-0.9860	-1.6575	-2.2866	-3.8697
2013	-0.2550	-0.5687	-0.9656	-1.6200	-2.2380	-3.7770
2014	-0.2500	-0.5554	-0.9433	-1.5796	-2.1847	-3.6765
2015	-0.2444	-0.5411	-0.9191	-1.5362	-2.1267	-3.5681
2020	-0.2083	-0.4540	-0.7689	-1.2740	-1.7693	-2.9079
2025	-0.1498	-0.3213	-0.5390	-0.8855	-1.2362	-1.9600
2030	-0.0870	-0.1791	-0.2910	-0.4671	-0.6597	-0.9159
2034	-0.0290	-0.0481	-0.0618	-0.0806	-0.1268	0.0583
2035	-0.0095	-0.0047	0.0135	0.0452	0.0459	0.3648
2036	0.0071	0.0327	0.0790	0.1554	0.1979	0.6427

Source:computed

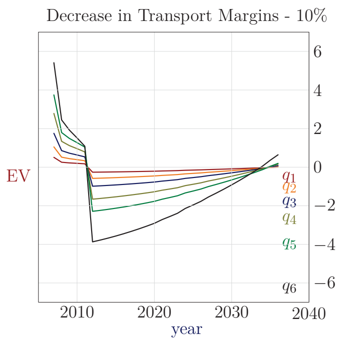

Fig. 7.1.: Equivalent Variation: Decrease in transport Margins by 10%

7.3.2. Increase in Income tax by 10%

A second simulation run involved a 10% income tax increase, as of 2012, in order to support the extra cost of reimbursing the debt to the private company that executed and financed the project.

The results are presented in table 7.5. From 2007 to 2011, the construction period, the welfare impact is positive for all years and for all household groups, as expected. As of 2012, the repayment of the debt is started and the impact is negative and stays negative for the full period.

Table 7.5.: Equivalent Variation: Increase in Income tax by 10%

t	q1	q2	q3	q4	q5	q6
2007	0.5094	1.0510	1.7552	2.7844	3.7454	5.4146
2008	0.2563	0.5188	0.8580	1.3401	1.7943	2.4493
2009	0.2224	0.4427	0.7262	1.1190	1.4921	1.9232
2010	0.1996	0.3897	0.6328	0.9590	1.2716	1.5146
2011	0.1756	0.3340	0.5348	0.7916	1.0405	1.0869
2012	-0.4994	-0.8632	-1.4951	-2.1643	-3.1404	-5.1761
2013	-0.5063	-0.8707	-1.5092	-2.1782	-3.1684	-5.2150
2014	-0.5132	-0.8781	-1.5233	-2.1919	-3.1956	-5.2531
2015	-0.5202	-0.8857	-1.5373	-2.2055	-3.2223	-5.2906
2020	-0.5567	-0.9258	-1.6106	-2.2758	-3.3564	-5.4829
2025	-0.5892	-0.9555	-1.6665	-2.3155	-3.4504	-5.6322
2030	-0.6397	-1.0194	-1.7813	-2.4456	-3.6713	-5.9932
2035	-0.7027	-1.1049	-1.9344	-2.6321	-3.9756	-6.5028
2036	-0.7179	-1.1270	-1.9738	-2.6828	-4.0556	-6.6372

Source: computed

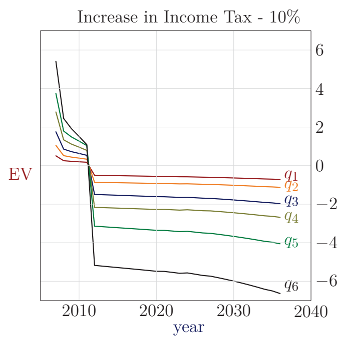

Fig. 7.2.: Equivalent Variation: Increase in Income tax by 10%

The increase in income tax adversely affects the households as it curtails the spending and leads to a substantial fall in the GDP as compared to other scenarios. The improvement of the current account balance captured by the excess of exports over imports is insufficient to compensate for the fall in GDP. The fall in output is accompanied with a rise in unemployment and falling wages. Thus an overall reduction in the incomes is largely responsible for the lower welfare.

The welfare losses are greater in the higher income household groups who bare a greater share of the taxes paid.

In the above scenarios we have separately simulated a fall in transport margins in addition to cut in taxes and transfers. However once the road has been completed, the benefits of a fall in transport margins will accrue. The government if so wishes then can choose to impose either additional taxes or cur transfers to meet its payment obligations. Thus the negative impacts of taxes and transfer cuts are over estimated. In the following simulations we combine the tax increase and transfer cuts with the accrued benefits of transport margins. As expected the welfare losses are lower when the implicit gains from road construction are accounted for but are not large enough to offset the additional payment burden.

If we simulate the tax increase, jointly with the fall in the transport margins, the results turn out slightly better as seen on table 7.6. The pattern is, however, still the same.

Table 7.6.: Equivalent Variation: Increase in Income tax by 10% and 10% Decrease in Transport Margins

t	q1	q2	q3	q4	q5	q6
2012	-0.4752	-0.8220	-1.4165	-2.0642	-3.0390	-5.0801
2013	-0.4813	-0.8275	-1.4280	-2.0730	-3.0596	-5.1051
2014	-0.4873	-0.8329	-1.4391	-2.0813	-3.0789	-5.1284
2015	-0.4933	-0.8382	-1.4500	-2.0891	-3.0972	-5.1505
2020	-0.5240	-0.8650	-1.5031	-2.1244	-3.1820	-5.2546
2025	-0.5486	-0.8774	-1.5311	-2.1175	-3.2122	-5.2925
2030	-0.5888	-0.9191	-1.6091	-2.1873	-3.3516	-5.5124
2035	-0.6387	-0.9767	-1.7149	-2.2968	-3.5525	-5.8431
2036	-0.6510	-0.9924	-1.7434	-2.3298	-3.6087	-5.9363

Source: computed

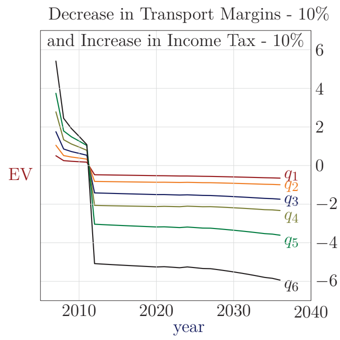

Fig. 7.3.: Equivalent Variation: Increase in Income tax by 10% and 10% Decrease in Transport Margins

7.3.3. Cut in Transfers by 10%

One other way of financing the cost of the road is through a reduction in government expenditures. We consider a cut in transfers to the private sector. Two simulations were run in this case: one without the impact of a cut in transport margins and one considering the cut.

There is no policy change in the initial years, until 2011, the welfare gains remain exactly the same as in table 7.5. Marginal adverse impact starts as the road payments begin. Since the government expenditures fall on account of the payments and that the households receive less is a contributor to the fall in equivalent variation. The cut in expenditure leads to a fall in the GDP and since the foreign transfers are fixed, only the current account changes. The model adjusts with an increase in exports as compared to imports thus improving the current account balance with respect to the US, ROW and the EU. This export lead growth however is insufficient to compensate for the falling GDP on account of reduced expenditure and transfers by the government. As the economy grows, the share of the fixed repayments in the GDP falls and the export lead growth contributes, albeit marginally to a positive welfare of the richer households. Also the repayment amount in the latter years is a smaller fraction of the government expenditure and thus growth contributes to alleviating the decrease in welfare.

Table 7.7.: Equivalent Variation: Cut in transfers by 10%

t	q1	q2	q3	q4	q5	q6
2012	-1.0572	-0.6237	-1.0670	-1.7617	-2.3952	-3.9746
2013	-1.0741	-0.6125	-1.0478	-1.7264	-2.3500	-3.8879
2014	-1.0910	-0.6004	-1.0266	-1.6883	-2.3001	-3.7932
2015	-1.1079	-0.5872	-1.0037	-1.6472	-2.2457	-3.6907
2020	-1.1940	-0.5064	-0.8611	-1.3978	-1.9066	-3.0588
2025	-1.2754	-0.3808	-0.6410	-1.0242	-1.3933	-2.1385
2030	-1.3725	-0.2466	-0.4049	-0.6230	-0.8384	-1.1223
2035	-1.4778	-0.0812	-0.1143	-0.1303	-0.1564	0.1297
2036	-1.5008	-0.0458	-0.0518	-0.0243	-0.0095	0.4017

Source: computed

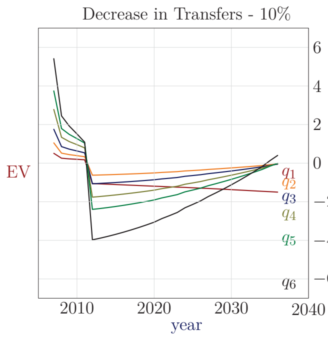

Fig. 7.4.: Equivalent Variation: Cut in transfers by 10%

The welfare impact is negative for the full period for most household groups. Only the higher household groups become better off toward the end of the period. Lower income groups bear a greater share of the burden since they are the main recipients of the transfers that were cut.

Running the simulation considering, simultaneously the change in transport margins and the fall in transfers produces the results presented in table 7.8. The pattern described before is maintained but equivalent variation becomes positive sooner, for higher income households, as expected. The impact is always negative for the lowest income group.

Table 7.8.: Equivalent Variation: Cut in transfers by 10% and 10% Decrease in Transport Margins

t	q1	q2	q3	q4	q5	q6
2012	-1.0331	-0.5825	-0.9884	-1.6616	-2.2938	-3.8789
2013	-1.0491	-0.5692	-0.9664	-1.6211	-2.2410	-3.7780
2014	-1.0651	-0.5550	-0.9423	-1.5775	-2.1832	-3.6685
2015	-1.0811	-0.5396	-0.9162	-1.5306	-2.1203	-3.5505
2020	-1.1613	-0.4454	-0.7533	-1.2460	-1.7316	-2.8297
2025	-1.2348	-0.3024	-0.5051	-0.8257	-1.1543	-1.7974
2030	-1.3216	-0.1461	-0.2322	-0.3640	-0.5177	-0.6395
2033	-1.3784	-0.0392	-0.0451	-0.0479	-0.0815	0.1606
2034	-1.3977	-0.0003	0.0229	0.0671	0.0772	0.4517
2035	-1.4139	0.0473	0.1057	0.2056	0.2676	0.7915
2036	-1.4339	0.0891	0.1790	0.3293	0.4382	1.1045

Source: computed

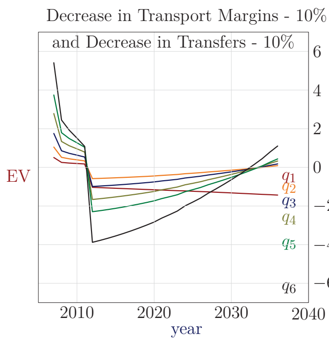

Fig. 7.5.: Equivalent Variation: Cut in transfers by 10% and 10% Decrease in Transport Margins

7.3.4. Scenario Comparison

In order to compare the various scenarios constructed we can look at the qualitative results or we can calculate the present value of the equivalent variation impact of each scenario. The positive impacts of the first five years are compared to the subsequent impact until the investment is reimbursed. Table 7.9 presents the net present value of the equivalent variation for each scenario. The total represents the residual value the project should have in order to make the net present value zero.

Table 7.9.: NPV of Equivalent Variation (million €)

	q1	q2	q3	q4	q5	q6	Total
Scenario 1	-0.5	-1.3	-2.3	-4.1	-6.0	-11.7	-25.88
Scenario 2	-3.8	-5.9	-10.4	-14.1	-21.6	-37.3	-92.94
Scenario 3	-9.1	-1.8	-3.1	-5.2	-7.2	-13.0	-39.30
Scenario 4	-3.4	-5.3	-9.4	-12.7	-19.9	-35.0	-85.60
Scenario 5	-8.8	-1.2	-2.1	-3.7	-5.5	-10.6	-31.94

Source: computed

The most favourable scenarios are those where the road is financed by a decrease in general government expenditures, followed by those in which it is financed by a reduction in transfers even though the lower income households might bear a relatively higher burden.

7.4. Conclusions

The current paper set out to measure the welfare impact of the construction of a road under a public/private partnership in the Azores. The issue facing the government was not to gage the willingness to pay for a new road but to create infrastructure. Hence individuals had no choice between a road with and without toll, whose pricing would have been based on marginal cost principles. The next question than was to address the issue of the cost of infrastructure as the burden of payment would be shifted across time. Since the road introduces distortions in the system, individuals use the new road based on their propensities to consume transport services. Since the model does not factor in externalities like reduced commuting time, lower fuel expenditure and emissions and less maintenance costs, the positive welfare impacts are understated. Also on the production side, the expenditure is specifically in the road sector and thus affects the return to capital only in the road sector. The model behaviour would suggest an increased return to capital in the construction sector on account of demand for road services and thus distort the results in favour of additional capital in the construction sector. In reality the foreign firms have already invested in capital and do not incur expense on capital expenditure. To neutralise this effect the model incorporates the additional expenditure on the road as a general increase in expenditure.

For this purpose a dynamic CGE model was used. The calibration of the model used a SAM matrix constructed with 2001 data with a considerable level of detail which was not fully reported in this exercise.

The main concern here was to analyse the welfare impact of the project as it impacted on the equivalent variation.

As expected, the project has an initial positive impact given that, during the first five years, there are no payments made to the private partner. The negative impact sets in after construction is completed and the investment has to be repaid. Account

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is taken of the fact that trade margins should be reduced because of improvement of land transportation.

Overall, it does not seem that the initial positive impacts on welfare compensate the subsequent welfare loss due to the payments that have to be made either through additional taxes or through reductions in transfers from the public budget to the private sector.

There is an unambiguous fall in welfare by the imposition of an additional income tax. The welfare impact is not the same for different household groups. Lower income households tend to bear a greater burden if the payment is done by reducing transfers as opposed to increasing taxes. In this case it is the higher income groups that pay more.

No reasonable scenario provides sufficient efficiency gains to justify the investment undertaken. Only if we admit certain residual values for the road can we make a case for its positive impact.

8

Impacts of Closure of a Military Base on a Small Island Open Economy

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Military bases are commonplace in many countries. Their economic impact on the neighbouring communities depends on their location and the level of integration of their activities on the local economies. Base closures or base activity reductions are also frequent as a consequence of military strategy alterations. The current paper seeks to analyse the economic impact of a US base located in the island of Terceira in the Azores. The base has been an important element of economic life in this island since the end of WWII. The changing geo-strategic map of the world has, along the second half of the twentieth century, led to changing roles of this base and consequent changes in the intensity of its activity. On the other hand discussions over the importance of the base for the local economy are recurrent in an attempt, on the part of the participants, to set forth arguments in favour or against the presence of military forces in the location. The current paper tries to contribute with a quantification of the economic impact of the base using a dynamic CGE model of the Azorean economy. A closure scenario is created and the impacts traced through various economic indicators including some sector detail. Estimates are made for the overall impact and for the impact on the island that houses the base.

Contents

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8.1. Introduction

Military bases are commonplace in many countries. Their location has important economic impacts on the neighbouring communities and depends on the level of integration of their activities with the local economies. Base closures or base activity reductions are also frequent as a consequence of military strategy alterations. The current paper seeks to analyse the economic impact of a US base located in the island of Terceira in the Azores. The base has been an important element of economic life on this island since the end of WWII. The activity of the base reflects the changing geo-political situation in the second half of the twentieth century.

The improvements in defence technology coupled with a new equilibrium in global power politics have impacted the American strategy and perhaps their perception of the Lajes air base. This is reflected in the strength of the military personnel on the base and their approach to integration in the local community. Housing problems in the past were solved via rentals of the local housing but in the recent times there has been a shift to housing in the air base, this reducing the dependence on the local economy.

During the 1980's, in addition to the direct impact of the base's activity, its presence justified aid given to the regional government and other given directly to the national authorities. The monetary compensation for the use of the base is immediately quantified. Not so obvious are the impacts on local economic activity that results from the presence of the base.

Discussions over the importance and the impact of the base for the local economy are recurrent in an attempt, on the part of the participants, to advance arguments in favour or against its presence. The current paper tries to contribute with a quantification of the economic impact of the base using a dynamic CGE model of the Azorean economy. A closure scenario is created and the impacts traced through various economic indicators including some sector detail. Estimates are made for the overall impact and for the impact on the island that houses the base.

In what follows section 2 reviews the body of the literature on base closures using CGE models. Section 3 presents the main variables that characterize the impact of the base on the local economy. Section 4 reviews the main characteristics of a CGE model of the Azores. Section 5 reviews the results of calibration of the model and the results of the closure scenario developed. Section 6 presents some of the main conclusions that can be drawn from application of the model.

8.2. Analysing Base Closures with CGE Models

[Hoffmann *et al.* (1996)] analyse the impact of defence cuts on the economy in California using a computable general equilibrium (CGE) model. Their focus is on the migration of factors from California to other states and the impact of this migration on the economy. CGE models are better suited to analyse the economy wide impact of these defense cuts and their study shows that the impacts are highly sensitive to the assumption of inter-state mobility.

The current research paper can be classified as a cut in defence expenditure and is more focussed on base closure on an island economy where the economic conditions are not very conducive to factor mobility, especially labour migration.

8.3. The Military Base in Terceira/Azores

The base in Terceira/Azores houses both US and Portuguese military activities. It comprises an airport adequate for landing any known type of aircraft, fuel storage tanks and port facilities. This base has been extensively used in various international conflicts, namely those that have occurred in the last half century and in the Middle East during recent times.

The impact of the American component of the base can be simulated by the model using data on the main variables. In the simulation undertaken here the relevant data collected characterizes expenditures on construction works and repair, employment and private consumption by the US military, servicemen and civilians.

Access of locals to purchases in the base's stores can also be taken into consideration. It is common for some locals to make their purchases in the base stores at prices that are lower than those practiced in the local stores, for a wider variety of products. There are no good estimates for the total value of the purchases made in these stores, which is equivalent to purchasing the goods abroad. Given that there are no good estimates of the values involved, two scenarios will be created to test the impact of these "imports": one where the import effect is zero, the reference scenario and one in which 50% of the income is spent on these "foreign" stores.

The main elements of the date on the activity of the US military are summarized in tables 8.1 and 8.2. Table 8.1 provides an estimate of the value (in US Dollars) of the construction works and repair commissioned by the Lajes Field Base for 2004 and for 2005.

Table 8.1.: Construction works and repair commissioned by the Lajes Field base US$

Projects	2004	2005
Repair breakwater	14,400,000	7,000,000
Construct housing, phase 3	13,392,000	0
Add/renovate fitness center	4,086,000	5,689,000
Community Improvements	3,865,644	7,644,000
Airfield improvements	407,592	150,000
Housing improvements	833,241	663,550
Fuel System improvements	556,046	4,010,000
DoDDS improvements	568,117	615,000
TOTAL	38,110,644	25,771,550

Source: U.S. Air force

An evaluation of the local consumption expenditure by the US base staff in Azores is given in Table 8.2. To estimate the local impact of the Lajes Field Base two scenarios are created. On the first scenario it is assumed that 30% of the payroll of active duty personnel living on base is spent outside the base. For the active duty personnel living off base this figure is estimated at 50 per cent, and for US civilians living it is assumed at 55%. For the Portuguese civilians working on base, two scenarios are considered: one in which they spend 100 per cent of their income off base and one in which they spend only 50 per cent off base. This second scenario tries to take into account the mentioned fact that many civilians purchase goods in the base stores supplied by the US military.

Table 8.2.: Annual payroll and estimates regarding the loss in terms of private consumption

	Annual Payroll		Impact Factor	Local Impact	
	2004	2005		2004	2005
Reference Scenario	57,509,059	61.247.015		34,710,940	37,311,796
Active duty on base	19,814.147	19,287.261	0.30	5,944.244	5,786.178
Active duty off base	13,209.431	12,858.174	0.50	6,604.716	6,429.087
US civilians	5,163.335	8,900.109	0.55	2,839.834	4,895.060
Portuguese civilians	19,322.146	20,201.471	1.00	19,322.146	20,201.471

Source: U.S. Air force

The closure of the US component of the Lajes Field base would have direct and indirect impacts on the economy of the Azores through the following four channels:

(1) The reduction in the demand for construction works and repair;
(2) The employment loss of the Portuguese civilians working on the base, which leads to a loss in the labour income and consumption demand both domestic and foreign, namely the demand of goods from the base's stores;
(3) The loss in the consumption demand from the US active duty personnel living on base and off base;
(4) The loss of the rents of local lodging contracted quarters. The difference of financial impact between the two scenarios is about US$10 million, a variation of about 28%.

8.4. Simulation of various base closure policies

8.4.1. Main results of the policy measure

This simulation exercise aims at evaluating the economic impacts of the Lajes Field base removal from Azores.

The setup of the policy scenario relies on the evaluation in terms of construction works and repair, employment and private consumption, provided by the U.S. Air Force. The main assumptions are summarized in tables 8.3 and 8.4. Table 8.3

provides an estimation of the losses in terms of construction works and repair caused by the removal of the Lajes Field base, expressed in US $.

Table 8.3.: Construction works and repair commissioned by the Lajes Field base

Major FY04 Projects and US$ value	US$
Repair breakwater	14,400,000
Construct Nascer do Sol, phase 3	13,392,000
Add/renovate fitness center, phase 1	4,086,000
Community improvements	3,865,644
Airfield improvements	407,592
Housing improvements	833,241
Fuel system improvements	556,046
DoDDS improvements	568,117
TOTAL	38,108,640

Source: U.S. Air force

An evaluation of the loss in terms of private consumption, expressed in US $, due to the removal of the NATO base is given in table 8.4. To estimate the local impact on private consumption it has been assumed that 30 % of the consumption of active duty personnel living on base originates from the domestic economy. For the active duty personnel living off base the share of private consumption originating from the domestic economy is 50 %, while for the US civilians 55 per cent and for the Portuguese civilians 100 %.

Table 8.4.: Estimates regarding the loss in terms of private consumption

Base Personnel	Annual Payroll ($)	Local Impact*($)
Active duty on base	19,814,147	5,944,244
Active duty off base	13,209,431	6,604,716
US Civilians	5,163,335	2,839,834
Portuguese Civilians	19,322,146	19,322,146
Total impact of annual payroll	57,509,059	34,710,940
Local lodging contracted quarters		1503

Source: U.S. Air force
Note: *Estimated pay spent in local economy: 30% of active-duty pay living on base; 50% of active-duty pay, living off base; 55% civilian pay.

The facility replacement value ($1,075,649,430) has not been taking into account in this policy scenario.

The closing of the Lajes Field base will have direct and indirect impacts on the Azores economy through the following four channels:

(1) The reduction in the demand for construction works and repair;
(2) The employment loss of Portuguese civilians working on the base, which leads to a loss in factor income;

(3) The loss in demand from the US active duty personnel living on base and off base;
(4) The loss of rents of local lodging contracted quarters.

The decline in the demand for construction works and repair by the Lajes Field base leads to a decline in investment demand which further affects the production of construction work sector (see table 10). Both profitability and employment in the sector reduce (see tables 11 and 19).

The loss of employment by Portuguese civilians generates a fall in labour income for the Portuguese households. Thus, consumption demand for all products declines (see table 7). Consequently, production of most sectors goes down leading to a downwards adjustment in employment by the sectors.

All these effects are strengthened by the fact that demand from US active duty personnel living on base and off base as well as the US civilians falls. As a consequence, consumption demand for commodities drops by about 2 per cent (see table 7 in the Appendix).

The domestic currency depreciates to maintain the fixed trade deficit thus giving a boost to the external competitiveness by increasing exports to both Mainland and Rest of the World. Therefore, the negative impact induced by the private demand on the production and employment in the agriculture, hunting and forestry sector (sec1), fish (sec2), manufacture products (sec4), transport and communication services (sec9) and financial intermediation services (sec10) is reversed (see tables 10-11). Furthermore, imports from both Mainland and Rest of the World decline due to the relative increase of world prices of imports compared with the domestic prices and the drop in domestic sales.

The fall in domestic and foreign savings, i.e. supply of loanable funds, generates a reduction in the demand for investment goods (see table 16).

At the macro level, GDP drops by 0.89 %, due to the retrenchment of the private and investment demand. Furthermore, the negative impact on employment accounts for about 0.1 percentage points (see table 8.5).

Table 8.5.: Macroeconomic effects (% changes compared with the baseline)

Macroeconomic Variables	Value
GDP (% change)	-0.89
Unemployment rate (%)	4.09
Change in Unemployment rate (% points)	1.16
Welfare gains/losses (000s €)	-27919
Welfare gains/losses (% of households income)	-2.13

The measure generates a loss in households' welfare of about 28 million €, which is equivalent to 2.13 % of households' income. The detailed sector results on output and prices are depicted in the Appendix.

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8.5. Conclusions

The closure of the Lajes air base does adversely affect the economy. The mechanism is largely driven through the exogenous cut in American expenditure affecting the demand for services, construction and rental housing, besides increase in unemployment on account of job losses. The job loss leads to a fall in income and demand for both domestic and imported commodities forcing the producers to export more to the mainland and ROW.

Appendix

Detailed sector results of the policy measure

Table 8.6.: Changes in private consumption compared with the baseline (%)

Cdty	Sector Name and Description	% change
1	Agriculture, Hunting & Forestry, Logging	-1.95
2	Fishing	-1.88
3	Mining and Quarrying	-2.62
4	Manufactured products	-2.68
5	Electrical energy, gas, steam and hot water	-1.51
6	Construction work	-2.05
7	Wholesale and retail trade services; repair services of motor vehicles, motorcycles and personal and household goods	-2.03
8	Hotel and restaurant services	-2.71
9	Transport, storage and communication services	-3.00
10	Financial intermediation services	-2.73
11	Real Estate, renting and business services	-2.66
12	Public administration and defence services, compulsory social security services	-2.36
13	Education services	-2.37
14	Health and social services	-2.32
15	Other community, social and personal services	-2.53
16	Private household with employed persons	-2.58

Table 8.7.: Changes in government purchase of goods and services w.r.t. baseline (%)

Cdty	Sector Name and Description	% change
4	Manufactured products	-2.12
5	Electrical energy, gas, steam and hot water	-1.49
9	Transport, storage and communication services	-2.04
12	Public administration and defence services, compulsory social security services	-1.25
13	Education services	-1.31
14	Health and social services	-1.12
15	Other community, social and personal services	-1.35

Table 8.8.: Changes in domestic sales by sector compared with the baseline (%)

Cdty	Sector Name and Description	% change
1	Agriculture, Hunting & Forestry, Logging	-0.56
2	Fishing	-1.34
3	Mining and Quarrying	-0.84
4	Manufactured products	-1.97
5	Electrical energy, gas, steam and hot water	-1.04
6	Construction work	-1.04
7	Wholesale and retail trade services; repair services of motor vehicles, motorcycles and personal and household goods	-2.15
8	Hotel and restaurant services	-1.97
9	Transport, storage and communication services	-1.07
10	Financial intermediation services	-0.82
11	Real Estate, renting and business services	-1.53
12	Public administration and defence services, compulsory social security services	-1.27
13	Education services	-1.35
14	Health and social services	-1.57
15	Other community, social and personal services	-1.95
16	Private household with employed persons	-2.58

Table 8.9.: Changes in sectoral gross output compared with the baseline (%)

Cdty	Sector Name and Description	% change
1	Agriculture, Hunting & Forestry, Logging	0.07
2	Fishing	0.42
3	Mining and Quarrying	-0.11
4	Manufactured products	0.47
5	Electrical energy, gas, steam and hot water	-1.03
6	Construction work	-2.15
7	Wholesale and retail trade services; repair services of motor vehicles, motorcycles and personal and household goods	-1.18
8	Hotel and restaurant services	-1.41
9	Transport, storage and communication services	0.62
10	Financial intermediation services	0.11
11	Real Estate, renting and business services	-0.02
12	Public administration and defence services, compulsory social security services	-1.27
13	Education services	-1.35
14	Health and social services	-1.57
15	Other community, social and personal services	-0.99
16	Private household with employed persons	-2.58

Table 8.10.: Changes in employment by sector compared with the baseline (%)

Cdty	Sector Name and Description	% change
1	Agriculture, Hunting & Forestry, Logging	0.78
2	Fishing	0.96
3	Mining and Quarrying	-0.22
4	Manufactured products	0.70
5	Electrical energy, gas, steam and hot water	-1.18
6	Construction work	-3.09
7	Wholesale and retail trade services; repair services of motor vehicles, motorcycles and personal and household goods	-2.39
8	Hotel and restaurant services	-1.54
9	Transport, storage and communication services	1.25
10	Financial intermediation services	0.16
11	Real Estate, renting and business services	-0.10
12	Public administration and defence services, compulsory social security services	-1.40
13	Education services	-1.45
14	Health and social services	-1.88
15	Other community, social and personal services	-1.04
16	Private household with employed persons	-2.58

Table 8.11.: Changes in exports to Mainland compared with the baseline (%)

Cdty	Sector Name and Description	% change
1	Agriculture, Hunting & Forestry, Logging	0.73
2	Fishing	1.81
3	Mining and Quarrying	5.60
4	Manufactured products	1.65
8	Hotel and restaurant services	1.10
9	Transport, storage and communication services	1.63
11	Real Estate, renting and business services	2.96
15	Other community, social and personal services	1.88

Table 8.12.: Changes in exports to ROW compared with the baseline (%)

Cdty	Sector Name and Description	% change
1	Agriculture, Hunting & Forestry, Logging	0.73
2	Fishing	1.81
4	Manufactured products	1.65
5	Electrical energy, gas, steam and hot water	3.41
7	Wholesale and retail trade services; repair services of motor vehicles, motorcycles and personal and household goods	6.74
8	Hotel and restaurant services	1.10
9	Transport, storage and communication services	1.63
10	Financial intermediation services	2.74
11	Real Estate, renting and business services	2.96
15	Other community, social and personal services	1.88

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Table 8.13.: Changes in imports from Mainland compared with the baseline (%)

Cdty	Sector Name and Description	% change
1	Agriculture, Hunting & Forestry, Logging	-0.64
3	Mining and Quarrying	-4.88
4	Manufactured products	-2.46
8	Hotel and restaurant services	-5.01
9	Transport, storage and communication services	-3.08
10	Financial intermediation services	-3.06
11	Real Estate, renting and business services	-5.40

Table 8.14.: Changes in imports from ROW compared with the baseline (%)

Cdty	Sector Name and Description	% change
1	Agriculture, Hunting & Forestry, Logging	-0.64
2	Fishing	-1.71
3	Mining and Quarrying	-4.88
4	Manufactured products	-2.46
5	Electrical energy, gas, steam and hot water	-3.33
7	Wholesale and retail trade services; repair services of motor vehicles, motorcycles and personal and household goods	-11.15
8	Hotel and restaurant services	-5.01
9	Transport, storage and communication services	-3.08
10	Financial intermediation services	-3.06
11	Real Estate, renting and business services	-5.40
15	Other community, social and personal services	-6.58

Table 8.15.: Changes in demand for investment commodities w.r.t. baseline (%)

Cdty	Sector Name and Description	% change
1	Agriculture, Hunting & Forestry, Logging	-4.86
4	Manufactured products	-4.33
6	Construction work	-2.31
7	Wholesale and retail trade services; repair services of motor vehicles, motorcycles and personal and household goods	-1.73
11	Real Estate, renting and business services	-4.02
15	Other community, social and personal services	-3.58

Table 8.16.: Changes in commodities prices net of taxes w.r.t. the baseline (%)

Cdty	Sector Name and Description	% change
1	Agriculture, Hunting & Forestry, Logging	1.58
2	Fishing	1.21
3	Mining and Quarrying	0.07
4	Manufactured products	1.01
5	Electrical energy, gas, steam and hot water	0.37
6	Construction work	-1.08
7	Wholesale and retail trade services; repair services of motor vehicles, motorcycles and personal and household goods	-1.66
8	Hotel and restaurant services	0.79
9	Transport, storage and communication services	0.93
10	Financial intermediation services	0.96
11	Real Estate, renting and business services	0.69
12	Public administration and defence services, compulsory social security services	0.13
13	Education services	0.18
14	Health and social services	-0.01
15	Other community, social and personal services	0.22
16	Private household with employed persons	0.39

Table 8.17.: Changes in price of domestic output compared with the baseline (%)

Cdty	Sector Name and Description	% change
1	Agriculture, Hunting & Forestry, Logging	1.92
2	Fishing	1.97
3	Mining and Quarrying	0.26
4	Manufactured products	1.13
5	Electrical energy, gas, steam and hot water	0.37
6	Construction work	-1.08
7	Wholesale and retail trade services; repair services of motor vehicles, motorcycles and personal and household goods	-1.66
8	Hotel and restaurant services	0.69
9	Transport, storage and communication services	1.27
10	Financial intermediation services	0.66
11	Real Estate, renting and business services	0.29
12	Public administration and defence services, compulsory social security services	0.13
13	Education services	0.18
14	Health and social services	-0.01
15	Other community, social and personal services	-0.53
16	Private household with employed persons	0.39

Table 8.18.: Rental rate of capital services compared with the baseline (%)

Cdty	Sector Name and Description	% change
1	Agriculture, Hunting & Forestry, Logging	3.47
2	Fishing	4.01
3	Mining and Quarrying	-0.14
4	Manufactured products	4.27
5	Electrical energy, gas, steam and hot water	-6.76
6	Construction work	-12.95
7	Wholesale and retail trade services; repair services of motor vehicles, motorcycles and personal and household goods	-9.55
8	Hotel and restaurant services	-5.69
9	Transport, storage and communication services	5.17
10	Financial intermediation services	1.38
11	Real Estate, renting and business services	0.14
12	Public administration and defence services, compulsory social security services	-8.13
13	Education services	-10.81
14	Health and social services	-10.16
15	Other community, social and personal services	-5.90

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9

Measuring the Impacts of Personal and Corporate Income Tax Cuts on a Small Island Open Economy

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In 1999, subsequent to a legislative review, the authorities of the Azores, an autonomous region of Portugal, decided to reduce income tax rates applicable locally by 30% in the case of corporate income and by 20% in the case of personal income. There was no debt or transfer compensation for this tax reduction, meaning that the regional budget was reduced by the equivalent amount of the tax reduction. The current paper analyses the impact of such a shock on various macro and micro variables pertaining to the Azorean economy, including social welfare, using a dynamic CGE model comprising forty five sectors, six household groups, three government levels and four trading partners. It is concluded that the short run impact on GDP is, as expected, negative, given that the marginal propensity to save of the private sector is positive and there was no compensating policy. There is an initial increase in unemployment due to the cut in government expenditures. In the long run, however, the impact becomes positive due to increased investment and private consumption. The stronger effect comes from the reduction in personal income taxes, a much greater proportion of all taxes collected in the region. Real wages net of personal income taxes rise as does the labour supply. The impact of the policy is shown to be positive for all household income groups, as evaluated through equivalent variation. The lowest income group ends up benefiting the most, in relative terms.

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9.1. Introduction

In 1999, subsequent to a legislative review, the authorities of the Azores, an autonomous region of Portugal, decided to reduce income tax rates applicable locally by 30% in the case of corporate income and by 20% in the case of personal income. Part of the rationale for the tax reduction was that, being an outermost region, far from the continent, with lagging economic development and with higher costs of living, it would be fair to reduce the tax burden on firms and on families. Prior to this, since 1986, the value added tax applicable in this region was already reduced by 30% relative to the national rates. In essence, the local authorities, under cover of legislation that allowed for the tax rate adaptation, lowered the tax rates pegging them to the national rates. The 1999 adaptation of the tax law occurred with the approval of a new regime of intergovernmental transfers whereby the local government kept all the tax revenues that were generated by economic activity undertaken in the region plus transfers to a cohesion fund and additional transfers arising from national solidarity, based on a pre determined formula. For its financing, the government could also resort to debt, a prerogative that was later suspended when Portugal approached the upper deficit limit established by the stability and growth pact. Under the established tax regime, any tax reduction undertaken by the regional authorities had no compensation in other transfers from the central or other levels of government. As such, a tax reduction meant a transfer of financial resources from the government budget to firms and to families. The current paper analyses the impact of such a shock on various macro and micro variables pertaining to the Azorean economy, including GDP, employment, social welfare and household income group distribution, using a CGE model comprising forty five sectors, six household groups and four trading partners. In what follows, in section two we proceed to characterize the tax and transfer system that applies in the Azores and the changes that were introduced with the 1999 tax reduction bills. In section three we present the main features of a CGE model of the Azorean economy and the expected impacts of a corporate and personal income tax reduction. Section four we analyses the results of the tax reduction package on various relevant variables. Finally, section five presents some concluding remarks and suggestions.

9.2. The Azorean Tax and Transfers Systems

The Azores, like Madeira, is an autonomous region of Portugal. The statute of autonomy was established in 1976 creating local authorities, including a regional assembly and a regional government, with extensive powers over the application of its own financial resources coming, mainly, from taxes, transfers and debt.

Until 1998, with the publication of a clarifying law, the regional government had no legal basis to adapt the national tax system to its own policy preferences. As such the tax system and the tax rates applicable in the Azores were those applicable in the rest of the country. The only exception was the rates of VAT which were 30% lower since 1986, by deliberation of the national authorities. In this casa, even though the tax rates on VAT were lower and even though the economic base of the Azores was considerably weaker than that of Portugal, VAT revenues were attributed to this region on the basis of the nation per capita VAT revenue. This implies, of course, that registered VAT revenues were in fact a combination of two components: one that reflected the effectively generated tax on the basis of the transactions undertaken in this economy and a subsidy component, given that the national economy had a stronger average tax base and paid according to higher rates.

As of 1998, the regional authorities were empowered to either increase existing corporate and personal income taxes by a maximum of 10% or reduce them by a maximum of 30%. They were also empowered to create other taxes they considered necessary.

In 1999, the regional authorities deliberated a corporate and personal income tax reduction of, respectively,30% and 20%, the latter one in two steps, 15% in 1999 and 20% in 2000.

The revenue formula for the regional budget is described in the following paragraphs.

The main revenue sources of the regional budget, previous to the tax change can be represented by the following expression

$$R_i^* = \sum_{j=1}^{J} t_j^* B_j^i + T_i(1+\eta_z) + TROW \tag{9.1}$$

	Description
R_i^*	represents total normal revenues of region i, where i can be the Azores,
t_j^*	is the national tax rate for each tax base j
B_j^i	is the tax base j, in region i
T_i	are transfers to region i, established by a predetermined formula
z	is the rate factor that multiplies by the basic transfers to determine the additional transfers for investment (national cohesion funds)
$TROW$	are transfers from the rest of the world, mainly EU funds

Two restrictions apply to the above formula, one establishing a lower bound for transfers and another establishing a lower bound for VAT revenues.

The restriction on transfers safeguards that nominal transfers in any year is at least equal to the transfers of the previous year adjusted for the growth of current expenditures of the national budget.

$$T_i \geq (1+\gamma)T_{i,(t-1)} \tag{9.2}$$

	Description
γ	is the growth rate of current expenditures in the national budget

The restriction on VAT contemplates the fact that the revenue should be, at minimum, according to the national per capita values.

$$t^i_{VAT}B^i_{VAT} + Y \geq \frac{P_r}{P_n}VAT_{national} \tag{9.3}$$

	Description
t^i_{VAT}	is the regional vat rate,
B^i_{VAT}	is the regional VAT base
Y	is the implicit transfer
P_r	is the regional population
P_n	is the national population
$VAT_{national}$	is the national VAT revenue

With the tax reduction the first term of the revenue expression becomes

$$\sum_{j}^{J}(t^j_i - t^*_j)B^j_i \tag{9.4}$$

for $j \neq VAT$,

	Description
t^j_i	is the regional tax rate
t^*_j	is the national tax rate

Revenues are therefore given by the following expression

$$R^*_i = \sum_{j=1}^{J} t^*_j B^i_j - \sum_{j=1}^{J}(t^j_i - t^*_j)B^i_j + Y + T_i(1+\eta_z) + TROW \tag{9.5}$$

Given this expression, a tax reduction has no interference with other tax revenue sources or with other transfer sources or, for that matter, with any debt financing criteria. It becomes a simple transfer of resources from the government to the public.

9.3. Calibration of the Model and Simulation of Tax Changes

The model was calibrated using a SAM matrix constructed for the year 2001 for the Azorean economy.

The scenario created, based on the policies effectively implemented, presumed a corporate income tax cut of 30% and a personal income tax cut of 20%. The simulation was initiated in 2002 and impacts traced up to 2013. The main results, representing percentage changes relative to the base results, are presented in the following table. As expected, in the short run, there is a negative impact on GDP. Because the marginal propensity to save is positive, not all the extra money left in private hands is channelled to expenditures. The negative impact, however, tapers off and by the year 2012 becomes positive. Private consumption increases steadily while public consumption decreases. Gross fixed investment increases as does private GDP. In the end, the economic outcome tends to recover the short

Table 9.1.: Impacts of a 30% drop in CIT and a 20% drop in PIT

% change to the BAU	2002	2003	2004	2005	2006	2007
GDP	-0.23	-0.21	-0.18	-0.16	-0.14	-0.12
Private consumption	0.37	0.39	0.42	0.45	0.47	0.50
Government consumption	-1.65	-1.67	-1.69	-1.71	-1.73	-1.75
Gross fixed investment	1.10	1.19	1.27	1.36	1.44	1.53
Foreign balance	0.76	0.81	0.85	0.90	0.95	0.99
Exports	-0.01	0.02	0.04	0.07	0.10	0.12
Imports	0.47	0.51	0.55	0.59	0.63	0.67
Private GDP	0.54	0.58	0.63	0.67	0.72	0.76
% change to the BAU	2008	2009	2010	2011	2012	2013
GDP	-0.09	-0.07	-0.04	-0.02	0.01	0.03
Private consumption	0.52	0.55	0.57	0.60	0.63	0.65
Government consumption	-1.77	-1.78	-1.80	-1.82	-1.83	-1.85
Gross fixed investment	1.61	1.69	1.78	1.86	1.94	2.03
Foreign balance	1.04	1.09	1.13	1.18	1.22	1.27
Exports	0.15	0.18	0.21	0.24	0.27	0.31
Imports	0.71	0.74	0.78	0.82	0.86	0.90
Private GDP	0.81	0.85	0.90	0.95	1.00	1.04

term losses in GDP with gains in the private component of the economy and when compared to the public component. Table 9.1 presents the aggregate results of the exercise.

A scenario was created to isolate each of the two taxes. Table 9.2 reports the results of the 20% personal income tax reduction. As it turns out, the reduction of this tax has the bigger effect in GDP. In fact, more than 70% of the impact on GDP comes from this component and it is the driving effect on the turn of the variation on GDP. While the negative impact of the corporate income tax reductions lingers for the full period, the impact of the personal income tax becomes positive as of 2011.

These results are in line with what would be expected since corporate income taxes represent a small percentage of personal income taxes. The tax reduction policy implemented in the Azores in the early years of the XXI century led, according to the model specified, and assuming nothing else changed, to a short term reduction in GDP. In the long run, however, the tendency is for a recovery in the growth of this variable. The private sector grew relative to the public sector.

To assess the redistributive impact of the policy we can look at what it implied for the different household categories considered. Overall, real wages before tax decreased due to a decrease in employment. Real average wages net of taxes, however, increased as did the real average return to capital. Table 9.3 shows the results.

At a more disaggregated level, we find that, for all household groups, there is a negative impact on gross income due to the fact that unemployment increased (Table 9.4). The measure of final welfare, the equivalent variation, comes out positive for all income groups with higher relative gains registered for the lower income

Table 9.2.: Impacts of a 20% drop in PIT

% change to the BAU	2002	2003	2004	2005	2006	2007
GDP	-0.17	-0.15	-0.13	-0.11	-0.10	-0.08
Private consumption	0.33	0.35	0.37	0.39	0.41	0.43
Government consumption	-1.30	-1.31	-1.33	-1.34	-1.36	-1.37
Gross fixed investment	0.85	0.93	1.00	1.07	1.14	1.21
Foreign balance	0.62	0.67	0.71	0.75	0.79	0.84
Exports	-0.01	0.01	0.02	0.03	0.05	0.06
Imports	0.38	0.42	0.45	0.48	0.51	0.54
Private GDP	0.44	0.48	0.51	0.55	0.58	0.62
% change to the BAU	2008	2009	2010	2011	2012	2013
GDP	-0.06	-0.04	-0.02	0.00	0.02	0.04
Private consumption	0.46	0.48	0.50	0.52	0.54	0.57
Government consumption	-1.38	-1.40	-1.41	-1.42	-1.43	-1.44
Gross fixed investment	1.28	1.35	1.42	1.50	1.57	1.64
Foreign balance	0.88	0.92	0.96	1.00	1.04	1.08
Exports	0.08	0.10	0.12	0.14	0.16	0.18
Imports	0.58	0.61	0.64	0.68	0.71	0.74
Private GDP	0.66	0.69	0.73	0.77	0.81	0.84

Table 9.3.: Impacts of a Cut in PIT (20%) and CIT (30%) on Wages and Returns to Capital

Effects on Real Wage and RTC	2002	2003	2004	2005	2006	2007
Real average wage (Before tax)	-0.33	-0.30	-0.26	-0.23	-0.20	-0.16
Real average wage net of PIT	0.38	0.41	0.45	0.48	0.51	0.55
Real average return to capital	0.91	0.86	0.81	0.76	0.71	0.66
Effects on Real Wage and RTC	2008	2009	2010	2011	2012	2013
Real average wage (Before tax)	-0.13	-0.09	-0.06	-0.03	0.01	0.04
Real average wage net of PIT	0.58	0.62	0.65	0.69	0.72	0.76
Real average return to capital	0.61	0.56	0.50	0.44	0.38	0.32

groups, a result that is desired but was uncertain given that the tax reduction did not change the progressivity of the tax system (Table 9.5).

The results for equivalent variation are consistent with the registered increases in real consumption (Table 9.6). In fact, the greater increases were found in the lower income groups. Only the fourth group presents a diversion from an otherwise clear pattern.

Table 9.4.: Tax Cut Impact on Total Household Income before Taxes

HH	HHgrp	2002	2003	2004	2005	2006	2007
HH1	q1	-0.12	-0.10	-0.09	-0.07	-0.05	-0.03
HH2	q2	-0.24	-0.21	-0.19	-0.17	-0.15	-0.12
HH3	q3	-0.21	-0.19	-0.17	-0.15	-0.13	-0.10
HH4	q4	-0.28	-0.25	-0.23	-0.20	-0.18	-0.16
HH5	q5	-0.22	-0.20	-0.18	-0.15	-0.13	-0.11
HH6	q6	-0.14	-0.12	-0.10	-0.08	-0.06	-0.04
HH	HHgrp	2008	2009	2010	2011	2012	2013
HH1	q1	-0.02	0.00	0.02	0.04	0.06	0.07
HH2	q2	-0.10	-0.08	-0.06	-0.03	-0.01	0.01
HH3	q3	-0.08	-0.06	-0.04	-0.02	0.01	0.03
HH4	q4	-0.13	-0.11	-0.09	-0.06	-0.04	-0.01
HH5	q5	-0.09	-0.07	-0.04	-0.02	0.00	0.02
HH6	q6	-0.02	0.00	0.02	0.04	0.06	0.08

Table 9.5.: Tax Cut Impact on Total Equivalent Variation in Income

EV*	HHgrp	2002	2003	2004	2005	2006	2007
HH1	q1	0.61	0.63	0.64	0.65	0.67	0.68
HH2	q2	0.42	0.44	0.46	0.48	0.50	0.52
HH3	q3	0.43	0.45	0.47	0.49	0.51	0.53
HH4	q4	0.25	0.28	0.30	0.32	0.34	0.37
HH5	q5	0.31	0.33	0.35	0.38	0.40	0.42
HH6	q6	0.23	0.25	0.27	0.29	0.31	0.33
* EV in income %(in % of hh income)							
EV*	HHgrp	2008	2009	2010	2011	2012	2013
HH1	q1	0.70	0.71	0.73	0.74	0.76	0.77
HH2	q2	0.54	0.56	0.58	0.60	0.62	0.64
HH3	q3	0.55	0.57	0.59	0.61	0.64	0.66
HH4	q4	0.39	0.41	0.44	0.46	0.48	0.51
HH5	q5	0.44	0.46	0.48	0.50	0.53	0.55
HH6	q6	0.35	0.37	0.39	0.41	0.43	0.45

* EV in income %(in % of hh income)

Table 9.6.: Tax Cut Impact on Household Real Consumption

HH	HHgrp	2002	2003	2004	2005	2006	2007
HH1	q1	0.59	0.61	0.63	0.65	0.66	0.68
HH2	q2	0.41	0.43	0.45	0.48	0.50	0.52
HH3	q3	0.44	0.47	0.49	0.51	0.54	0.56
HH4	q4	0.28	0.30	0.33	0.35	0.38	0.41
HH5	q5	0.37	0.39	0.42	0.44	0.47	0.49
HH6	q6	0.35	0.38	0.41	0.44	0.46	0.49
HH	HHgrp	2008	2009	2010	2011	2012	2013
HH1	q1	0.70	0.72	0.74	0.76	0.77	0.79
HH2	q2	0.55	0.57	0.59	0.62	0.64	0.66
HH3	q3	0.58	0.60	0.63	0.65	0.67	0.70
HH4	q4	0.43	0.46	0.48	0.51	0.54	0.56
HH5	q5	0.52	0.54	0.57	0.60	0.62	0.65
HH6	q6	0.52	0.55	0.58	0.61	0.64	0.67

Table 9.7.: Propensity to save (μ) of each household group

μ	HHgrp	2002	2003	2004	2005	2006	2007
a	q1	0.71	0.71	0.71	0.71	0.71	0.71
b	q1	0.00	0.00	0.00	0.00	0.00	0.00
a	q2	1.85	1.85	1.85	1.85	1.85	1.85
b	q2	0.01	0.01	0.01	0.01	0.01	0.01
a	q3	3.86	3.86	3.86	3.86	3.86	3.86
b	q3	0.02	0.02	0.02	0.02	0.02	0.02
a	q4	7.81	7.81	7.81	7.81	7.81	7.80
b	q4	0.05	0.05	0.04	0.04	0.04	0.04
a	q5	12.07	12.07	12.06	12.06	12.06	12.06
b	q5	0.08	0.07	0.07	0.07	0.07	0.06
a	q6	29.54	29.53	29.53	29.52	29.52	29.51
b	q6	0.20	0.19	0.19	0.18	0.17	0.17
μ	HHgrp	2008	2009	2010	2011	2012	2013
a	q1	0.71	0.71	0.71	0.71	0.71	0.71
b	q1	0.00	0.00	0.00	0.00	0.00	0.00
a	q2	1.85	1.85	1.85	1.85	1.85	1.85
b	q2	0.01	0.01	0.01	0.01	0.01	0.01
a	q3	3.86	3.86	3.86	3.86	3.86	3.85
b	q3	0.02	0.02	0.02	0.02	0.02	0.02
a	q4	7.80	7.80	7.80	7.80	7.80	7.79
b	q4	0.04	0.04	0.03	0.03	0.03	0.03
a	q5	12.05	12.05	12.05	12.05	12.04	12.04
b	q5	0.06	0.06	0.06	0.05	0.05	0.05
a	q6	29.50	29.50	29.49	29.48	29.48	29.47
b	q6	0.16	0.16	0.15	0.14	0.14	0.13

Note: a: %; b: % points difference with BAU

The propensity to save of each household group is also affected in an expected manner. The impact of the tax cut should be greater for higher income groups. That is, in fact, what happens. The lower income group, with a low savings propensity, does not register any significant change in its savings behaviour. The highest income group registers the highest increase.

9.4. Conclusions

The current paper set out to measure the impact of a corporate and personal income tax cut undertaken in the Azores, an autonomous region of Portugal. For this purpose a dynamic CGE model was used. The calibration of the model used a SAM matrix constructed with 2002 data with a considerable level of detail which was not fully reported in this exercise.

The main concern here was to analyse the impact of the measure on a few major economic indicators, particularly GDP.

As expected, the reduction in taxes with a corresponding reduction in government expenditures led to a reduction in GDP in the short run. This result is, however, inverted in the longer run.

Government expenditures are reduced for the full period while private expenditures are increased, when compared to the base scenario of no tax cut. In the end, the private sector tends to become relatively bigger and GDP to recover its growth path.

The impact of the policy benefits relatively more the lower income families that get a bigger increase in their wellbeing as measured by equivalent variation.